Elaboration of a scattering matrix measurement procedure for higher order acoustic duct modes propagation conditions.

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Introduction

Fluid-machines are normally coupled to a duct or pipe system which often can be described as a network of duct elements. When linear theory is valid, the acoustical two-port theory [1] leads to an efficient way to analyse sound transmission. The concept of scattering-matrix was shown to be the basic description of a wave interaction problem [2] and a measurement method for characterisation of the 4 coefficients of the matrix for a plane wave acoustic propagation condition was developed [3]. In this paper progress in the development of a method for measuring the scattering matrix of duct discontinuities for higher order acoustic modes propagation condition is presented. It is based on previous works [4] on measurement of reflection and transmission matrices.

Theoretical basis of the experiment

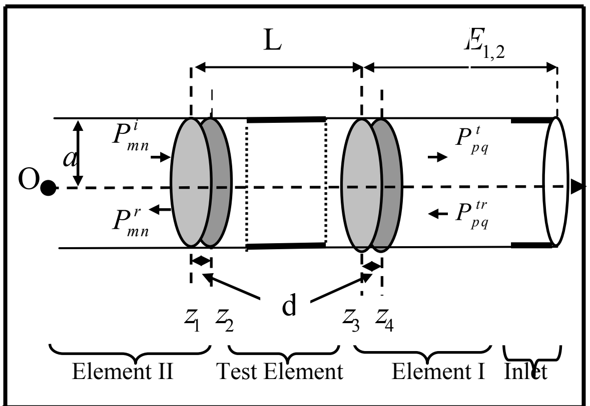

Figure 1. Duct configuration diagram.
$L = z_3 - z_1 = 100cm, \quad d = 5.12cm$
$a = 7.5cm, E_1 = 21cm, E_2 = 26cm.$

Assume a discontinuity located between two circular elements of a cylindrical duct of radius a with hard wall (figure 1). In no flow conditions, the fluid being assumed to be ideal and linear acoustic theory valid the acoustic pressure distribution in the duct is written in cylindrical coordinates[1]:

$$P(r,\theta,z) = \sum_{m=-\infty}^{+\infty}\sum_{n=1}^{+\infty} p_{mn}(z)\Psi_{mn}(r,\theta)e^{-i\omega t} \qquad (1)$$

$\Psi_{mn}(r,\theta)$ are the eigenfuctions associated with mode (m,n), and $P_{mn}(z)$ the total modal coefficients in cross section located at axial coordinate z.. In the elements II and I respectivly in $z = z_1$ and in $z = z_3$:

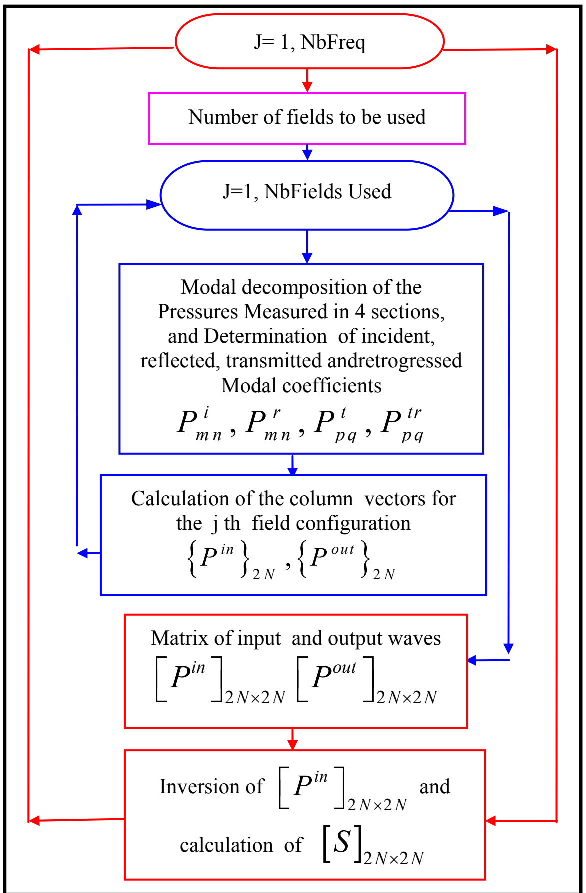

Figure 2. Flow chart of data processing

$$P_{mn}(z_1) = P^i_{mn}(z_1) + P^r_{mn}(z_1) \qquad (2)$$

$$P_{pq}(z_3) = P^t_{pq}(z_3) + P^{tr}_{pq}(z_3) \qquad (3)$$

where $P^i_{mn}, P^r_{mn}, P^t_{pq}$ *and* P^{tr}_{pq} are respectively incident, reflected, transmitted and retrogressed modal coefficients, which are deduced after a Fourier Lommel's transform of pressures measured in two closed cross sections which separates incident and reflected waves [4]. N being the number of cut-on modes in the frequency band, if the duct can be regarded as a linear time invariant and passive system in the positive direction, a linear relationship between waves incoming the test element $\left\{P^{in}\right\}_{2N} = \left\{P^{i}_{pq}, P^{tr}_{pq}\right\}^{t}$ and output waves $\left\{P^{out}\right\}_{2N} = \left\{P^{r}_{mn}, P^{t}_{mn}\right\}^{t}$ exists:

$$\begin{Bmatrix} P^{r}_{mn} \\ P^{t}_{mn} \end{Bmatrix}_{2N} = \begin{bmatrix} \left[S^{11}\right]_{NxN} & \left[S^{12}\right]_{NxN} \\ \left[S^{21}\right]_{NxN} & \left[S^{22}\right]_{NxN} \end{bmatrix}_{2N\times 2N} \times \begin{Bmatrix} P^{i}_{pq} \\ P^{tr}_{pq} \end{Bmatrix}_{2N} \quad (4)$$

where the $(2N)^2$ coefficients $S^{kl}_{mn,pq}$ k;l=1,2 are the scattering matrix coefficients which characterise the discontinuity. These $(2N)^2$ coefficients are solution of the following system:

$$\left[S^{kl}_{mn,pq}\right]_{2N\times 2N} = \left[P^{out}\right]_{2N\times 2N} \times \left[P^{in}\right]^{-1}_{2N\times 2N} \quad (5)$$

The 2N columns of $\left[P^{out}\right]_{2N\times 2N}$ and $\left[P^{in}\right]_{2N\times 2N}$ are filled with terms issued from the measurement of 2N linearly independent pressure fields. To solve equation (5), the experiment was carried out with N various configurations [4] sources for two inlet charges. Calculation of matrix $\left[S\right]_{2Nx2N}$ is performed with a "selective" method [1] for a total wave number ka variation from 0 to 3 and a maximum N=3 modes [(0,0);(+1,0); (-1,0)].

Hardware and various steeps of experiment are described in detail in [4].

Results

To verify the validity of the method a test case with a uniform straight duct was used. The theoretical scattering matrix is given by:

$$\left[S\right]_{6x6} = \begin{bmatrix} \left[0\right]_{3x3} & \left[diag(e^{jk_{mn}L})\right]_{3x3} \\ \left[diag(e^{jk_{mn}L})\right]_{3x3} & \left[0\right]_{3x3} \end{bmatrix}_{6x6} \quad (6)$$

The results of $\left|S^{11}_{00,00}\right|$, $\left|S^{12}_{00,00}\right|$, $\left|S^{12}_{10,10}\right|$, $\left|S^{11}_{00,10}\right|$ and $\left|S^{21}_{-10,-10}\right|$ are presented respectively in figures 3 and 4.

$S^{11}_{00,00}$ gives the relation between P^{i}_{00} and P^{r}_{00}.

$S^{12}_{00,00}$ gives the relation between P^{r}_{00} and P^{tr}_{00}. As there is no discontinuity, no reflection through the element tested, measured values are closed to the theoretical ones: $\left|S^{11}_{00,00}\right| = 0$ and $\left|S^{12}_{00,00}\right| = 1$ except near the cut-off frequencies of modes (±1,0).
$\left|S^{12}_{00,00}\right|$ is decreasing from 1 to 0.75 , and $\left|S^{11}_{00,00}\right|$ is increasing to 0.25. This discrepancy is due to the measurement problem in section II caused by stationary waves, and by the distance d between microphones [4]. $S^{1,2}_{10,10}$ gives the relation between P^{r}_{+10} and P^{tr}_{+10}. $S^{2,1}_{-10,-10}$ gives the relation between P^{t}_{-10} and P^{i}_{-10}. $\left|S^{1,2}_{10,10}\right|$ and $\left|S^{2,1}_{-10,-10}\right|$ are converging to 1 when the frequency increases away from cut-off frequency. $S^{1,1}_{00,10}$ represents the conversion of P^{i}_{+10} into P^{r}_{00}, except near cut-off frequencies of mode (±1,0). Indeed, its modulus is lower than 0.1, the assumption on the axi-symetry is verified. Also except near the cut-off frequencies, phases of $S^{12}_{00,00}$, $S^{2,1}_{-10,-10}$ and $S^{1,1}_{00,10}$ agree with the theory.

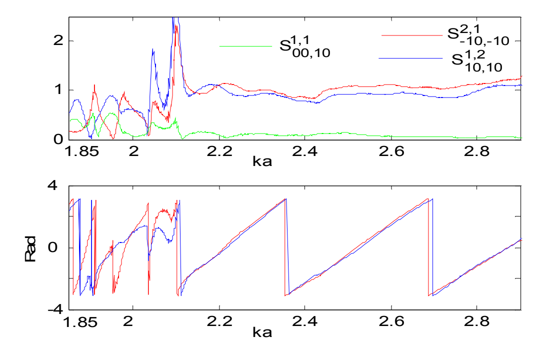

Figure 3. $S^{1,1}_{00,10}$, $S^{12}_{10,10}$ and $S^{21}_{-10,-10}$ versus ka.

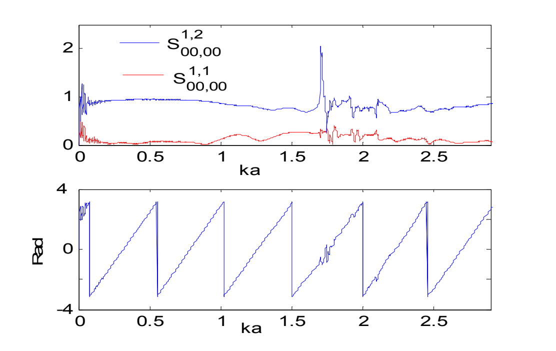

Figure 4. $S^{11}_{00,00}$ and $S^{12}_{00,00}$ versus ka.

Conclusions

In this work a method for measuring the scattering-matrix in higher order modes propagation conditions was presented and tested with a uniform straight duct. Except near the cut-off frequencies experimental results agree with the theory. More complicated configurations and association of discontinuities will be tested during future works.

References

[1] A.F Pierce Acoustics: An introduction to its physical principles and applications. NY Mc Graw-Hill, 1981.

[2] P.O.A.L Davies, Practical flow duct acoustics. Journal of Sound and Vibration 124 (1988), 91-115.

[3] M.Åbom, Measurement of the scattering-matrix of acoustical two ports. Mechanical Systems and Signal Processing 5(2) (1991), 89-104.

[4] A.Sitel, J.M.Ville, F.Foucart: An experimental facility of acoustic transmission matrix and acoustic power dissipation of duct discontinuity in higher order modes propagation conditions. Acta acustica united with acustica Vol.89 (2003) 586-594.