Fast BEM – FEM Coupling for the Simulation of Acoustic-Structure Interaction

Matthias Fischer[1], Lothar Gaul[2]

[1] *Institut A für Mechanik, Universität Stuttgart, D-70550 Stuttgart, Germany, Email: fischer@mecha.uni-stuttgart.de*

[2] *Institut A für Mechanik, Universität Stuttgart, D-70550 Stuttgart, Germany, Email: gaul@mecha.uni-stuttgart.de*

Introduction

BEM – FEM coupling is widely used for the simulation of the interaction between structural vibrations and radiated acoustic fields. Employing the FEM for the structure and the BEM for the acoustic field exploits the specific advantages of the two methods. However, the efficiency of BEM – FEM simulations suffers from the fully populated BEM matrices. In recent years, fast algorithms have been developed that allow a sparse representation of the BEM systems. In the presented paper, a coupling scheme is developed that accounts for the properties of the fast multipole BEM. The coupling algorithm is based on Lagrange multipliers and provides high flexibility in the choice of discretizations. For the solution of the resulting saddle point problem, an approximate Uzawa algorithm is employed.

Acoustic-Structure Interaction

A structure fully submerged in an acoustic fluid as displayed in Figure 1 is modeled as a thin Kirchhoff plate on the interaction boundary Γ^{int}. The out-of-plane displacement is denoted by w, the loading $f = f^0 + f^{\mathrm{e}}$ consists of surface forces due to the acoustic field f^0 and externally applied forces f^{e}. The time-harmonic pressure p in the acoustic field Ω_{f} is governed by the Helmholtz equation $\triangle p + \kappa^2 p = 0$ with the circular wavenumber $\kappa = \omega/c_0$. The acoustic flux on the boundary is defined as $q = \partial p/\partial \vec{n}_{\mathrm{f}}$. For simplicity of presentation, the boundary $\partial\Omega_{\mathrm{f}} = \Gamma = \Gamma^{\mathrm{int}} \cup \Gamma^{\mathrm{N}}$ is composed of acoustic-structure interface and Neumann boundary. Dirichlet boundary conditions or computations on exterior domains can be implemented without difficulties. On the acoustic-structure interface Γ^{int} the coupling conditions enforce equilibrium $p = f^0$ and continuity $q = -\rho_0\omega^2 w$.

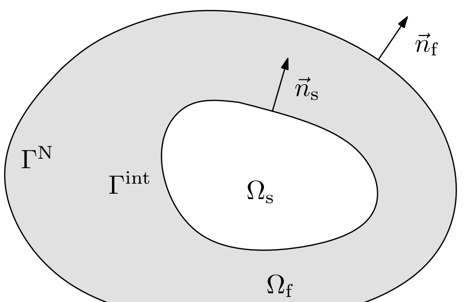

Figure 1: Structure and acoustic domain.

Mortar Coupling

A mortar algorithm is employed for the BEM–FEM coupling that allows a non-conforming discretization of the sub-domains. The BEM mesh is chosen as mortar side and the interface pressure is interpolated as Lagrange multiplier $\lambda = p = f^0$.

The FEM for the Kirchhoff plate is derived from the variational formulation

$$a(\nabla w, \nabla v^w) - \int_{\Gamma^{\mathrm{int}}} v^w \lambda \, \mathrm{d}s_x = \int_{\Gamma^{\mathrm{int}}} v^w f^{\mathrm{e}} \, \mathrm{d}s_x \,. \tag{1}$$

A detailed introduction to the BEM can be found in [1]. For the coupling algorithm, the method is derived from the boundary integral equation

$$p(x) = \frac{1}{2}p(x) + \underbrace{\int_{\Gamma} P^*(x,y)\, q(y)\, \mathrm{d}s_y}_{(Vq)(x)} - \underbrace{\int_{\Gamma} \frac{\partial P^*(x,y)}{\partial n_y}\, p(y)\, \mathrm{d}s_y}_{(Kp)(x)}\,, \quad x \in \Gamma\,, \tag{2}$$

and the hyper-singular boundary integral equation

$$q(x) = \frac{1}{2}q(x) + \underbrace{\int_{\Gamma} \frac{\partial P^*(x,y)}{\partial n_x}\, q(y)\, \mathrm{d}s_y}_{(K'q)(x)} - \underbrace{\int_{\Gamma} \frac{\partial^2 P^*(x,y)}{\partial n_x \partial n_y}\, p(y)\, \mathrm{d}s_y}_{-(Dp)(x)}\,, \quad x \in \Gamma\,. \tag{3}$$

The single layer potential $(Vq)(x)$, double layer potential $(Kp)(x)$, adjoint double layer potential $(K'q)(x)$ and the hyper-singular operator $(Dp)(x)$ are the well-known boundary integral operators with the fundamental solution $P^*(x,y) = \mathrm{e}^{\mathrm{i}\kappa|x-y|}/(4\pi|x-y|)$ defining the integration kernels.

The pressure and flux fields on the boundary are decomposed as $p = p^{\mathrm{int}} + \tilde{p}$ and $q = q^{\mathrm{int}} + \bar{q}$, where $\bar{q}$ are the prescribed Neumann boundary conditions and $\tilde{p} = \bar{q} = 0$ on Γ^{int}.

Using Equation (2) tested with v^q on Γ^{int} and Equation (3) tested with v^p on the entire boundary Γ, one obtains the system

$$\int_{\Gamma^{\mathrm{int}}} v^q (V q^{\mathrm{int}})(x)\,\mathrm{d}s_x - \int_{\Gamma^{\mathrm{int}}} v^q (K\tilde{p})(x)\,\mathrm{d}s_x + \int_{\Gamma^{\mathrm{int}}} v^q \left[-\frac{1}{2}p^{\mathrm{int}}(x) - (Kp^{\mathrm{int}})(x)\right]\,\mathrm{d}s_x + \int_{\Gamma^{\mathrm{int}}} v^q \left[p^{\mathrm{int}}(x) - \lambda(x)\right]\,\mathrm{d}s_x = -\int_{\Gamma^{\mathrm{int}}} v^q (V\bar{q})(x)\,\mathrm{d}s_x\,, \quad (4)$$

$$\int_{\Gamma} v^p (Dp^{\mathrm{int}})(x)\,\mathrm{d}s_x + \int_{\Gamma} v^p (D\tilde{p})(x)\,\mathrm{d}s_x + \int_{\Gamma} v^p \left[-\frac{1}{2}q^{\mathrm{int}}(x) + (K'q^{\mathrm{int}})(x)\right]\,\mathrm{d}s_x = \int_{\Gamma} v^p \left[\frac{1}{2}\bar{q}(x) - (K'\bar{q})(x)\right]\,\mathrm{d}s_x\,. \quad (5)$$

The term $p^{\mathrm{int}}(x) - \lambda(x)$ in Equation (4) was introduced to enforce equilibrium on the interface. Continuity is enforced by

$$\int_{\Gamma^{\mathrm{int}}} v^\lambda \left(\rho_0\omega^2 w + q^{\mathrm{int}}\right)\,\mathrm{d}s_x = 0\,. \quad (6)$$

Discretizing plate equation (1), fluid equations (4), (5) and continuity equation (6), one obtains the system of equations

$$\begin{pmatrix} \rho_0\omega^2\mathbf{A} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{C}_{\mathrm{F}} \\ \mathbf{0} & \mathbf{V} & \frac{\mathbf{1}}{\mathbf{2}} - \mathbf{K}_{\mathrm{int,int}} & -\mathbf{K}_{\mathrm{int,N}} & -\mathbf{C}_{\mathrm{B}} \\ \mathbf{0} & -\frac{\mathbf{1}}{\mathbf{2}} + \mathbf{K}^{\mathrm{T}}_{\mathrm{int,int}} & \mathbf{D}_{\mathrm{int,int}} & \mathbf{D}_{\mathrm{int,N}} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{\mathrm{N,int}} & \mathbf{D}_{\mathrm{N,int}} & \mathbf{D}_{\mathrm{N,N}} & \mathbf{0} \\ \mathbf{C}^{\mathrm{T}}_{\mathrm{F}} & \mathbf{C}^{\mathrm{T}}_{\mathrm{B}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{w} \\ \mathbf{q}^{\mathrm{int}} \\ \mathbf{p}^{\mathrm{int}} \\ \tilde{\mathbf{p}} \\ \lambda \end{pmatrix} = \begin{pmatrix} \rho_0\omega^2 \int_{\Gamma^{\mathrm{int}}} \varphi^w f^{\mathrm{e}}\,\mathrm{d}s_x \\ -\int_{\Gamma^{\mathrm{int}}} \varphi^q (V\bar{q})(x)\,\mathrm{d}s_x \\ \int_{\Gamma} \varphi^p \left[\frac{1}{2}\bar{q}(x) - (K'\bar{q})(x)\right]\,\mathrm{d}s_x \\ \mathbf{0} \end{pmatrix}. \quad (7)$$

For the solution of the system (7) an approximate Uzawa type algorithm is employed. GMRES iterations are performed on the reduced equation for the Lagrange multiplier λ. The inverse of the FEM matrix is approximated by conjugate gradient iterations, while the BEM Dirichlet-Neumann map for the acoustic domain is evaluated using inner GMRES iterations. Special attention must be paid to the preconditioning of the sub-systems. Standard diagonal scaling already reduces the required iterations significantly, but in particular for the plate system adopted techniques should be employed.

In the approximate Uzawa algorithm, matrix-vector products of discretized boundary integral operators must be evaluated. Using a standard BEM formulation, the computing time and memory requirements are of order $\mathcal{O}(N^2)$ and the method is thus not feasible for large scale simulations. Using the fast multipole BEM, the numerical cost can be reduced to $\mathcal{O}(N \log^2 N)$. For a description of the multipole algorithm and the fast realization of the boundary integral operators it is referred to a paper by the authors [2].

Numerical Example

The proposed coupling algorithm is demonstrated on the example of an acoustic cavity backed by an elastic panel. An analytic series solution is used as a reference solution. The elastic panel considered has the dimensions $1\,\mathrm{m} \times 1\,\mathrm{m}$ and a thickness of $t = 0.01\,\mathrm{m}$. It is made from steel ($E = 2.1 \times 10^{11}\,\mathrm{N/m^2}$, $\nu = 0.3$, $\rho = 7900\,\mathrm{kg/m^3}$) and is simply supported on all edges. The panel is coupled to a closed acoustic cavity with dimensions $1\,\mathrm{m} \times 1\,\mathrm{m} \times 1\,\mathrm{m}$. The remaining surfaces of the cavity are reverberant walls, *i.e.* homogeneous Neumann boundary conditions ($\bar{q} = 0$) are applied. The acoustic fluid is water ($c_0 = 1481\,\mathrm{m/s}$, $\rho = 1000\,\mathrm{kg/m^3}$).

Figure 2 plots the frequency response at the point $(0.2\,\mathrm{m}, 0.3\,\mathrm{m})$ on the plate due to a force of $F = 1\,\mathrm{N}$ at the same position. The FEM-BEM results computed using 316 boundary elements on the interface and 20×20 finite plate elements agree completely with the series solution.

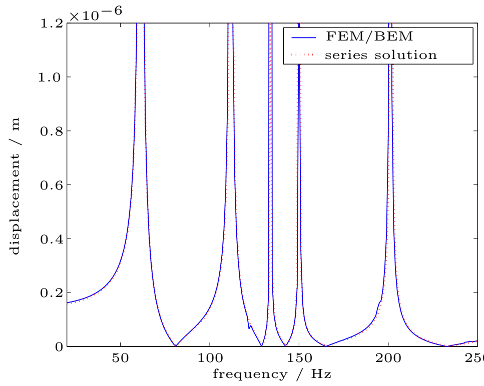

Figure 2: Frequency response of coupled system.

To demonstrate the flexibility of the algorithm with respect to mesh refinement in the sub-domains, the system is studied at a frequency of $f = 180\,\mathrm{Hz}$ with forcing by a single force as before. The resulting displacement field is depicted in Figure 3. A rather fine FEM discretization is required for the spatial resolution of the displacement field. Thus, the error $e_2^{\mathrm{fluid}} = ||\mathbf{p}^{\mathrm{int}}_{\mathrm{BEM}} - \mathbf{p}^{\mathrm{int}}_{\mathrm{series}}||_2 / ||\mathbf{p}^{\mathrm{int}}_{\mathrm{series}}||_2$ reduces quickly with FE mesh refinement until the maximum accuracy for the chosen fluid mesh is obtained (see Figure 3).

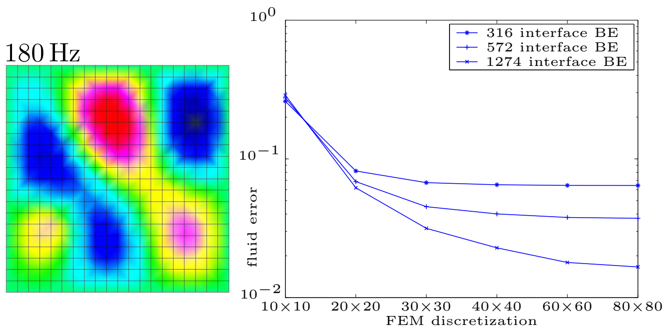

Figure 3: Displacement field and fluid error e_2^{fluid} at 180 Hz

References

[1] Gaul, L, Kögl, M., and Wagner, M. (2003): *Boundary Element Methods for Engineers and Scientists*, Springer.

[2] Fischer, M., Gauger, U., and Gaul, L. (2004): "A multipole Galerkin boundary element method for acoustics", *Engineering Analysis with Boundary Elements*, Vol. 28, pp. 155-162.