Enhanced Simulation of Hydroacoustics in Flexible Structures by Substructuring and Model Reduction

Matthias Maess, Lothar Gaul

Institute A of Mechanics, University of Stuttgart, Pfaffenwaldring 9, 70550 Stuttgart, Germany
Email: {maess,gaul}@mecha.uni-stuttgart.de

Introduction

The paper discusses aspects of simulating hydroacoustic vibrations and wave propagation of compressible acoustical fluids enclosed in flexible containers such as pipe assemblies or tanks. The Finite-Element two-field simulation models in three dimensions are fully coupled at fluid-structure interfaces resulting in both nonsymmetric mass and stiffness matrices [6], if acoustical pressure p is used as field variable in the fluid. By introducing a potential-like fluid variable, it is possible to rearrange the system matrices such that mass and stiffness matrices become symmetric while a skew-symmetric "damping" matrix $\boldsymbol{D}$ exists [4]. For the further methodology, it is possible to use both formulations to generate reduced component models for segments of more complex fluid-filled systems such as flexible piping systems filled with water or oil. The reduced component models are used as superelements and become part of a design toolbox. Afterwards, they can be assembled in the desired geometry of the overall system, satisfying the need for a flexible and efficient simulation tool for hydroacoustics in flexible assemblies.

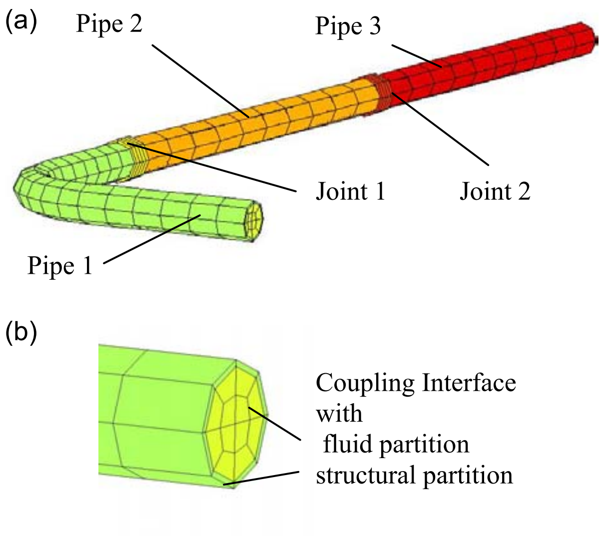

Figure 1: Piping system assembled by five substructures (a) and the two-field component interface (b).

Reduced Component Models

The present method uses reduction bases according to Craig and Bampton [3]. Hereby, constrained interface modes are assembled with dominant eigenvectors of the constrained system. In general, high frequency eigenvectors are truncated from the reduction base. For FSI-coupled problems in nonsymmetric representation according to [6]

$$\begin{bmatrix} \boldsymbol{M}_s & \boldsymbol{0} \\ \rho \boldsymbol{Q}^T & \boldsymbol{M}_a \end{bmatrix} \begin{pmatrix} \ddot{\boldsymbol{u}} \\ \ddot{\boldsymbol{p}} \end{pmatrix} + \begin{bmatrix} \boldsymbol{K}_s & -\boldsymbol{Q} \\ \boldsymbol{0} & \boldsymbol{K}_a \end{bmatrix} \begin{pmatrix} \boldsymbol{u} \\ \boldsymbol{p} \end{pmatrix} = \begin{pmatrix} \boldsymbol{f}_s \\ \boldsymbol{q}_a \end{pmatrix}, \quad (1)$$

reduction bases $\boldsymbol{T}$ are separated according to fluid and structural degrees of freedom

$$\begin{pmatrix} \boldsymbol{u} \\ \boldsymbol{p} \end{pmatrix} = \begin{bmatrix} \boldsymbol{T}_u & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{T}_p \end{bmatrix} \begin{pmatrix} \boldsymbol{u}_R \\ \boldsymbol{p}_R \end{pmatrix}. \quad (2)$$

The block matrices $\boldsymbol{T}_u$ and $\boldsymbol{T}_p$ contain constrained interface modes along with the eigenspace of either the uncoupled problem or of the coupled problem. An iterative partial eigensolver [1] is adopted to accelerate the computation of the eigenpairs. It is worth noticing that reduction bases might be applied similarly to the system with symmetric mass and stiffness matrices [5]. However, the skew-symmetric matrix $\boldsymbol{D}$ requires an eigensolver handling complex eigenvectors.

Substructuring

The reduced component models are assembled in the desired geometry, as depicted in Fig. 1a for five pipe segments. On the component interfaces (Fig. 1b), implicit scleronomic conditions hold between the nodal degrees of freedom. These coupling conditions are transformed to modal space and afterwards, the null space resulting from a QR-decomposition is used as explicit coupling matrix [2]. As a result, the conditioning of the system matrices does not deteriorate. Both nodal C^0 and C^1 – continuity hold on the fluid and the structural interface. Modal analysis is performed for the global system based on reduced component models. Hereby, the adopted iterative partial eigensolver can be used again, since the structure of the global dynamic equation has still the form from Eq. (1). CPU times are decreased, if the global piping system is assembled by reduced component models, which are part of a design toolbox. The computational effort linked to the generation of the reduction base is only made once per substructure. In Fig. 2, the relative frequency error Δf of the eigenvalues is plotted over the mode number covering the frequency range up to 4300 Hz. Reduction bases of the five substructures are built up by only 40 normal modes and the constrained interface modes, such that the eigenvalue problem is decreased from 3547 to 440 degrees of freedom, while the frequency error stays less than 1.2%.

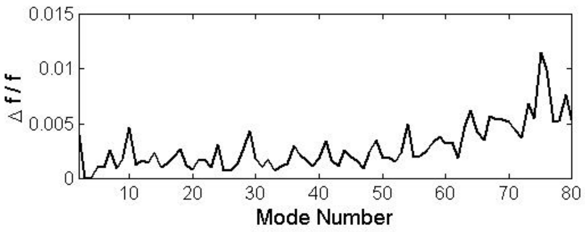

Figure 2: Frequency error between the full model and the reduced model in Fig. 1.

In Fig. 3, the pressure field caused by a pipe bending mode is depicted within the structural displacement of the pipe body and of the joints. The result gives evidence of the coupling nature of the two fields. The elastic joint component model includes the pipe body and the fluid inside, such that the coupling interface is uniform throughout the whole piping system.

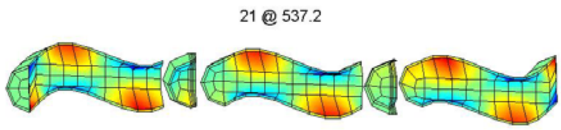

Figure 3: Pressure field and structural displacements of a bending mode of a straight piping system build up by three straight pipe segments and two flexible joints.

Transient Wave Simulation

For the simulation of transient hydroacoustic wave propagation in flexible piping system, the FSI-coupled models are reduced with respect to their excitation specified by the input matrix $\boldsymbol{b}$. Therefore, attachment modes $\boldsymbol{T}_s = \boldsymbol{K}^{-1}\boldsymbol{b}$ enrich the reduction base consisting of the low-frequency eigenspace of the fully coupled problem according to Eq. (2)

$$\boldsymbol{T} := [\boldsymbol{T} \quad \boldsymbol{T}_s] \, . \tag{3}$$

Since this matrix is used for both reduction and projection, the resulting formulation does not decouple the normal modes. However, a further treatment of the reduced-order model by projection with left eigenvectors allows for modal decoupling. Figure 4 shows the time simulation of a plane wave in water enclosed in a straight brass pipe segment. The pressure boundary condition $p(0,t)$ at the left end is depicted on top. The acoustic pressure time histories at $x = 0.8\,\text{m}$ for the full model (13480 DOF's) and for the reduced model (101 DOF's) are compared and the error Δp is shown at bottom.

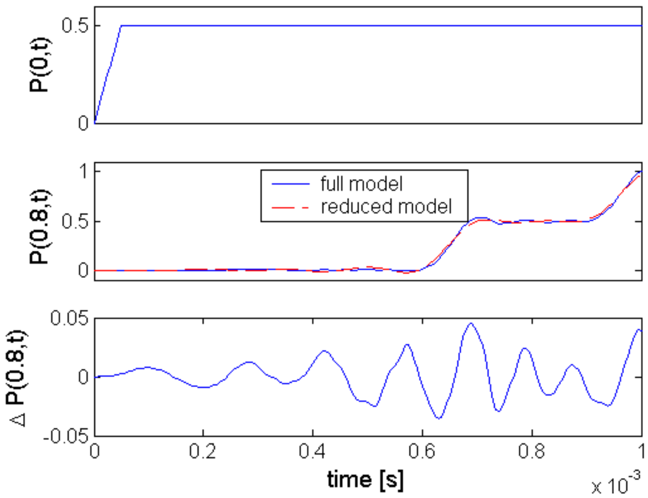

Figure 4: Time histories of the acoustic pressure at the boundary (top) and of the acoustic pressure x=0.8m of the full and of the reduced model (middle) and error time history.

Further model reduction in the sense of the input-output behaviour can be performed, if the controllability and observability Grammians are considered to further neglect non-dominant modes from the reduction base or even to balance the dynamics.

Acknowledgements

Funding of this project by the German Research Society DFG in the Collaborative Research Center SFB 412 “Computer Aided Modeling and Simulation for Analysis, Synthesis and Operation in Process Engineering” is gratefully acknowledged.

References

[1] Bobillot, A.; Balmes, E.: Iterative technique for eigenvalue solutions of damped structures coupled with fluids, *AIAA* -**1391** (2002), 1-9.

[2] Brechlin, E.; Gaul, L. Two methodological improvements for component mode synthesis. In: *Proc. 23. International Seminar on Modal Analysis*, Leuven 2000.

[3] Craig, R.J.; Bampton, M.C.C.: Coupling of substructures for dynamic analysis. *AIAA Journal* **6** (1968),1313-1319.

[4] Everstine, G.C.: A symmetric potential formulation for fluid-structure interaction, *Journal of Sound and Vibration* **79** (1981),157-160.

[5] Kropp, A.; Heiserer, D.: Efficient broadband vibro-acoustics analysis of passenger cars using an FE-based component mode synthesis approach. *Journal of Computational Acoustics* **11** (2003),139-158.

[6] Zienkiewicz, O.C.; Taylor, R.L.: *The Finite Element Method*, Butterworth-Heinemann, Oxford, 2002.