Some remarks on the uncertainties associated with the laboratory measurement of noise from waste water installations

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1 Introduction

Measurements of noise from waste water installations [1] focus on the determination of quantities describing the emission of airborne and structure-borne sound. In this paper, the GUM [2] is applied to assess the uncertainty of the quantity describing structure-borne sound.

2 Measurement principle

The measurement is carried out in a usual test suite according to ISO 140-1 (Figure 1) and the waste water system is mounted in the sending room on the wall between sending and receiving room.

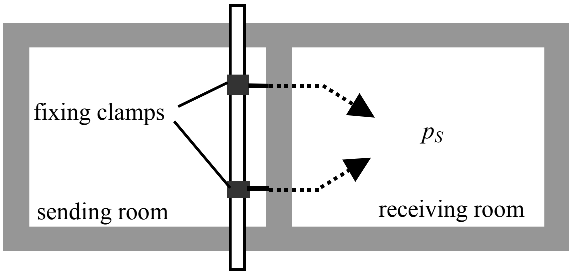

Figure 1 Laboratory measurement setup

The quantity directly measured is the volume-averaged sound pressure in the receiving room p_S, which is assumed to be transmitted as structure-borne sound. This sound pressure is normalised to a reference absorption A_0 = 10 m² with the absorption of the receiving room A

$$p_{\mathrm{sn}}^2 = p_{\mathrm{s}}^2 \frac{A}{A_0} \quad . \tag{1}$$

Since this value still depends on the properties of the mounting wall, the so-called structural sensitivity α serves to characterise the ability of the wall to receive, transmit and radiate structure-borne sound. This is done by an inverse method in which a reference sound source of constant airborne sound power W is placed in the receiving room and the vibration velocity v is measured at the fixing clamp (see [1]):

$$\alpha = \frac{\rho\, c}{16\pi} \frac{A\, v^2}{W} \tag{2}$$

where ρ and c are the density and the speed of sound, respectively. The so-called characteristic sound pressure p_{sc} is a measure of the ability of the waste water system to emit structure-borne sound into a receiving structure. It is

$$p_{\mathrm{sc}}^2 = \frac{16\pi\, \alpha_0}{\rho\, c\, A_0} p_{\mathrm{s}}^2 \frac{W}{v^2} \tag{3}$$

with the reference sensitivity α_0 as defined in [1].

3 Mathematical model for the measurement

According to [2], an uncertainty analysis requires a mathematical model of the measurement which will be derived in this chapter.

It is a prerequisite for the applicability of the method that the source acts as a force source. The excitation by such a source can be modeled using a two-terminal network (see e.g. [3]) consisting of a force source F_S, an internal impedance Z_{int} and an input impedance Z of the receiving structure (Figure 2).

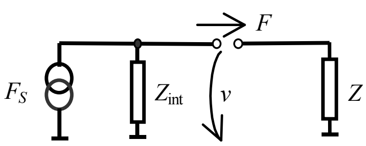

Figure 2 Excitation model

The input force F is

$$F = v\, Z \tag{4}$$

and the force source

$$F_S = v\left(Z_{\mathrm{int}} + Z\right) . \tag{5}$$

The aim is that the input force F is independent of the receiving structure which can be expressed by

$$F = F_S \Delta_F \tag{6}$$

with

$$\Delta_F = \frac{Z}{Z_{\mathrm{int}} + Z} . \tag{7}$$

A mandatory test is described in [1] to verify whether the waste water system really acts as a force source. The measurement of the vibration velocity at the clamps during excitation with the reference sound source is carried out twice, once with the clamp open and once with the clamp fixed. The difference in the velocity levels must be smaller than 3 dB. Since this difference is due to the impedance ratio, Δ_F is restricted to values between 0.71 and 1 by this test.

$$\Delta_F = 0.71\ldots 1 \tag{8}$$

The sound is transmitted from the excitation point to the surface of the mounting wall in the receiving room. This can be expressed by

$$\overline{v_{r,1}} = v_1\, H_1 \tag{9}$$

with the transfer function H_1 and the time- and surface-averaged vibration velocity $\overline{v_{r,1}}$. Usually, two clamps are used which leads to a vibration velocity on the surface of the mounting wall in the receiving room

$$v_r^2 = \left(v_1\, H_1\right)^2 + \left(v_2\, H_2\right)^2 \tag{10}$$

the cross correlation between the two forces at the excitation points being assumed to be negligible. The radiated sound power then is

$$W = \rho\, c\, S\, v_r^2\, \sigma \tag{11}$$

with radiation efficiency σ and surface area S. This sound power is measured by the diffuse-field method

$$W = \frac{p_s^2}{\rho\, c} \frac{A}{4} . \tag{12}$$

Combining Eqs. (4) to (12) yields

$$p_s^2 = \left(\rho\, c\right)^2 \sigma \frac{4\,S}{A} \left[\left(H_1 \frac{F_1}{Z_1} \right)^2 + \left(H_2 \frac{F_2}{Z_2} \right)^2 \right] \tag{13}$$

and normalisation with respect to absorption leads to

$$p_{\mathrm{sn}}^2 = \left(\rho\, c\right)^2 \sigma \frac{4\,S}{A_0} \left[\left(H_1 \frac{F_1}{Z_1} \right)^2 + \left(H_2 \frac{F_2}{Z_2} \right)^2 \right] . \tag{14}$$

For normalisation with respect to the structural sensitivity, the sound pressure caused by only one clamp is considered, which is

$$p_{\mathrm{sn},1}^2 = \left(\rho\, c\right)^2 \sigma \frac{4\,S}{A_0} \left(H_1 \frac{F_1}{Z_1} \right)^2 . \tag{15}$$

Eq. B.1 from Annex B of [1] is

$$W_1 = \frac{1}{\rho\, c} k^2\, F_1^2\, \alpha_1 \tag{16}$$

(k is the wave number) which can be combined with Eq. (15) to obtain

$$p_{\mathrm{sn},1}^2 = k^2\, F_1^2 \alpha_1 \frac{4}{A_0} \tag{17}$$

and finally

$$\alpha_1 = \left(\frac{\rho\, c}{k} \right)^2 S \sigma \left(\frac{H_1}{Z_1} \right)^2 . \tag{18}$$

This means that the structural sensitivity is mainly a function of the acoustic properties of the mounting wall, namely surface area S, radiation efficiency σ, transfer function H and input impedance Z. Combining Eqs. (14) and (17) yields

$$p_{\mathrm{sn}}^2 = k^2 \frac{4}{A_0}\left(F_1^2\alpha_1 + F_2^2\alpha_2 \right). \tag{19}$$

This equation is now normalised with respect to the structural sensitivity, which is done using the mean value of the two structural sensitivities determined for the two clamps according to Eq. (2)

$$p_{\mathrm{sc}}^2 = k^2 \frac{8}{A_0}\left(F_1^2\alpha_1 + F_2^2\alpha_2 \right)\frac{\alpha_0}{\alpha_1+\alpha_2}. \tag{20}$$

Eq. (20) can be transformed into

$$p_{\mathrm{sc}}^2 = k^2 \frac{4}{A_0}\left(F_1^2 + F_2^2 \right)\alpha_0\, \Delta_\alpha . \tag{21}$$

The first part of Eq. (21) is the ideal measurement result which is free from any influence of the structural sensitivity, whereas the term

$$\Delta_\alpha = \frac{2\left[\left(F_1/F_2\right)^2 \alpha_1/\alpha_2 + 1\right]}{\left(1+\alpha_1/\alpha_2\right)\left[1+\left(F_1/F_2\right)^2\right]};\quad \delta_\alpha = 10\,\lg\Delta_\alpha\ \mathrm{dB} \tag{22}$$

assumes the value of 1 (or 0 dB) only if either the forces F_1 and F_2 or the structural sensitivities α_1 and α_2 are equal (Figure 3). Otherwise, errors of some dB can occur.

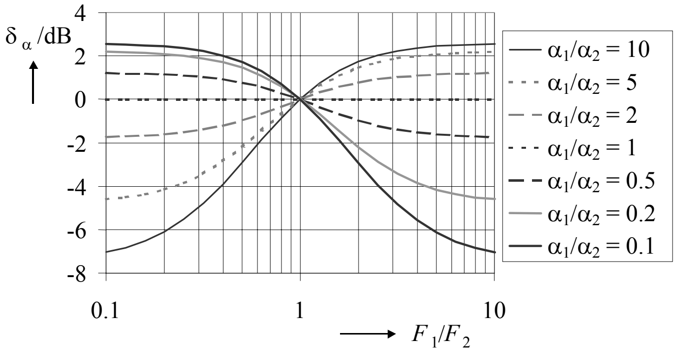

Figure 3 Sensitivity normalisation error

Introducing Eq. (6) into Eq. (21) yields

$$p_{\mathrm{sc}}^2 = k^2 \frac{4}{A_0}\left(F_{S1}^2\Delta_{F1}^2 + F_{S2}^2\Delta_{F2}^2 \right)\alpha_0\, \Delta_\alpha \tag{23}$$

which can be rearranged to

$$p_{\mathrm{sc}}^2 = k^2 \frac{4}{A_0}\left(F_{S1}^2 + F_{S2}^2 \right)\alpha_0\, \Delta_\alpha \Delta_S \tag{24}$$

with

$$\Delta_S = \frac{\left(F_{S1}/F_{S2}\right)^2 \Delta_{F1}^2 + \Delta_{F2}^2}{\left(F_{S1}/F_{S2}\right)^2 + 1};\quad \delta_S = 10\,\lg\Delta_S\ \mathrm{dB}. \tag{25}$$

So the model equation is (see Eq. (3)):

$$p_{\mathrm{sc}}^2 = \frac{16\pi\,\alpha_0}{\rho\, c\, A_0}\, p_{\mathrm{s}}^2 \frac{W}{0.5\left(v_1^2+v_2^2\right)}\Delta_\alpha\Delta_S \tag{26}$$

or in levels

$$L_{\mathrm{sc}} = L_s + L_{\mathrm{W}} - 10\,\lg\left[\frac{1}{2}\left(10^{0{,}1 L_{v,1}/\mathrm{dB}} + 10^{0{,}1 L_{v,2}/\mathrm{dB}}\right)\right]\mathrm{dB}$$
$$+ L_{\mathrm{SSR}} + \delta_\alpha + \delta_S - 10\,\lg\left(\frac{A_0\, v_0^2}{16\pi}\frac{\rho\, c}{W_0}\right)\mathrm{dB} \tag{27}$$

with

$$L_{\mathrm{SSR}} = 10\,\lg\alpha_0\ \mathrm{dB}. \tag{28}$$

4 Calculation of the uncertainty

When a measured value y is determined by n input quantities x_i, the combined uncertainty $u(y)$ can be calculated by (see [2])

$$u\left(y\right) = \sqrt{\sum_{i=1}^{n} c_i^2\, u^2\left(x_i\right)} \tag{29}$$

where the sensitivity coefficients are

$$c_i = \frac{\partial f}{\partial x_i} \tag{30}$$

and the function f describes how quantity y is determined from the input quantities. Application of (29) and (30) to (27) yields the uncertainty budget shown in Table 1 where the reasonable assumption

$$u^2\left(L_{v1}\right) = u^2\left(L_{v2}\right) = u^2\left(L_v\right) \tag{31}$$

has been made. Stated uncertainties are valid for third-octave values in the central frequency range from approximately 250 Hz to 2 kHz. Outside this range, the uncertainties will be larger. The uncertainty of the sound power level of the reference sound source L_W is taken from [4], whereas the uncertainty of the vibration velocity L_v is estimated from experience. The uncertainty of sound pressure level L_S is derived from internal data from sets of measurements for which approximately 15 different teams used different measurement equipment and averaging procedures to determine a nominally constant volume-averaged sound pressure level in a usual receiving room. The uncertainty of δ_α is calculated assuming a rectangular distribution between –2 and 2 dB (see Figure 3) thus excluding large differences F_1-F_2 and α_1-α_2 and, finally, the uncertainty of δ_S is calculated on the assumption of rectangular distribution between –3 and 0 dB which is based on Eqs. (25) and (8). Addition of all the contributions leads to a combined uncertainty of 2.0 dB for the measurement result L_{SC}.

The uncertainty index

$$I_i = \frac{\left(c_i\, u_i\right)^2}{u^2\left(L_{SC}\right)} \tag{32}$$

also shown in Table 1 describes the percentage an uncertainty component contributes to the combined uncertainty. The largest contribution is the sensitivity normalisation but most of the other components are only slightly smaller. Only the contribution from the sound power level L_W can be considered to be of no relevance.

Table 1 Uncertainty budget

quantity	distribution	u_i / dB	c_i	$c_i\, u_i$ / dB	I_i / %
L_S	Gaussian	1.0	1	1.0	24
L_W	Gaussian	0.3	1	0.3	2
L_v	Gaussian	1.0	1	1.0	24
δ_α	rectangular	1.2	1	1.2	31
δ_S	rectangular	0.9	1	0.9	18

5 Conclusion and future work

The uncertainty of the characteristic quantity L_{SC} turns out to be in the order of 2 dB. The most important uncertainty contribution is the sensitivity normalisation. This could be eliminated by twice measuring the sound pressure level, once with the upper and once with the lower clamp fixed. Both sound pressure levels could then separately be normalised and energetically added. This change would reduce the uncertainty of the characteristic quantity L_{SC} to about 1.7 dB.

Future work will encompass experimental investigations, e.g. on the scatter of the structural sensitivity of usual mounting walls and on the uncertainty of measured sound pressure and vibration velocity levels.

Acknowledgements

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Literature

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