Propagation of nonlinear acoustic signals through inhomogeneous moving media

P. Blanc-Benon[1], V.A. Khokhlova[2], M.V. Averianov[2], L. Dallois[1], R.O. Cleveland[3]

[1] *Laboratoire de Mecanique des Fluides et d'Acoustique, U.M.R. C.N.R.S. 5509, Ecole Centrale de Lyon,B.P. 163, 69131 Ecully Cedex, France, Email: Philippe.Blanc-Benon@ec-lyon.fr*

[2] *Department of Acoustics, Faculty of Physics, M. V. Lomonosov Moscow State University, Leninskie gory, Moscow 119992, Russia, Email: vera@acs366.phys.msu.ru*

[3] *Department of Aerospace and Mechanical Engineering, Boston University, 110 Cummington Street, Boston, Massachusetts 02115, Email: robinc@bu.edu*

Introduction

Nonlinear propagation of intense acoustic waves through inhomogeneous medium is an important problem for many modern applications including sonic booms in a turbulent atmosphere, explosive waves in a fluctuating ocean, and intense ultrasound and shock waves in biological tissue. Two different types of inhomogeneities are of importance: scalar inhomogeneities (spatial distribution of sound speed and density), for example, due to temperature variations in the medium or variations in tissue type; and vector inhomogeneities (spatial distribution of particle velocity), for example, due to the presence of vortices or flow in the medium [1]. Sound scattering by vortices is still an open problem despite a lot of theoretical analyses and experimental studies. In all of these problems the combined effects of inhomogeneities, diffraction, and nonlinear propagation determine the peak and average characteristics of the acoustic field. A complete theoretical model that includes all the above mentioned phenomena is very complicated for analysis, thus most results to date have been obtained for simplified models. Nonlinear geometrical acoustics has been applied [2, 3], parabolic equations have been derived for linear sound propagation in inhomogeneous moving media [4] and for nonlinear waves in media with scalar inhomogeneities [5]. In this work, a nonlinear parabolic wave equation with inclusion of both scalar and vortex inhomogeneities is presented.

Theoretical model

Propagation of the plane wave along the x direction through vortex in linear approximation is governed by the modified KZ equation:

$$\frac{\partial}{\partial \tau}\left[\frac{\partial p}{\partial x} - \frac{\varepsilon}{c_0^3 \rho_0} p \frac{\partial p}{\partial \tau} - \frac{v_x}{c_0^2}\cdot\frac{\partial p}{\partial \tau} + \frac{1}{c_0}\left(v_y \frac{\partial p}{\partial y} + v_z \frac{\partial p}{\partial z}\right)\right] = \frac{c_0}{2}\Delta_\perp p , \quad (1)$$

where p is the acoustic pressure, x the principal propagation direction, c_0 the sound speed in non-perturbed medium, τ the retarded time, v_x the components of the vortex along the coordinate x, v_y and v_z – along the transverse directions y and z. The third term on the left side of Eq. (1) corresponds to the scalar type of inhomogeneities and the forth term corresponds to the vector type inhomogeneities.

In dimensionless coordinates and axially symmetric geometry the modified KZ equation can be represented as:

$$\frac{\partial}{\partial \theta}\left[\frac{\partial P}{\partial z} - NP\frac{\partial P}{\partial \theta} - 2\,\pi\delta\gamma V_z \frac{\partial P}{\partial \theta} + \gamma V_r \frac{\partial P}{\partial R}\right] = \frac{\gamma \Delta_\perp P}{4\pi\delta} \quad (2)$$

The coordinates are normalized by the following characteristic vortex length scales: Δ_r vortex size along radial direction, Δ_x vortex size along x direction. Dimensionless coordinates $R = r/\Delta_r$ and $z = x/\Delta_x$, $\theta = \omega\tau$ dimensionless time, $P = p/p_0$ acoustic pressure, $N = \Delta_x \varepsilon p_0 \omega_0 / c_0^3 \rho_0$ nonlinear parameter, $\delta = \Delta_r/\lambda$ is a ratio between Δ_r and acoustic wavelength, $\gamma = \Delta_x/\Delta_r$ is a ratio between length and thickness of the vortex. It is assumed that the dimensions of the vortex could be different.

Numerical results

We consider the propagation of an initially plane harmonic wave through a sequence of two axisymmetric vortices. The first vortex with both axial and radial velocity components introduces a disturbance of the acoustic field that scatters on the second vortex. In dimensionless coordinates the first axisymmetric vortex can be represented as [6]:

$$\begin{aligned} V_z &= A\frac{R_0}{R}(R-R_0)\sqrt{\gamma}\exp\left(-(z-z_0)^2-(R-R_0)^2\right) \\ V_R &= -A\frac{R_0}{R}(z-z_0)\frac{1}{\sqrt{\gamma}}\exp\left(-(z-z_0)^2-(R-R_0)^2\right) \end{aligned} \quad (3)$$

where A is the velocity amplitude normalized by the sound speed c_0. Here we scaled the amplitude of V_z и V_r so that $div(\vec{V}) = 0$. The center of the vortex is located at $z = z_0$, $R = R_0$.

The second vortex with a forced linearly growing core and a free exterior has only a radial velocity component given by:

$$\begin{aligned} V_R &= V_c\cdot\frac{R}{R_c}\exp\left(-\left(\frac{z-z_c}{b}\right)^4\right), \quad if\ R < R_c \\ V_R &= V_c\cdot\frac{R_c}{R}\exp\left(-\left(\frac{z-z_c}{b}\right)^4\right), \quad if\ R > R_c \end{aligned} \quad (4)$$

The parameters of the first vortex were $A = 0.03$, $R_0 = 4$, $z_0 = 3$, $\gamma = 1.0$, $\delta = 3.0$. The parameters of the second vortex were: $V_c = 0.3$, $R_c = 1.5$, $z_c = 30$, $b = 10$. Spatial distribution of radial (a) and axial (b) velocity components of the vortices is shown in Figure 1.

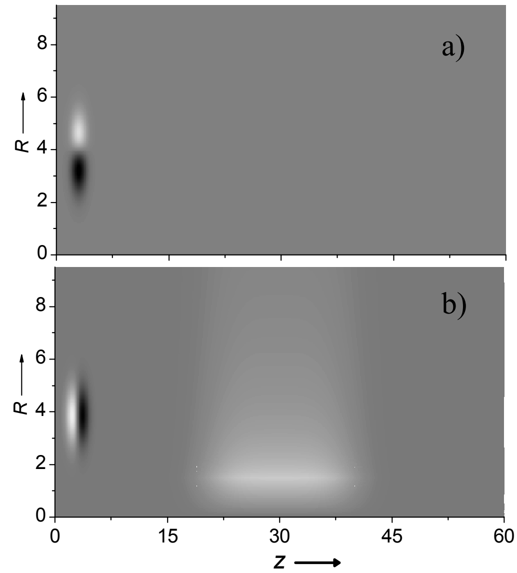

Figure 1: Geometry of inhomogeneities. Axial (a) and radial (b) components of mean velocity field.

In order to show the importance of the inclusion of the transverse component of inhomogeneity into the KZ model, the problem was considered first in the linear approximation

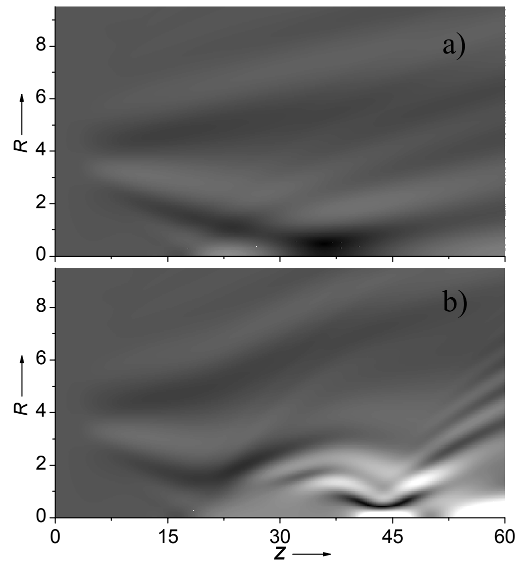

Figure 2: Acoustic pressure field calculated with account of only axial *z* component of inhomogeneity (a) and including both axial *z* and radial *R* components (b)

both with and without the transverse component V_R of the vortices. The results are presented in Figure 2 for the acoustic field of an initially plane linear wave transmitted through the sequence of these two vortices. Simulations were performed with (Fig. 2b) and without (Fig. 2a) account of the radial component of velocity inhomogeneities, that is neglecting the components shown in Fig 1b.

In both cases we can see scattering from the first turbule located at $z = 3$. The flow in the first turbule is directed towards the boundary $z = 0$ closer to the axis (dark area in Fig. 1a), that results in focusing effect, and opposite from the boundary further away from the axis (white area in Fig. 1a), that results in defocusing of the wave. Because of the homogeneous distribution of the acoustic pressure in a plane wave passing through this first turbule, no visible effect of radial flow velocity is seen until the appearance of the second turbule. Then the scattering is much stronger when the second turbule V_r ($z = 30$) is included in the simulation. This illustrates the importance of including the radial velocity component of vortex inhomogeneities in the prediction of the scattered field for further consideration of nonlinear problems

Acknowledgments

This work was partly supported by the NATO, ONRIFO, and CRDF grants.

References

[1] V.E. Ostashev. Acoustic in moving inhomogeneous media, Ed. E&Fn Spon, London, 1997

[2] O.V. Rudenko, A.K. Sukhorukova, A.P.Sukhorukov. Full solutions of equations of geometrical acoustics in stratified moving media. Acoustical Physics, 1996; **43**(3), pp. 339-343

[3] A.N. Dubrovskii, V.A. Khokhlova, and O.V. Rudenko. Fluctuation characteristics of a sonic boom after passing through randomly inhomogeneous layer. Acoustical Physics, 1996; **42**(5), pp. 550-554

[4] Ph. Blanc-Benon, B. Lipkens, L. Dallois, M.F. Hamilton, D.T. Blackstock. Propagation of finite amplitude sound through turbulence: modelling with geometrical acoustics and the parabolic approximation. J. Acoust. Soc. Amer. **111**(1), Pt. 2 (2002), 487-498

[5] L. Dallois, Ph. Blanc-Benon, D. Juvé. A wide angle parabolic equation for sound waves in inhomogeneous moving media: applications to atmospheric sound propagation. J. Comp. Acoustic **9**(2) (2001) 477-494

[6] C.Bogey, C Bailly. Three-dimensional non-reflective boundary conditions for acoustic simulations: far field simulation and validation test cases. Acta Acustica united with Acustica **88** (2002), 463-471