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Measuring the Mobility Matrix at the Bridge of Stringed Instruments by the Wire Breaking Method

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The couplings between the strings and the body of a bowed or plucked string instrument are described by 2-D mobility matrices. In this paper, a low cost method for measuring such mobility matrices is presented. It is based on the so called wire-breaking technique, which is a simple mechanism for driving the points where the strings make contact with the bridge : a thin copper wire placed around the string in a position very close to the saddle is pulled aside in the direction of interest until it breaks abruptly imparting a step function force to the driving point. When carried out with damped strings, the acceleration of the bridge measured with a miniature sensor provides a good estimation of transfer mobilities without using any force sensor. A calibration method for absolute mobility measurement is proposed. The limits of the technique in terms of repeatability and signal-to-noise ratio are investigated in the cases of the classical guitar making use of comparisons with results obtained by the classical impact hammer method.

1 Introduction

The sound produced by bowed or plucked stringed instruments is the result of interactions of various subsystems: the excitation mechanisms, the strings, the instrument body, the external medium and the listener. Once the strings are played, the most of the energy that will be converted into radiated sound is transferred to the body through the bridge. The ratio of this energy transfer depends on how strong the strings and the body are coupled: the stronger the couplings are, the higher is the energy transfer ratio, so the tone produced is relatively powerful but with a short duration. Conversely, weak strings/body couplings leads to a low energy transfer ratio resulting in a less powerful tone with a longer duration. Therefore, the degrees of couplings between the strings and the body, which can be described by 2-D mobility (or mechanical admittance) matrices measured at the bridge, are determinant on the instrument sound quality. In addition, bridge mobility measurements may be helpful for instrument makers to characterize and compare objectively different stringed instruments [1, 2, 3, 4, 5, 6, 7, 8]. This study is a part of the PAFI project, which aims to develop a set of tools dedicated to instrument makers [1, 2, 6, 9]. In this context, the main goal of this paper is to present and investigate the wire breaking method, which is a low cost method for measuring the mobility matrix at the bridge of stringed instruments.

1.1 Bibliography review

The wire breaking method is based on the analysis of the response of a structure to a step force. This technique has been investigated for example for some civil engineering systems such as wind turbine where mechanical excitations for modal testing are not so easy to produce. In the musical acoustics context, such technique has been introduced by Woodhouse [5] who presented two different applications. First, it was used for obtaining controlled pluck responses on classical guitars: the wire provided at the pluck position a repeatable excitation in terms of level of stress in a direction of interest and the vibration responses were recorded using a microphone and accelerometer, which allowed comparisons with synthesized plucks obtained by different methods. Second, the wire breaking method was used for measuring co-located admittances at the bridge of classical guitars, which were used to feed different synthesis models. Unlike the present study, the wire breaking force was not determined so that the measurements were calibrated with respect to a calibrated hammer/laser measurement. In [12], it has been presented a methodology for guitar synthesis based on constructing passive admittance matrix models from measured co-located admittance matrix at the guitar bridge. The bridge was also excited by the wire breaking technique at a position close to the saddle and the bridge response was measured by a miniature accelerometer. In [10], mobility measurements on cellos using the wire breaking method were carried out. A pickup system mounted on the bridge in order to collect the input force signals at the string notches. The results were compared with hammer excitation and normal bowing: results indicated that there is nothing fundamentally different between those methods. The wire technique was also used in [11] for measuring the bridge impulse response on violins with damped strings: the string was excited at the bowing position leading the breaking wire to impart an impulse that runs along the string and hits the bridge. In [14] the wire excitation allows a controllable pluck at different positions, in different directions: a high resolution analysis was used for the extraction of the body-mode contribution from the recorded sound. Finally, in [15], the wire technique was used to pluck isolated strings in order to study the influence of the string damping on the decay times of electric guitar tones.

1.2 Statement of the problem

For a linear system, the mobility transfer function $Y_{ij}(\omega)$ is defined in the Fourier domain as the ratio between the velocity response $V_i(\omega)$ at the point i due to the force $F_j(\omega)$ applied at the point j,

$$Y_{ij}(\omega) = \frac{V_i(\omega)}{F_j(\omega)}, \qquad (1)$$

where ω is the angular frequency.

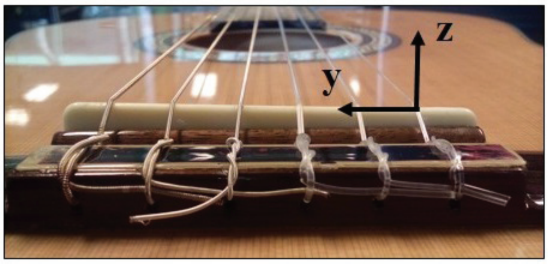


Figure 1: Directions assumed for string forces and bridge velocities: two orthogonal components, parallel and perpendicular to the soundboard, represented as y and z directions, respectively.

For bowed and plucked string instruments, the mobility measured at the bridge quantifies the conversion of string force into bridge velocity. In the present study, both string forces and bridge velocities are assumed to be composed by two orthogonal components, parallel and perpendicular to the soundboard, represented in Figure 1 as y and z directions, respectively, so that

$$\begin{bmatrix} V_y(\omega) \\ V_z(\omega) \end{bmatrix} = \mathbf{Y} \begin{bmatrix} F_y(\omega) \\ F_z(\omega) \end{bmatrix}, \quad (2)$$

where the indexes y and z represent respectively the parallel and the perpendicular directions to the soundboard and $\mathbf{Y}$ is the 2×2 mobility matrix defined as

$$\mathbf{Y} = \begin{bmatrix} Y_{yy}(\omega) & Y_{yz}(\omega) \\ Y_{zy}(\omega) & Y_{zz}(\omega) \end{bmatrix}. \quad (3)$$

The description above neglects both string and bridge longitudinal motions since the parallel and perpendicular components are much higher. It is also assumed that no torque is exerted on the body when forces are applied to the driving points, so that the component in the x direction is ignored (*cf.* [16]).

Mobility matrices measured at the string/bridge contact points of string instruments are usually used to feed models aiming at the instrument sound synthesis [7, 12, 13]. The classical method used for measuring these transfer functions is based on the so-called hammer method: an impulse force is imparted at the point in direction of interest by means of a miniature hammer and the resulting acceleration is measured by a laser vibrometer or a lightweight accelerometer mounted on the bridge. Figure 2 shows typical experimental setups used for measuring mobilities at the bridge of banjos, Brazilian guitars, classical guitars and violins via the hammer method.

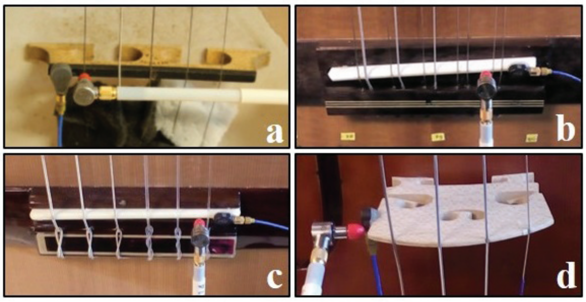


Figure 2: Typical experimental setups used for measuring mobilities at a bridge of banjo (a), Brazilian guitar (b), classical guitar (c) and a violin (d) via the hammer method.

Although being a classic procedure, doubts regarding the reproducibility of the hammer method may be raised since it can be hard to perform repeatable measurements in terms of excitation directions and impact positions. In addition, it is impractical to perform with hammer an excitation at the strings/bridge contact points in the direction y, parallel to the soundboard. Finally, despite the hammer method is a well-adapted procedure for measurements in the laboratory environment, it may prove unsuitable for applications in the context of the instrument manufacturing, mainly due to the high cost of the experimental setup. In this sense, the present paper is devoted to study the wire breaking method, which can be a low cost and well-adapted procedure for measurements in the environment of a instrument maker workshop.

The main aspects of our approach are highlighted by the organization of the paper. Section 2 presents the principles of the wire breaking method and describes one application on the estimation of unknown mobilities in the case of two strings. In Section 3, the limits of the wire breaking method are investigated in the case of the classical guitar making use of comparisons with results obtained by the classical impact hammer technique. Finally, a calibration method for absolute mobility measurement is proposed in Section 4.

2 Wire breaking method

The method is based on the the so-called wire breaking technique which can be a suitable mechanism for exciting the points where the strings make contact with the bridge. It consists in driving the string/bridge contact points by means of a thin copper wire: first, the wire is placed around the string in a position as close as possible to the saddle and then is pulled aside in the direction of interest until it breaks abruptly imparting a step function force to the excitation point. The measurement of the bridge response without the effect of string motion is feasible when the strings are completely damped. Under those conditions, the acceleration response to the wire excitation measured with a miniature sensor mounted on the bridge provides a good estimation of bridge mobilities without using any force sensor as detailed in Subsection 2.1.

The choice of the wire diameter is determinant in the reliability of the results: if the wire is too thick, the instrument may move when the wire is pulled aside leading to distorted measurements. Conversely, if the wire is too thin, a low signal to noise ratio may invalidate the measurements. The wire diameter is also related to its breaking force f_0, whose magnitude is measured using an experimental procedure described in Subsection 4.1. Finally, since the breaking force f_0 is expected to be invariable for samples from the same reel and the choice of the excitation angles are controllable, the method allows the measurements to be reproducible in different environments, by manipulation of different operators.

2.1 Relation of equivalence between *v*-impulse response and *a*-step response

Let us consider a system described by N degrees of freedom, a mass matrix $\mathbf{M}$, a damping matrix $\mathbf{C}$, a stiffness matrix $\mathbf{K}$, a displacement $N \times 1$ vector $\mathbf{x}$(t), excited by a force $N \times 1$ vector $\mathbf{f}$(t). The Laplace transform of the motion equation

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \quad (4)$$

leads to

$$\begin{aligned} X(s) =& [s^2\mathbf{M}\ddot{\mathbf{x}} + s\mathbf{C} + \mathbf{K}]^{-1}\mathbf{f}(s) \\ &+ [s^2\mathbf{M}\ddot{\mathbf{x}} + s\mathbf{C} + \mathbf{K}]^{-1}((s\mathbf{M} + \mathbf{C}))\mathbf{x}(0) + \mathbf{M}\dot{\mathbf{x}}(0)). \end{aligned} \quad (5)$$

The Laplace transform of velocity resulting from a Dirac excitation $f_D(t) = [0...0 \quad \delta(t) \quad 0...0]^t$ applied at one single degree of freedom is given by

$$L\{\dot{\mathbf{x}}(t)\} = s[s^2\mathbf{M}\ddot{\mathbf{x}} + s\mathbf{C} + \mathbf{K}]^{-1}[0...0 \quad 1 \quad 0...0]^t. \quad (6)$$

It can be also shown that the Laplace transform of the acceleration resulting from a step force excitation

$f_S(t) = [0...0 \quad H(t) \quad 0...0]^t$ applied at one single degree of freedom ($H(t)$ being a Heaviside function), is given by

$$L\{\ddot{\mathbf{x}}(t)\} = s[s^2\mathbf{M}\ddot{\mathbf{x}} + s\mathbf{C} + \mathbf{K}]^{-1}[0...0 \quad 1 \quad 0...0]^t. \quad (7)$$

Since the right-hand side of Equations (6) and (7) are the same, it is clear that the velocity resulting from a Dirac excitation is equal to the acceleration response resulting from a step excitation. Thus, the mobility of the system can be obtained from the Fourier transform of the acceleration response resulting from a step force excitation. In the present issue, the amplitude of the step function force is equivalent to the wire breaking force f_0 rather than unitary so that the mobility of the system in physical units is obtained from the the acceleration response to the wire excitation divided by f_0, which can be seen as a calibration factor.

2.2 Estimation of unmeasured mobilities: case of 2 strings

The velocity response measured at a point A due to a force applied at a point E, leads to the mobility $Y(A, E, \omega)$,

$$Y(A, E, \omega) = i\omega \sum_{k=1}^{M} \frac{\varphi_k(A)\varphi_k(E)}{m_k(\omega_k^2 + 2i\omega\omega_k\zeta_k - \omega^2)}, \quad (8)$$

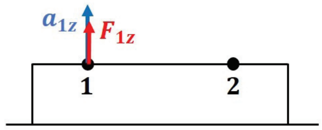
where M is the number of modes and φ_k, m_k, ω_k, ζ_k, are respectively the modal shape, the modal mass, the modal angular frequency and the modal damping factor of the k^{th} mode. A typical application for estimating unknown mobilities by the wire breaking method is described in Table 1, in the case of 2 strings: the accelerometer is fixed at the coupling point 1, in the direction z, denoted by point ($1z$) and the excitation points are changed so that we can measure the four mobilities expressed by the Equations from (9) to (12). Then, the modal parameters of each mode can be estimated over a large frequency range by means of a suitable high-resolution method, for example the subspace ESPRIT method (*cf.* [1]). Once extracted the modal parameters, it is possible the computation of unmeasured mobilities such as: $Y(1y, 1y, \omega)$, $Y(2z, 2z, \omega)$, $Y(2y, 2z, \omega)$, $Y(2y, 2y, \omega)$. Therefore, since the reciprocity principle is assumed, the estimation of the co-located mobility matrices at the points 1 and 2 is possible by using only a fixed accelerometer without force sensor.

2.3 Experimental setup

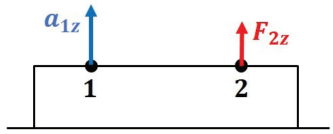
All the measurements reported in this study were performed in the same laboratory environment. The results presented in Section 4 regard to measurements performed on a classical guitar. The instrument is hanged by means of an appropriate support so that free boundary conditions are approached. The strings are tuned before any measurement to their usual static tensions. Since the present study focuses on a procedure which is not addressed to the fabrication of instrument strings, but to the manufacturing of instrument bodies, all the measurements were carried out with damped strings.

For measurements made with the hammer method, the force signal is provided by a miniature impact hammer PCB Piezotronics 086E80 whose head is mounted on a flexible beam clamped at its extremity. Such setup is a convenient way to control precisely the impact location and to avoid multiple hits. The excitation position was chosen on the saddle, as close as possible to the contact point of the e-string with the bridge as shown in Figure 2c.

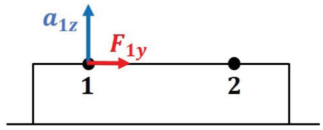
Table 1: A typical configuration for mobility measurements at 2 string/bridge contact points by the wire breaking method.



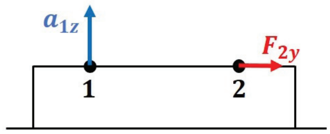
$$Y(1z, 1z, \omega) = i\omega \sum_{k=1}^{M} \frac{\varphi_k(1z)\varphi_k(1z)}{m_k(\omega_k^2+2i\omega\omega_k\zeta_k-\omega^2)} \quad (9)$$



$$Y(1z, 2z, \omega) = i\omega \sum_{k=1}^{M} \frac{\varphi_k(1z)\varphi_k(2z)}{m_k(\omega_k^2+2i\omega\omega_k\zeta_k-\omega^2)} \quad (10)$$



$$Y(1z, 1y, \omega) = i\omega \sum_{k=1}^{M} \frac{\varphi_k(1z)\varphi_k(1y)}{m_k(\omega_k^2+2i\omega\omega_k\zeta_k-\omega^2)} \quad (11)$$



$$Y(1z, 2y, \omega) = i\omega \sum_{k=1}^{M} \frac{\varphi_k(1z)\varphi_k(2y)}{m_k(\omega_k^2+2i\omega\omega_k\zeta_k-\omega^2)} \quad (12)$$

For measurements made with the wire breaking method, the force signal is provided by a thin copper wire with diameter of 0.1mm placed around the e-string in a position very close to the saddle as shown in Figure 3. Measurements using wires with diameters of 0.056mm, 0.1mm and 0.15mm were carried out and the results using wires with diameter of 0.1mm proved to be more suitable for the case of the classical guitar. For both the hammer and wire breaking methods, acceleration signals are collected by a lightweight accelerometer PCB Piezotronics 352C23 (0.2 g) mounted on the bridge, close to the excitation point. A typical experimental setup for mobility measurements via wire breaking method is shown in Figure 3.

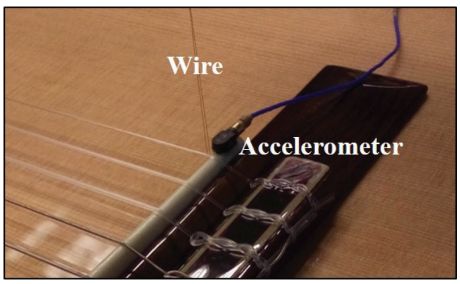


Figure 3: The experimental setup for mobility measurements via wire breaking method.

3 Results and discussion

3.1 Mobilities obtained with the hammer and wire breaking methods

The comparison between the typical mobility measurements at the bridge of banjos, Brazilian guitars, classical guitars and violins via the hammer method is shown in Figure 4, which highlights the difference of profiles of those four instruments. The respective experimental setups are shown in Figure 2. All the mobilities are characterized by numerous resonances, which induced variations around the averaged value over the useful frequency range. The values of the averaged mobility and the modal densities are important features of a soundbox [4]. Since the soundboard of the banjo is a membrane, its mobility is the highest up to 1500 Hz. On the other hand, the violin mobility is amplified in the vicinity of 2500 Hz, presenting the highest values: this feature is often refereed as the *Bridge Hill* [8, 17, 18]. The guitar soundboards' (classical and Brazilian) have been shown to plate-like systems: their mean mobilities and the modal densities are nearly independent on the frequency. This property is the one of a plate, whose equivalent parameters can be computed (*cf.* [4]).

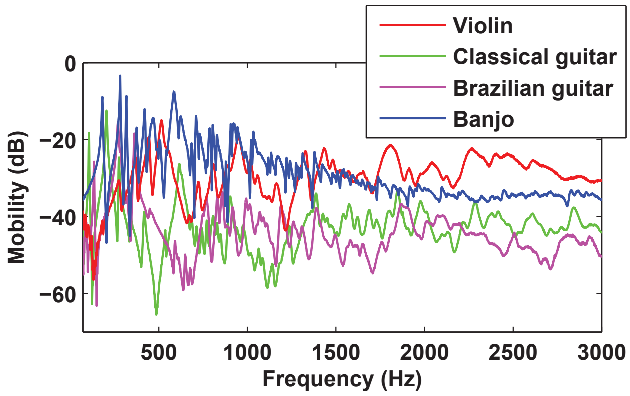


Figure 4: Mobility curves measured via the hammer method at the bridge of different instruments: classical guitar (green), Brazilian guitar (magenta), violin (red) and banjo (blue).

Figure 5 shows the comparison between calibrated and uncalibrated mobilities, from 0 to 2000 Hz, obtained with the hammer and wire breaking methods, respectively. Figure 6 shows the same comparison in a frequency range from 2000 Hz to 7000 Hz. In general, both curves present similar patterns except for the difference in level, which is about 10 dB. As highlighted above, these discrepancies are expected since the measurements obtained by the wire method are not calibrated, i.e. the factor f_0 is not taken into account. It is also observed that at frequencies higher than 4000 Hz the hammer method produces noisier results, revealing another advantage of using the wire breaking method in the present context. The results presented in Figures 5 and 6 confirm the mathematical statement demonstrated in Subsection 3.1 and clearly illustrate that the determination of the factor f_0 is crucial to validate mobility measurements obtained with the wire breaking method.

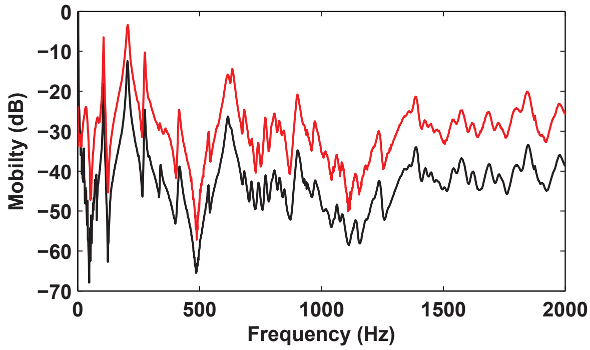


Figure 5: Comparison between mobility curves, from 0 to 2000 Hz, obtained with the hammer (black) and wire breaking (red) methods.

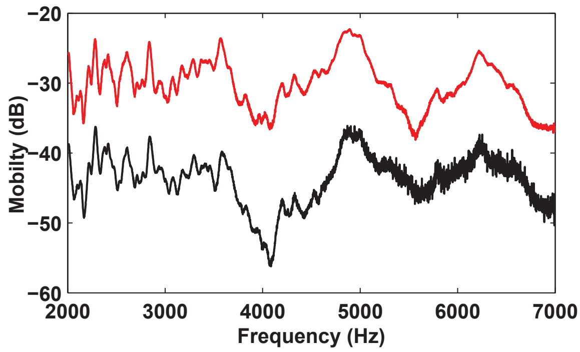


Figure 6: Comparison between mobility curves, from 2000 Hz to 7000 Hz, obtained with the hammer (black) and wire breaking (red) methods.

3.2 Repeatability of the wire breaking method

In order to assess the repeatability of the wire breaking method, 5 mobility curves are measured under the same measurement conditions and compared in Figure 7. It can be observed that all the curves present substantially the same profile, which confirms the breaking wire method is repeatable.

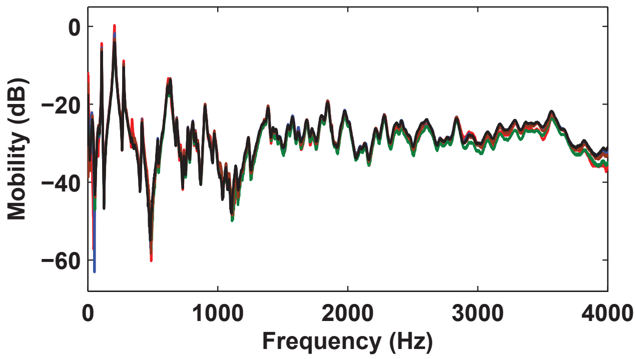


Figure 7: Comparison between 5 measures of mobility curves obtained in the same conditions with the wire breaking method.

4 Calibration of the wire breaking method

As demonstrated in Subsection 2.1, the wire breaking method offers the advantage with respect to the hammer method because it requires only one acceleration sensor for measuring mobilities on the bridge of the instruments. On the other hand, it is necessary to determine preliminarily the calibration factor f_0, which converts uncalibrated mobility curves into transfer functions with real physical meanings.

4.1 Use of a fixed impact hammer for wire force measurement

Figure 8 shows the experimental setup used for estimating the wire breaking force. The measurements consist in threading the wire through a rigid holder attached at the head of an impact hammer PCB Piezotronics 086C03, while the opposite hammer end is clamped onto a flat surface. In this way, the magnitude of the force measured by the hammer when the wire is pulled quasi-statically until it breaks is equivalent to the force exerted on the string by the wire in a mobility measurement. The value of f_0, therefore, is given by the maximum magnitude of the force curve measured in function of time, called in this study the *wire breaking force curve*.

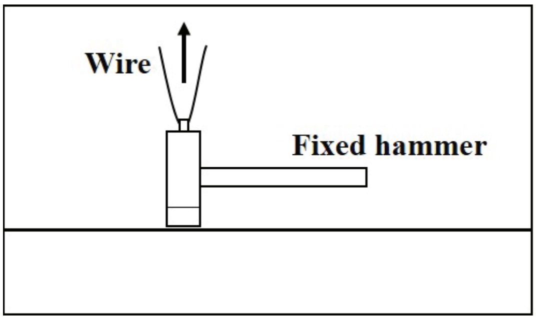


Figure 8: Experimental setup used for estimating the wire breaking force curves: the wire is attached to a fixed impact hammer and pulled until it breaks.

Figure 9a shows a typical wire breaking force curve. For the sake of better visualization the signal of the measured force was inverted. At first, an upward force region is observed, which corresponds to the time interval the wire is stretched. Then, the wire breaks and the measured force falls abruptly since no tension is exerted by the wire. Finally, the measured force features a damped oscillatory behavior that fades out progressively, whose reasons are not clear but may be related to the fact we are dealing with a quasi-statically test while the impact hammer used is designed to perform dynamic tests. Figure 9b shows the comparison between 10 measures of the wire breaking force curves obtained at the same measurement conditions. Although all the 10 resulting curves exhibit the same profile, small differences can be observed, which can be due to slight variations of the gesture made by the operator while pulling the wire. Thus, a more detailed study concerning the gesture and its possible influences on the reliability of the wire breaking method may be investigated in further works. Since the factor f_0 is given by the maximum magnitude of the wire breaking force curves, a value of $f_0 = (4.32 \pm 0.14)N$ is obtained.

Figure 10 compares bridge mobilities measured with the hammer method and the wire breaking method after calibration via the procedure described above. It can be observed a satisfactory agreement between both curves, which indicates that the experimental procedure used for measuring the wire breaking force curves provides a suitable calibration for the wire breaking method.

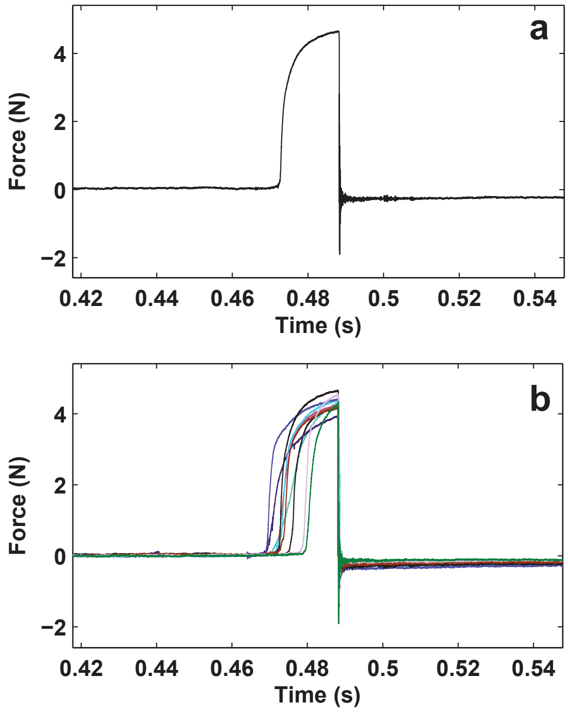


Figure 9: Typical measurement of force obtained by the setup with only impact hammer.

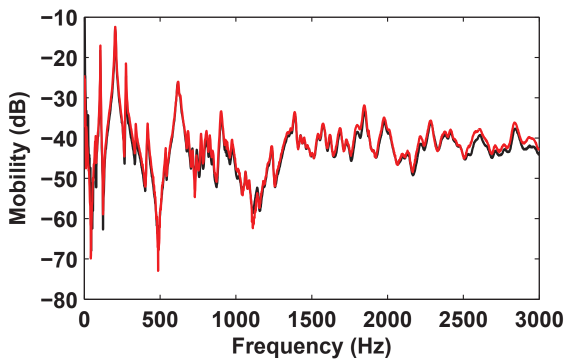


Figure 10: Comparison between calibrated mobility curves obtained with the hammer (black) and wire breaking (red) methods.

Finally, it is worth mentioning that the wire breaking force may be determined by statical tests by gradually attaching weights to the wire until its rupture. This procedure may be more feasible since a possible influence of the hammer on the results can be avoided.

5 Conclusion

This paper has presented and investigated the wire breaking method, which is used for measuring mobility matrices at the bridge of string instruments. was used for measuring mobilities at the bridge of string instruments without using any force sensor. In addition, it can be a low cost and well-adapted procedure for measurements in the environment of a instrument maker workshop since no force sensor is required. The method was shown to be repeatable and provided results in reasonable agreement with the classical hammer method. A calibration method for absolute mobility measurement was proposed and validated.

Finally, it was also shown an application of the wire breaking method: a modal analysis of the mobility curves measured using only a fixed accelerometer without force sensor allows the computation of unmeasured mobilities at the string/saddle contact points. In further works, such application can be used to feed sound synthesis models.

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