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Maîtrise de l'accord et du timbre d'instruments de percussion à lames par modifications structurales optimales

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The tuning of mallet bar instruments, such as vibraphones and marimbas, is usually based on the carving of the bar profile, a manufacturing process which is oftenly performed on the basis of empirical knowledge even today. In practice, the removal of material from a bar modifies both the inertia and stiffness characteristics of its vibrational modes, and this provides one possible strategy for adjusting the modal frequencies of the first partials in relation to a given fundamental. The relevant aspect of this work is to propose a new methodology for the multi-modal tuning of bar instruments, which combines structural modifications and optimization techniques, with the virtue of being non-destructive. The idea is to constrain the original dynamics of the bar by adding point masses, which change the modal frequencies of the original system according to a target tuning. This leads to address a problem of optimization, aiming at defining the characteristics of the correcting masses, i.e their masses and respective positions. In practice, it becomes viable not only to correct the tuning deficiencies of existing bars, but also to change their timbre, or even to finely tune plain bars with constant cross-section. The modeling approach includes Finite-Element modeling of the bar constrained locally by the masses, as well a reduced-order model based on a modal formulation. The comparison betwen the numerical and experimental results attests the validity and the feasability of the proposed tuning approach, which appears as a pratical solution towards the design of mallet bar instruments with predefined timbral features.

1 Introduction

The tonal quality of a struck bar is strongly dependent on the frequency ratios of its lowest order flexural modes [1, 2]. For a bar with uniform cross-section, these modes fall in inharmonic frequency relationships, and thus leads to an ambiguous definition of the pitch of the musical tone. Bar tuners have attempted to develop methods to adjust the frequencies of the first partials in order to approximate them to harmonic series, typically in the frequency ratios 1:4:10 or 1:3:9. Currently, these methods are classically based on the removal of material from the bar using precision machining tools, which physically change the mass and stiffness properties along the bar, and thus ultimately alter the modal frequencies. Mainly based on empirical knowledge acquired through trial and error procedures, this is not only a sensitive task which requires high levels of specialisation, but also is often costly and inefficient for manufacturers.

Although in recent years, backed by the exponential growth in computers performance, researchers have made significant advances in improving these methods [1, 2, 3, 4, 5, 6], the common practical approach still consists in the removal of bar material, a destructive process which irreversibly alters the bar profile. A different approach to the problem has also been proposed in [7] by using active control techniques. In this paper, we propose an original non-destructive method to address the tuning problem by adding to the bar, at specific locations, suitably designed masses. The positions and values of these tuning masses are determined by combining physical modeling techniques with optimization strategies, and as such, it seems that this approach has never been attempted, at least on a scientific basis. The proposed methodology uses a modal formulation of the bar constrained by discrete masses, feeded by the modal properties of the original unconstrained bar, stemming from a Timoshenko FEM beam model. The modal approach is particularly suitable for the objective of the work, as it provides a physical model with a reduced number of degree-of-freedom, and consequently requires small computational efforts for the optimization process. Here, to predict the optimal mass values and respective positions for achieving a given tuning, we implement a deterministic local optimization strategy and minimize a multivariable error function. Although these techniques are rather straightforward computationally, they are however prone to get stuck in a local minimum, and for that reason, some strategies have been developed in order to alleviate this problem. Two applications are then illustrated: (1) to correct a badly tuned vibraphone bar; (2) to tune an uniform cross-section aluminium bar.

Finally, the validation of the results through experiments allow to assess the adequacy and feasibility of the proposed tuning method. This provides encouraging results towards the development of reversible tuning strategies for mallet bar instruments.

2 Physical modeling of the bar dynamics

In order to predict the behaviour of the bar loaded with discrete masses, a constrained modal formulation of the system dynamics was used. As previously mentioned, this formulation was built from the modal properties of the unconstrained bar (without masses) computed through FEM.

2.1 Modal properties of the bar without tuning masses computed through FEM

Due to the geometry of the bars addressed here, we used the Timoshenko beam model which accounts for both the shear deformation and the rotary inertia effects. The coupling of this effects makes it suitable for describing the dynamical behaviour of thick bars with variable cross-section. The governing equations are given by [8]:

$$\rho A(x)\frac{\partial^2 Y}{\partial t^2} + kGA(x)\left(\frac{\partial \Phi}{\partial x} - \frac{\partial^2 Y}{\partial x^2}\right) = 0, \quad (1)$$

$$\rho I(x)\frac{\partial^2 \Phi}{\partial t^2} - EI(x)\frac{\partial^2 \Phi}{\partial x^2} + kGA(x)\left(\Phi - \frac{\partial Y}{\partial x}\right) = 0, \quad (2)$$

where $Y(x,t)$ is the flexural motion, $\Phi(x,t)$ is the slope of the cross-section due to bending, ρ is the density of the bar material, $A(x) = BH(x)$ is the cross-sectional area of the bar, k is the adjustment coefficient for the shear force, G is the shear modulus, $I(x) = \frac{BH(x)^3}{12}$ is the bar flexural moment of inertia and E is the Young modulus. Finite element discretization of Eq. (1) and (2) enables the computation of the elementary stiffness and mass matrices, which after assembling lead to the dynamical formulation of the model of the bar without additional masses (referred as to the original system). In terms of the physical coordinates, the bar transverse motion is given by:

$$[M_{os}]\{\ddot{Y}(x,t)\} + [K_{os}]\{Y(x,t)\} = \{0\}. \quad (3)$$

where $[M_{os}]$ and $[K_{os}]$ are the global mass and stiffness matrices of the original system and $\{Y\}$ is the vector of physical displacements. From Eq. (3), by assuming harmonic solutions of the form:

$$\{Y(t)\} = \{\varphi_m\}exp(i\omega_m t), \quad (4)$$

and solving the generalized eigenvalue problem:

$$\Big(-\omega_m^2[M_{os}] + [K_{os}]\Big)\{\varphi_m\} = \{0\}, \quad (5)$$

we can compute the modal frequencies of the original system ω_m and the corresponding modeshapes $\{\varphi_m\}$, to be used in the subsequent modal formulation.

2.2 Modal-based modeling of the bar with additional tuning masses

A physical model of the dynamics of the bar constrained by additional point masses, can be represented through the following formulation:

$$[M_{os}]\{\ddot{Y}(x,t)\} + [K_{os}]\{Y(x,t)\} = -[M_{ad}]\{\ddot{Y}(x,t)\}, \quad (6)$$

where $[M_{ad}]$ is the matrix of the additional point masses m_p, a diagonal matrix with the p-terms corresponding to the locations of the additional masses, such that $M_{ad}(p,p) = m_p$.

However, given the number of iterations required for the optimization process (see Section 3), a compact modal formulation seems better suited for our aims as it significantly reduces the number of equations, thus demanding less computation efforts. This formulation can be obtained by reformulating (6) using the coordinate transformation:

$$\{Y(x,t)\} = [\Phi_{os}(x)]\{Q(t)\}, \quad (7)$$

where $\{Q(t)\}$ is the vector of the modal amplitudes and $[\Phi_{os}] = [\{\varphi_{os1}\}\{\varphi_{os2}\},...,\{\varphi_{osn}\}]$ is the modal matrix built from the solutions of Eq. (5). Substituting (7) in (6), the latter equation now reads as:

$$[M_{os}][\Phi_{os}]\{\ddot{Q}(t)\} + [K_{os}][\Phi_{os}]\{Q(t)\} = -[M_{ad}][\Phi_{os}]\{\ddot{Q}(t)\}, \quad (8)$$

Then, pre-multiplying (8) by $[\Phi_{os}]^T$, and using the classical orthogonality properties between the mode shapes, we obtain the modal formulation:

$$[\mathcal{M}_{os}]\{\ddot{Q}(t)\} + [\mathcal{K}_{os}]\{Q(t)\} = -[\Phi_{os}]^T[M_{ad}][\Phi_{os}]\{\ddot{Q}(t)\}, \quad (9)$$

where:

$$[\mathcal{M}_{os}] = [\Phi_{os}]^T[M_{os}][\Phi_{os}] \quad (10)$$

$$[\mathcal{K}_{os}] = [\Phi_{os}]^T[K_{os}][\Phi_{os}], \quad (11)$$

are the diagonal modal mass and modal stiffness matrices of the original system respectively. From Eq. (9), we obtain:

$$\Big([\mathcal{M}_{os}] + [\Phi_{os}]^T[M_{ad}][\Phi_{os}]\Big)\{\ddot{Q}(t)\} + [\mathcal{K}_{os}]\{Q(t)\} = 0, \quad (12)$$

from which the bar modal frequencies ω_m and modeshapes $\{\varphi_m\}$ can be computed by assuming harmonic modal solutions as:

$$\{Q(t)\} = \{\varphi_m\}exp(i\omega_m t), \quad (13)$$

and solving the generalized eigenvalue problem for the mass-loaded system:

$$\Big(-\omega_m^2\Big([\mathcal{M}_{os}] + [\Phi_{os}]^T[M_{ad}][\Phi_{os}]\Big) + [\mathcal{K}_{os}]\Big)\{\varphi_m\} = \{0\}. \quad (14)$$

It is interesting to note that when we turn to the tuning of real bars, one possibility is to use the actual modal frequencies of the bar to be tuned, and thus compute the stiffness matrix $[\mathcal{K}_{os}]$ using the values stemming from a previous experimental modal identification, such as:

$$[\mathcal{K}_{os}] = [\mathcal{M}_{os}][\omega_{exp}^2], \quad (15)$$

where $[\omega_{exp}^2] = diag(\{\omega_1^2, \omega_2^2, ..., \omega_n^2\})$, with $\omega_n = 2\pi f_n$ the angular frequency of mode index n obtained experimentally on the original bar. Finally, notice that both formulations (6) and (14) are equivalent but (14) is more compact as it involves a reduced number of equations. This reduction is particularly welcome for peforming easy and fast computations, and largely compensates the effort for the proposed approach.

3 Optimization strategies

Our optimization problem consists in finding, for a given number n of masses, their optimal values $M_n^* = \{m_1^*, m_2^*, ..., m_n^*\}$, $(m_n \geq 0)$ and respective optimal locations along the bar $L_n^* = \{\ell_1^*, \ell_2^*, ..., \ell_n^*\}$ $(0 \leq \ell_n \leq L)$, that minimize the differences between the modal frequencies of the mistuned system and a predefined set of target frequencies. To that end, we used a deterministic local optimization approach [10] in order to minimize the error-function $\mathcal{E}(M_n, L_n)$ formulated as:

$$\mathcal{E}(M_n, L_n) = \sum_{j=1}^{J} \left| \frac{\omega_j^\star - \omega_j(M_n, L_n)}{\omega_j^\star} \right|, \quad (16)$$

where J is the number of modes to optimize, $\omega_j^\star$ are the target frequencies, and $\omega_j(M_n, L_n)$ are the computed modal frequencies for the mass values M_n and respective positions L_n. Likely, $\mathcal{E}(M_n, L_n)$ may present several local minima, and because the used algorithm is gradient-based, it can be trapped in one of them. In order to overcome this scenario, the optimization was successively performed using random initial solutions, and in general, converged results were consistently obtained. Fast results can still be achieved thanks to the efficiency of the optimization algorithm allied to the reduced model allowed by the modal formulation (see Subsection 2.2). Finally, notice that a semi-discrete scheme for the optimization process has also been recently proposed by the authors in [12]. Instead of assuming continuous values for the weight of the tuning mass, it allows for the use of discrete predefined sets of standard masses which appears more practical for real application.

4 Numerical results

In this section we present two illustrative cases where the optimization procedure is applied. First, we aim to improve the intonation of a vibraphone bar slightly out of tune. Then, we attempt to go further and objective is to tune a bar with an uniform cross-section. In both cases, a mesh comprising 64 elements was considered for the FEM bar model, and values of 710 GPa and 2750 Kg/m^3 were assumed for the Young modulus and density of aluminium respectively. For the modal formulation a low order model with seven equations was used.

4.1 Tuning a vibraphone bar

We present the optimization results for the first case, for which we address the tuning of a badly tuned real-life vibraphone bar. To that end, we model a laboratory vibraphone prototype bar which needs minor tuning corrections as it is classical for bar tuners. The bar dimensions are 0.45 m length, 0.05 m width and variable cross-section with heights between 0.01 and 0.025 m, with a total mass of 1.2 Kg. The target tuning was the musical note C_2 (which corresponds to 262 Hz for the fundamental frequency) with frequency ratios of 1:4:10 for the first three bar flexural modes. This corresponds to frequency corrections of about 8.8%, 7.8% and 5.6% respectively. A typical error value evolution during the optimization process is presented in Figure 1, illustrating the correct behaviour of the optimization process. As can be seen, the convergence of the error is achieved for a number of iterations of 25 for this case.

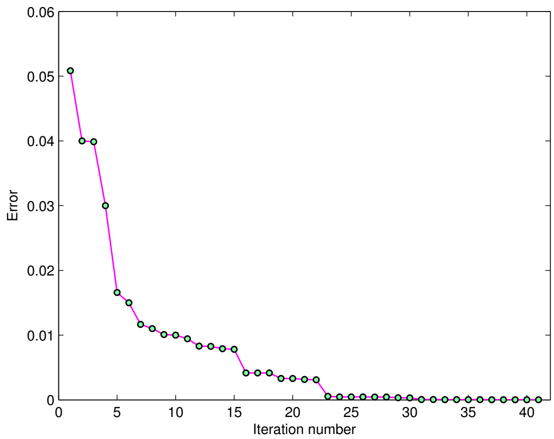

Figure 1: Error value $E(M_n, L_n)$.

Figure 2 represents the bar profile as modeled through FEM, in blue, and the optimized tuning masses and respective positions in order to obtain the target tuning, in red. The heights of the constraining masses were represented based on the assumption that their length and width are equal to those of each element of the mesh. Detailed values are presented in Table 1.

Mass nr.	1	2	3	4	5	6
L_n^* (m)	0.002	0.074	0.179	0.270	0.376	0.448
M_n^* (Kg)	0.008	0.028	0.028	0.028	0.028	0.008

Table 1: Computed optimal tuning mass values M_n^* and respective positions L_n^* corresponding to Figure 2.

Figure 3 shows a different optimal solution obtained for the same problem by using different initial values for the optimization, leading to a different set of masses and

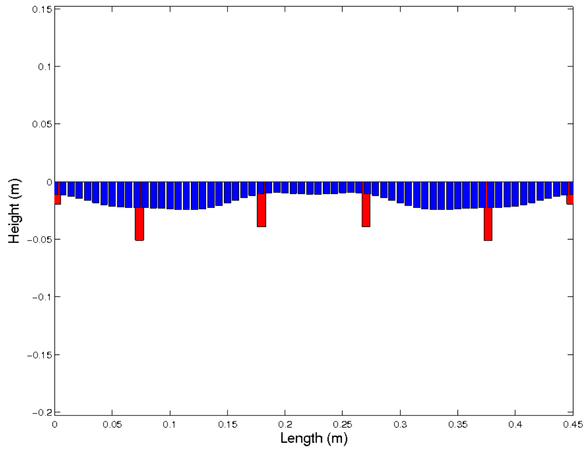

Figure 2: Optimization solution 1 (M_n^*, L_n^*). Blue: original bar profile; Red: additional tuning masses.

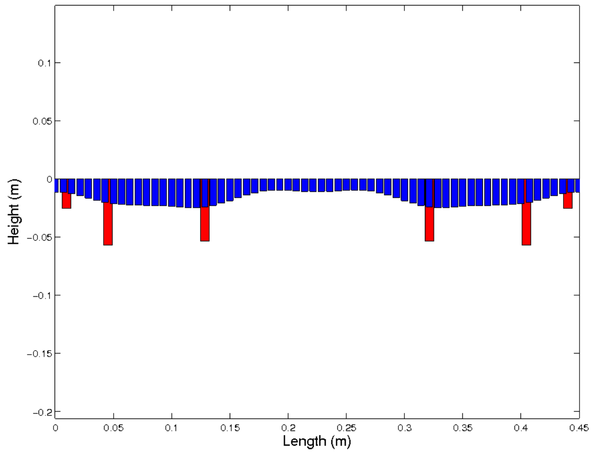

Figure 3: Optimization solution 2 (M_n^*, L_n^*). Blue: original bar profile; Red: additional tuning masses.

locations. In Table 2 we can compare the modal frequencies errors of the original system with the errors predicted for the bar with the optimized additional masses, relative to the target ratios. As we can see, negligible errors are obtained for both solutions shown in Figures 2 and 3, suggesting that very accurate tuning can be achieved with the proposed approach for a given set of target tuned modal frequencies. This also means that different local minima were accepted as the solution for each optimization and the existence of several solutions that comply with the target tuning.

4.2 Tuning a bar with uniform cross-section

Figure 4 shows, in blue, the profile of the modeled bar with 0.5 m length, 0.06 m width and uniform cross-section of 0.02 m. The frequency ratios of the unloaded bar for the first three flexural modes are 1:2.8:5.4, with the fundamental frequency 412 Hz. In this case, the objective was to achieve the target ratios of 1:3:9 by attaching masses to the bar. Since the mass increase only allows to lower the modal frequencies, the proposed task implies their substantial decrease of 106%, 42% and 11%, respectively. As in Figures 2 and 3, the red bars in Figure 4 represent the optimized additional masses and respective distribution along the bar in order to achieve

Mode nr.	Target ratios	Relative error (%)		
		Original	Opt. 1	Opt. 2
1	1	8.8	1.5 10^{-6}	7.8 10^{-6}
2	4	7.8	3.7 10^{-6}	1.8 10^{-5}
3	10	5.6	4.6 10^{-6}	5.4 10^{-5}

Table 2: Modal frequency errors for the first three flexural modes, relative to the tuning ratios 1:4:10, for the original vibraphone bar and the bar with additional masses. Opt. 1 and Opt. 2 correspond to the errors from the optimizations shown in Figures 2 and 3, respectively.

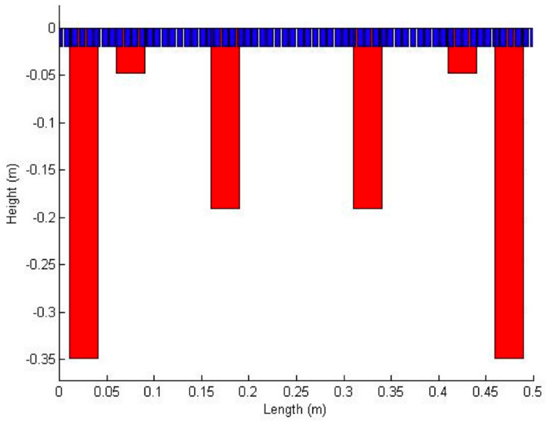

Figure 4: Optimization results (M_n^*, L_n^*). Blue: original bar profile; Red: additional tuning masses.

the target tuning. In this case, for representative purposes, we represent the bar heights assuming that they have the same width as a mesh element, and the length of four times the element length. The correspondent mass values are presented in Table 3.

Mass nr.	1	2	3	4	5	6
L_n^* (m)	0.025	0.075	0.175	0.325	0.425	0.475
M_n^* (Kg)	1.217	0.103	0.639	0.639	0.103	1.217

Table 3: Computed optimal tuning mass values M_n^* and respective positions L_n^*.

As expected, heavier masses are required for the tuning when compared with the previous case, resulting in a total added mass of 3.9 Kg. Nevertheless, as we can see in Table 4, despite the large frequency changes required, an accurate tuning appears to be viable for uniform cross-section bars, according to the optimization results.

5 Experimental results

We now validate the tuning approach experimentally. To that end, bars with similar profile as the ones considered in the previous section are investigated. Experimental modal analysis of the bars constrained and unconstrained are then performed, and the efficiency of the techniques can be easily assessed by examining the frequency changes caused by the tuning masses.

Mode nr.	Target ratios	Relative error (%)	
		Original	Optimized
1	1	106	2.2 10^{-6}
2	3	42	1.1 10^{-6}
3	9	11	2.4 10^{-7}

Table 4: Modal frequency errors for the first three flexural modes, relative to the tuning ratios 1:3:9, for the original uniform cross-sectional bar and the bar with additional masses.

5.1 Re-tuning a mistuned vibraphone bar

Figure 5 shows the prototype vibraphone bar modeled in Section 4.1, which was originally mistuned (see Table 2), and with the additional masses presented in Table 1. In order to comply the numerical model which assumes point masses, we used a set of spherical masses with weights as close as possible to the optimal mass values given by the optimization. Also for practical reasons, since the bar bottom surface was machined in steps, we opted to glue the spherical masses on the top flat surface. For validation, a

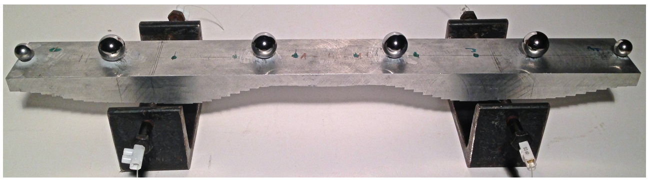

Figure 5: Vibraphone bar prototype with additional masses M_n^* in the L_n^* positions.

modal identification of the bar constrained by the masses was performed by impact testing, using an impact hammer to measure the input force and an accelerometer, glued to the bar at one extremity and aligned on the longitudinal axis, to measure the bar response. Modal identification was achieved by using a MDOF program based on the Eigensystem Realization Algorithm [13], which have been developed in [14]. Results are shown in Figure 6, where the

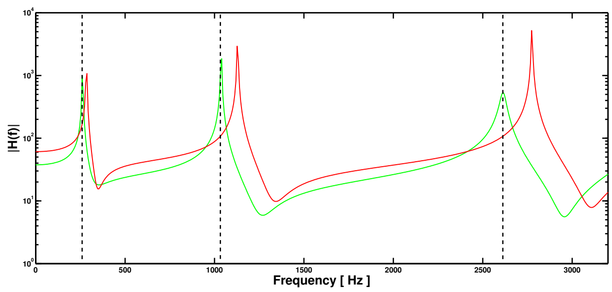

Figure 6: Transfer functions. Red: Unloaded bar; Green: Bar with optimized additional masses.

dashed lines represent the location of the target frequencies. As we can see, the original bar frequencies (in red) were correctly shifted in order to match the pre-defined target tuning. On the whole, negligible tuning errors of less than 1 % were obtained, which shows the effectiveness of the developed modeling and optimization strategies.

5.2 Tuning a bar with uniform cross-section

Here we will present the results for the second case, for which we attempted to tune an uniform cross-section bar. Figure 7 shows the original bar with the additional optimized tuning masses. In this case, because of the large mass values (see Table 3), threaded holes were made at the mass positions, so that the masses can be screwed to the bar. For the modal identification, we used the same methodology described in Section 5.1. As we can see in Figure 8, despite the large frequency changes demanded to accomplish with the proposed target ratios, successful results were obtained for the frequencies of interest.

Figure 7: Uniform cross-sectional bar with screwed optimized additional masses M_n^* at the L_n^* positions.

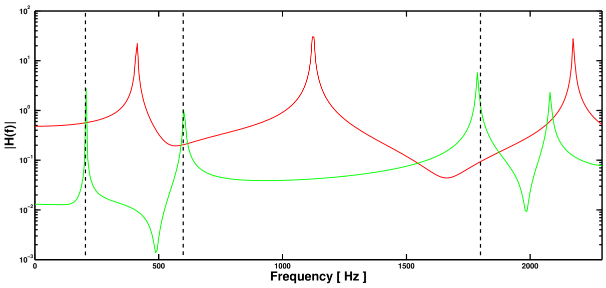

Figure 8: Transfer functions. Red: Unloaded bar; Green: Bar with optimized additional masses.

However, although error less than 1 % were obtained for the first two modes, the third modal frequency is shifted by an amount of about 4 %. Despite care was taken for the realization, this error can be probably explained by the stiffness constraints added by the screwing of the masses to the bar.

6 Conclusions

The aim of this work was to develop an innovative non-destructive method for the tuning of bars by attaching locally tuning masses. Our aim was accomplished through the coupling of physical modeling and optimization techniques, which proved to be effective for the proposed applications. The use of gradient-based optimization strategies combined with the modal formulation allows very fast computations which can be particularly advantageous for the application of the technique to more refined models. A possible difficulty with this kind of structural modification is an increase of the system damping due to the attachments of the masses. In order to minimize this problem, sub-system interface surfaces should be kept to a minimum. For the present bars, the increase in damping was found to be manageable. Nonetheless, being a reversible method, this work is a further step towards the non-destructive tuning of vibraphone bars as well as for other musical instruments.

Acknowledgments

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