Causality Constraints in Multi-Channel Active Control of Random Noise

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Introduction

The constraint of *causality* restrains the performances of systems for active control of largely non-predictible (i.e. random and broadband) noise. This causality constraint is easy to grasp and to check in the case of 1D acoustic propagation with one control loudspeaker: for perfect cancellation the incoming noise must be detected *before* it reaches the place where it has to be cancelled, with a time advance larger than the overall time lag in the control loop; this lag includes the time required for the computation and release of the loudspeaker input signal plus the propagation time between the loudspeaker and the minimization area (cf. [1]). The causality constraint is more difficult to interpret in real 3D cases with several control loudspeakers and many minimization locations; it seems that no theoretical or practical rule guarantees the causality of perfect control for a given arrangement of actuators and sensors. Indeed in some practical cases the time advance with which the incoming noise must be detected appears to be much larger than any propagation delay between the control devices (see [2]).

In this paper two distinct mechanisms are shown as imposing noise detection with a very large time advance for perfect active cancellation of random noise. Firstly, in the single-channel case of one minimization microphone and one control loudspeaker, perfect noise cancellation can require detection of the incoming noise with an infinitely large time advance simply *because of acoustic reflections*. Secondly it is shown in the multi-channel case that infinite non-causality can occur even in free-field *because of the algebraic inversion* of the matrix of secondary paths which is required for optimal control.

Causality constraint in the single-channel case

Figure 1 displays a typical feedforward arrangement for Active Noise Control in a rigid duct. Below the first cut-off frequency the transfer function for noise propagation from the detection sensor to the error sensor is ideally $F(\omega) = e^{-j\omega L/c}$ where c is the speed of sound; the secondary transfer function from the loudspeaker to the error sensor is $H(\omega) = e^{-j\omega l/c}$. Therefore, in order to cancel noise at the error sensor, the anti-noise that the loudspeaker has to generate is $-FH^{-1}x = -e^{-j\omega(L-l)/c}x$, which is a causal filtering of signal x detecting the incoming noise if the reference sensor is located *upstream* the loudspeaker (i.e. $L \geq l$).

In order to show that, in presence of noise reflections, the causality constraint can impose detection of the incoming noise with a much larger advance, suppose the response from the loudspeaker is made of a direct sound plus a reflected sound *louder* than the direct one:

$$H = e^{-j\omega\tau_1} + \alpha e^{-j\omega\tau_2} \quad \text{with} \quad \tau_2 > \tau_1 \quad \text{and} \quad \alpha > 1 \qquad (1)$$

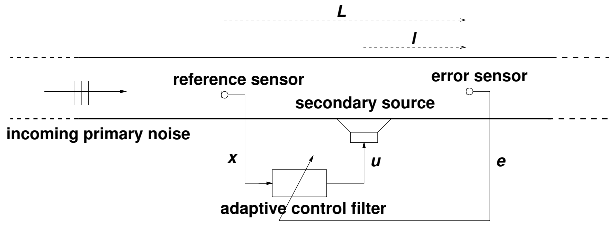

Figure 1: A typical feedforward Active Noise Control set-up.

In this case the inverse of the secondary transfer function is:

$$H^{-1} = \frac{1}{e^{-j\omega\tau_1}+\alpha e^{-j\omega\tau_2}} = \frac{\frac{1}{\alpha}e^{j\omega\tau_2}}{1+\frac{1}{\alpha}e^{-j\omega(\tau_1-\tau_2)}}$$
$$= \frac{1}{\alpha}e^{j\omega\tau_2}\left[1 - \frac{1}{\alpha}e^{j\omega(\tau_2-\tau_1)} + \frac{1}{\alpha^2}e^{2j\omega(\tau_2-\tau_1)} - \dots\right] \qquad (2)$$

which, as an infinite sum of time advances, has an infinite non-causal impulse response. For causal active noise cancellation, the incoming noise must therefore be detected with an infinite time advance (i.e. transfer function F including an infinite time lag) to compensate in the product $-FH^{-1}$ for the infinite non-causal response in H^{-1}.

No simple geometrical arrangement will give only one echo louder than the direct sound but figure 2 shows a possible set-up with two echoes. With rigid walls and notations from figure 2, the Green function accounting for the propagation from the point secondary source to the error sensor microphone can be computed by taking image sources into account:

$$H(\omega) = \frac{e^{-j\omega L_1/c}}{4\pi L_1} + 2\frac{e^{-j\omega L_2/c}}{4\pi L_2} + \frac{e^{-j\omega L_3/c}}{4\pi L_3}$$
$$\text{with} \quad L_1 = L, \;\; L_2^2 = (L_1+l)^2 + l^2, \;\; L_3 = L_1 + 2l \qquad (3)$$

and the first echo can have a larger level than the direct sound. With two echoes no series expansion easily shows as in equation 2 that the inverse response H^{-1} is infinitely non-causal but the impulse response can be computed numerically; figure 3 displays the impulse response when $L = 50cm$, $l = 25cm$, $c = 340m/s$ and with a sampling frequency of $32768Hz$ for computation of the response by Inverse Fast Fourier Transform. The non-causal part of the response appears to be much longer than the propagation time between the loudspeaker and the microphone, which illustrates the dramatic effect of noise reflections upon the causality requirements.

Causality constraint in the multi-channel case

Figure 4 shows an Active Noise Control device in free-field with two secondary sources and two minimization microphones. The incoming primary noise is a plane wave; the loudspeakers are idealized as point sources with volume velocity q_1 and q_2. Let ρ_0 denote the air density, x the primary

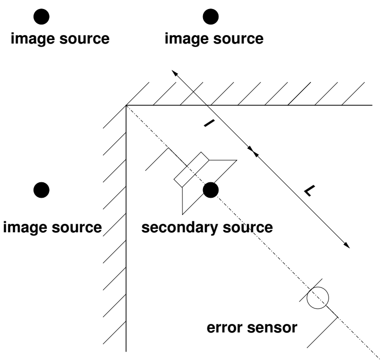

Figure 2: An Active Noise Control set-up with echoes.

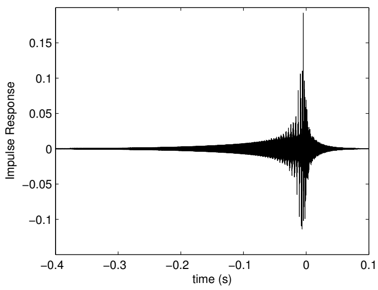

Figure 3: Inverse response of the acoustic path with echoes.

noise at microphone p_1 and W_1, W_2 linear control filters with input x and output q_1, q_2 (the dynamics of the loudspeakers is included in W_1 and W_2). The purpose of this section is now to show that W_1 and W_2 can be *infinitely non-causal*. With the notations of figure 4 and $k = \omega/c$ the noise at the minimization microphones can be written as:

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix}_{total} = \begin{bmatrix} 1 \\ e^{-jkd_5\cos\theta} \end{bmatrix} x + \frac{\rho_0 j\omega}{4\pi} \begin{bmatrix} \frac{e^{-jkd_1}}{d_1} & \frac{e^{-jkd_2}}{d_2} \\ \frac{e^{-jkd_3}}{d_3} & \frac{e^{-jkd_4}}{d_4} \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} x \qquad (4)$$

This total noise can be cancelled at both error sensors and at all frequencies if $d_1 d_4 \neq d_2 d_3$ and the optimum control filters are then given by:

$$\begin{bmatrix} W_1 \\ W_2 \end{bmatrix}_{optimal} = -\frac{4\pi}{\rho_0 j\omega} \frac{d_1 d_2 d_3 d_4}{d_2 d_3 e^{-jk(d1+d4)} - d_1 d_4 e^{-jk(d2+d3)}} \begin{bmatrix} \frac{e^{-jkd_4}}{d_4} & -\frac{e^{-jkd_2}}{d_2} \\ -\frac{e^{-jkd_3}}{d_3} & \frac{e^{-jkd_1}}{d_1} \end{bmatrix} \begin{bmatrix} 1 \\ e^{-jkd_5\cos\theta} \end{bmatrix} \qquad (5)$$

It can be assumed, without loss of generality because the indexes can be switched, that $d_1 d_4 < d_2 d_3$. After indroducing $\alpha = d_1 d_4 / d_2 d_3$ and $D = d2 + d3 - d1 - d4$, equation 5 can be re-written and expanded as:

$$\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = -\frac{4 d_1 d_4 \pi j\omega}{\rho_0} \left(1 + \alpha e^{-jkD} - \alpha^2 e^{-j2kD} + \ldots\right) \begin{bmatrix} \frac{e^{jkd_1}}{d_4} & -\frac{e^{jk(d_1+d_4-d_2)}}{d_2} \\ -\frac{e^{jk(d_1+d_4-d_3)}}{d_3} & \frac{e^{jkd_4}}{d_1} \end{bmatrix} \begin{bmatrix} 1 \\ e^{-jkd_5\cos\theta} \end{bmatrix} \qquad (6)$$

This expansion shows that the optimum control filters will have *an infinite non-causal impulse response* if $D < 0$ or, in the general case with possibly $d_1 d_4 > d_2 d_3$, if:

$$(d_1 d_4 - d_2 d_3)(d_1 + d_4 - d_2 - d_3) < 0 \qquad (7)$$

This can occur indeed, as an illustration figure 5 shows the impulse responses computed by IFFT at 12kHz of the optimal control for an trapezoidal arrangement with $d_1 = 2.8m$, $d_4 = 8.8m$, $d_2 = d_3 = 5m$ and a plane wave with incidence angle $\theta = 0^o$. The control filters have a long non-causal response which implies that noise cancellation is possible only if the incoming noise is detected far in advance.

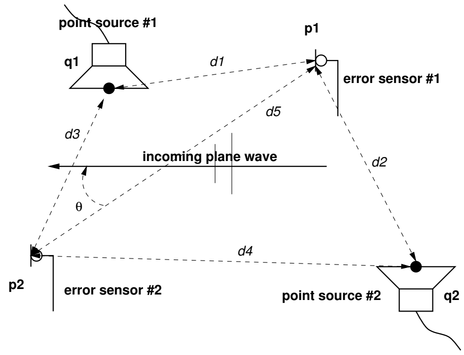

Figure 4: A 2-2 Active Noise Control device in free-field.

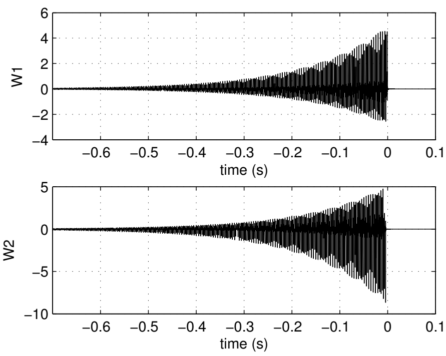

Figure 5: Time response of the optimal filters for a pathological 2-2 case in free-field.

However in the example of figure 5 the matrix of secondary paths is badly conditionned at all frequencies, which explains the slow decay of the responses when time tends towards $-\infty$. This suggests that the infinite non-causality which can happen in theory may still have limited effects in practice if the matrix of secondary paths is well conditionned, which is usually ensured to achieve a good convergence of the adaptative control algorithms such as the multi-channel Filtered-X Least Mean Squares.

References

[1] Active Noise Control. Academic Press, London, 1992

[2] Contrôle actif multivoies de bruits aléatoires large bande dans un grand volume : limitations physiques. Congrès Français d'Acoustique, Lille, on CD-ROM, 2002