Etude acoustique des mousses métalliques bicouches avec interface à gradient de propriétés

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In this article, an inhomogeneous fluid-saturated rigid frame porous layer is considered to belong to a certain class of inhomogeneous porous materials: materials for which the microstructural geometrical configuration is maintained throughout the material thickness. The material can be viewed as a material constituted out of a "mixture" of two materials with varying volume proportions from one material to another and only a porosity profile is needed for the description of the macroscopic inhomogeneity. In order to validate the theoretical approach proposed in this article, a special inhomogeneous sample was designed and built, implementing experimentally a conformal transformation from a tapered circular cross-section cylinder to a constant circular cross-section cylinder. This was done by forcing a circular cross-section flexible plastic foam (melamine) with tapered shape into a rigid constant circular cross-section hollow cylinder (the measurement impedance tube). The acoustic properties (absorption coefficient and transmission loss) of this sample were measured and compared to theoretical predictions involving the Johnson-Champoux-Allard (JCA) model with macroscopiccaly inhomogeneous parameters. To account for the inhomogeneous behavior, the classical series transfer matrix method involving multiple layers was used. In order to test the validity of the "mixture" approach, another approach involving recently developed parallel transfer matrices was also tested and compared to the more conventional series transfer matrices approach. It is thought that the study proposed in this article will apply to many inhomogeneous porous materials encountered in everyday life.

1 Introduction

The variations with depth of the macroscopic parameters of a porous layer can modify significantly its acoustic properties. The general study of rigid frame macroscopically inhomogeneous porous media saturated by air was carried out by De Ryck et al. [1, 2]. With the help of the state vectors formalism and of Peano series, this problem was extended to the 1D problem (in the thickness) of a macroscopically inhomogeneous poroelastic layer Gautier et al. [3]. In the present study, a certain class of inhomogeneous porous material are studied, namely those for which the microstructural geometrical configuration is maintained throughout the thickness of the material. In this case, the problem is greatly simplified as only a porosity profile is required to describe the macroscopic inhomogeneity in the material. The other parameter profiles can easily be determined with the help of the Carman-Kozeny relationship (see Bourbié et al. [4]) in which the geometrical factor involved is maintained constant throughout the inhomogeneity. It is thought that notwithstanding this restriction, many inhomogeneous porous materials will be fairly well described by our study. In our approach, the macroscopic inhomogeneity is viewed as variation of the volume ratio of a mixture of two materials A and B. This volume ratio is directly related to the local porosity and varies from 0% (material A) to 100% (Material B). The input data of our theoretical description are the Johnson-Champoux-Allard (JCA) parameters of material A (entry face A for the acoustic wave in the inhomogeneous porous layer) and also those of material B (exit face B for the wave) and also the porosity inhomogeneity profile. The JCA parameters involved are the (local) porosity, the tortuosity, the flow resistivity and the two characteristic lengths.

In order to validate this "mixture inhomogeneous material" approach, a special sample with well-controlled macroscopic parameters was created. A tapered circular cross section cylinder was created out of highly porous flexible polyurethane foam (melamine). This sample was then forced into a rigid hollow cylinder in which the acoustic tests where carried out, thus artificially creating an inhomogeneous material.

In a second part, a "mixture inhomogeneous material" approach based on parallel Transfer Matrix Method or P-TMM (Verdière et al. [8]) is proposed to validate the experimental results obtained on multilayer inhomogeneous cylinder of melamine foam of constant cross section. The series TMM classical approach and "mixture inhomogeneous material" based on P-TMM approach are also compared. Finally, this new approach approach is used on an inhomogeneous fluid-saturated rigid frame porous aluminum foam specially fabricated for this study.

2 Continuous gradient properties

2.1 Homogenous layer discretization

Figure 1 shows the experimental method used to create an inhomogeneous porous material. The static compression applied to the tapered cylinder induces the progressive modification of physical properties of the material. The Johnson-Champoux-Allard (JCA) model has been used in this study and 5 parameters are involved, namely the porosity, the tortuosity, the flow resistivity and the viscous and thermal characteristic lengths. The approximation of rigid skeleton is made and no modeling of the structural resonance effect is taken into consideration in this study.

Assuming that the microstructure microgeometry configuration is maintained throughout the thickness of the samples, the porosity, and characteristic lengths parameters are obtained from a homothetic behavior (associated with the conformal transformation). The flow resistivity variations are obtained from Carman-Kozeny's law (see Bourbié et al. [4], page 35). Assuming no variation of the shape factor involved, this factor cancels out and only the flow resistivity of the melamine need to be known. For the tortuosity, it is assumed to be constant throughout the inhomogeneous sample.

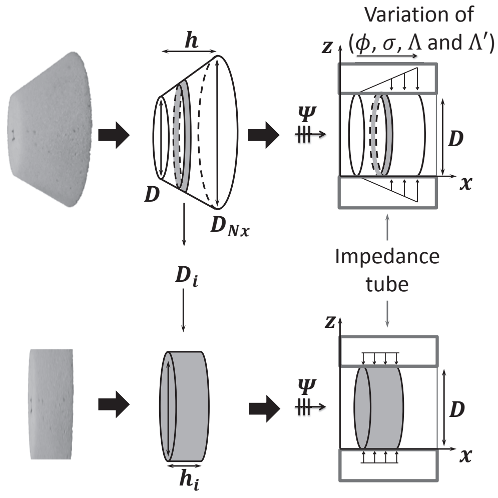

Figure 1: The shape and size parameters made on a truncated conic melamine foam sample

It is proposed that the truncated conic sample can be modeled as a series of homogenous layers, where the macroscopic parameters of each layer evolve (Fig. 1).

In Figure 1 a truncated conical sample of melamine is shown where h is the thickness, D the small diameter, D_{N_x} the base diameter, D_0 the impedance tube diameter where the samples are introduced, and x and z the coordinate axis. A representative melamine layer cylinder sample i of thickness is defined by a diameter D_i, such that $D < D_i < D_{N_x}$, being $D < D_i$. The wave incidence in the acoustical tube is normal to sample. The melamine conical shape sample is introduced into the acoustic tube, changing the macroscopic parameters ϕ, σ, Λ, Λ' continuously as a function of compression degree. The melamine foam transfer matrices will be developed for the studied rigid and limp frame.

2.2 JCA parameters under compression

This section presents the non-acoustic properties of a homogenous melamine foam with a tapered cylinder shape compressed to fit a constant cross section tube. A representative cylindrical layer i (from the compressed conical sample) defined by a diameter D_i and a thickness h_i is considered. The determination of JCA parameters of a volume sample under compression is possible assuming: (1) The cell undergoes a homothetic transformation when is compressed and (2) there are not variation of tortuosity under compression effect [5]. These assumptions are used to determine the porosity and the two characteristic lengths (viscous and thermal). The static air-flow resistivity is given by the Carman-Kozeny law where the shape factor is supposed constant.

The porosity ϕ is defined as the ratio of the fluid volume between V_f and the total volume V_T. The difference between V_f and V_T is the volume of the (solid phase) frame $V_s = V_T - V_f$, which is constant under compression. So, the porosity of layer i, can be expressed in function of porosity and diameter of layer 1 and of diameter of layer i (ϕ_i, ϕ, D and D_i consecutively) by the Eq. (1)

$$\phi_i = 1 - (1 - \phi)\left(\frac{D_i}{D}\right)^2 \tag{1}$$

Considering the assumption (1), a homothetic transformation is applied to the cell. Not accounting for the low compressibility of the solid with respect to the fluid, Λ'_i and Λ_i are calculated in a simplified approach as

$$\Lambda'_i = \left(\frac{D}{D_i}\right)\Lambda' \tag{2}$$

$$\Lambda_i = \left(\frac{D}{D_i}\right)\Lambda \tag{3}$$

For the static airflow resistivity, which can be defined as, $\sigma = \eta/\kappa$, with κ the permeability and η the dynamic viscosity, we use Carman-Kozeny's law (see Bourbié et al. [4], page 35)

$$\kappa = \frac{A}{\alpha_\infty} R_H^2\, \phi \tag{4}$$

R_H being the hydraulic radius of particles. In fact, it is well-known that the hydraulic radius corresponds to Λ'. A is a shape parameter. Replacing κ from Eq. (4) into equation of σ, it is possible to obtain for a given slice i a ratio σ_i/σ for compresed and uncompressed slice of material Considering the shape parameter and the tortuosity constants (assumptions (1) and (2)) σ_i is given by

$$\sigma_i = \left(\frac{\Lambda'}{\Lambda'_i}\right)^2 \left(\frac{\phi}{\phi_i}\right)\sigma \tag{5}$$

The density ρ of the foam is in function of air density ρ_{air} and of solid phase density ρ_{solid}

$$\rho = \phi\rho_{air} + (1 - \phi)\,\rho_{solid} \tag{6}$$

It is possible to obtain ρ_i from a ratio ρ_i/ρ before and after compression

$$\rho_i = \left(\frac{1 - \phi_i}{1 - \phi}\right)(\rho - \phi\rho_{air}) + \phi_i\rho_{air} \tag{7}$$

2.3 Validation on homogenous foam

In order to validate the hypotheses and the expressions for the physical parameters, the model has been validated with a compressed and not compressed homogenous cylindrical melamine foam sample. The Λ' , Λ and σ parameters have been compared between the approach proposed in this work (Eq. (2), (3) and (5) consecutively) and the indirect method [6, 7] (obtained from the transfer matrix measured). The method proposed by Iwase et al. (see Panneton and Only, [6]) to suppress or minimize the frame vibrations with needles of every samples has been used. The ϕ and ρ parameters have been calculated following, consecutively, the Eq. (1) and (7).

The diameter of the compressed homogenous cross section sample is the same as that of the acoustic tube ($D = 29\,mm$) with a thickness of $h = 23,3\,mm$. The dimensions of the homogenous "compressed" sample when it is not compressed are $D_i = 41,3 \pm 0,4\,mm$ and $h_i = 18,5\,mm$.

Table 1: JCA parameters for not compressed melamine foam sample (measured), for compressed melamine foam calculated by simplify approach (SA) Eq. (1-7) and found with the indirect method (IM).

	Not compressed (Measured)	Compressed (SA)	Compressed (IM)
ϕ	0,99±0,01	0,97±0,02	0,97±0,02
σ $(Pa \cdot s/m^2)$	11987±431	24698±1250	18674±2496
α_∞	1,0	1,0	1,0
$\Lambda\,(\mu m)$	80±9	56±6	70±13
$\Lambda'\,(\mu m)$	138±34	97±24	125±33
$\rho\,(kg/m^3)$	8±3	16±6	16±6

The determined values of σ, Λ and Λ' are in the limit error of indirect method estimation. Simulations and experimental results of transmission loss of compressed and not compressed homogenous melamine foam are presented in Figure 2.

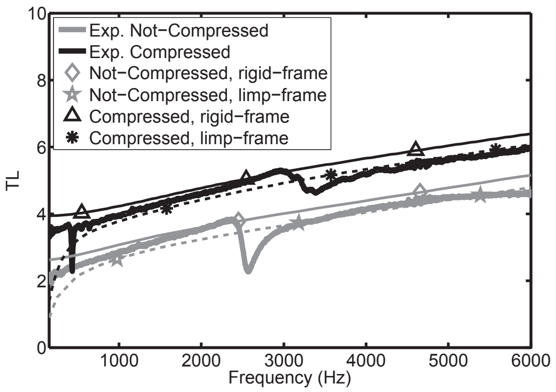

Figure 2: Transmission loss measurement and simulation of compressed (black lines) and not compressed (gray lines) for homogenous melamine foam. Corresponding ($\triangle$ line) to rigid-frame, ($*$ line) to limp-frame to compressed sample, ($\diamond$ line) to rigid-frame, ($\star$ line) to limp-frame to uncompressed sample.

The simulations have been derived from the JCA model using the JCA parameters methods for the homogenous sample and with the simple approach presented for the compressed sample. The good correlation between of experimental and calculated indicators validates the presented method for calculating the JCA parameters of a compressed melamine foam.

2.4 Model for the compressed truncated conic sample

2.4.1 Transfer Matrix Method with elements stacked in series

A melamine foam truncated cone sample as (Fig. 1) has been used to get a progressive variation of macroscopic parameters material. The TMM approach is mainly used to model the acoustic indicators of assemblies consisting of laterally infinite and homogeneous material layers stacked in series. The TMM is appliqued where T is the transfer matrix of the system, T_i the transfer matrix of layer i and N_x the number of layers in the x axis.

$$[T] = \prod_{i}^{N_x} [T_i] \tag{8}$$

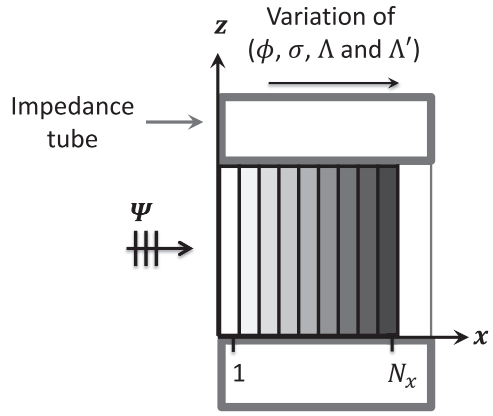

Figure 3: Truncated conic sample divided in N_x layers. The white element represents the smallest diameter D of the tapered cylinder. The bigger the diameter D_i becomes, the darker layer i is. The darkest element corresponds to the base diameter D_{N_x} of the trucated cone.

The simulation has included $N_x = 9$ layers homogenous materials stacked (Fig. 3) which thickness is $h_i = h/N_x$. Note that a short convergence analysis can help to determine N_x. The JCA parameters are calculated from the approach proposed in section 2.2 in Table 2. A sample of small diameter $D = 30,8 \pm 0,3\,mm$, base diameter $D_{N_x} = 57,4 \pm 0,3\,mm$ and thickness $h = 24,7\,mm$ is studied.

Table 2: JCA parameters for an uncompressed melamine homogeneous sample and for the first and the last equivalent layers of the compressed truncated conic sample.

	Not compressed	Layer 1	Layer N_x
ϕ	0,99±0,01	0,98±0,01	0,94±0,04
σ $(Pa \cdot s/m^2)$	11987±431	13548±712	49145±2849
α_∞	1,0	1,0	1,0
$\Lambda\,(\mu m)$	80±9	75±9	40±5
$\Lambda'\,(\mu m)$	138±34	130±32	70±17
$\rho\,(kg/m^3)$	8±3	9±3	29±12

It is worthy to notice that the value the static flow resistivity σ in Table 2 highly differs from the first equivalent layer to the last equivalent layer of the compressed truncated conic sample. Comparing the experimental results and the series MTT acoustics indicators, it is obtained the following behavior

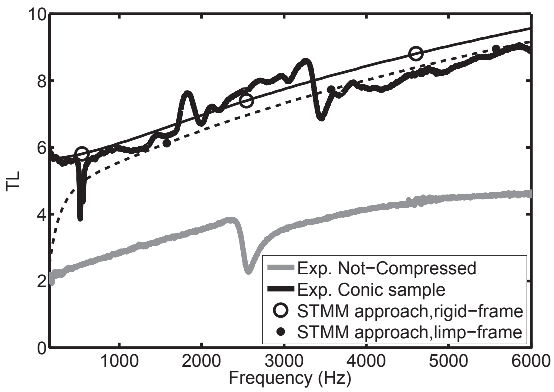

Figure 4: Transmission loss measurement and simulation of not compressed homogenous (color line) and truncated conic (black lines) melamine foam. Corresponding (◯ line) to limp-frame and (• discontinuous line) to rigid-frame.

The good estimation of transmission loss, showed in the Figure 4, validates the multilayer approach. It is showed that a truncated conic sample with gradient properties can be modeled as independent homogenous layers series.

2.4.2 Transfer Matrix Method with parallel elements

The approach suggest in the present work based in the P-TMM [8]. Since the elements are in parallel, it is more convenient to work with admittances (Y). r being the surface ratio and considering N_z elements in the z axis where the element j are stacked (Y) as

$$[Y] = \sum_{j}^{N_z} r_j \; [Y_j] \tag{9}$$

The transfer matrix is obtained from the JCA parameters using the approach explained in section 2.2 for the phase of diameter D and D_{N_x}. A progressive mixture is shown in the next Figure 5. The advantage of this simplified method is the capacity of prediction a properties gradient material knowing the JCA parameters of two external layers and only a "mixture law" on the porosity is necessary, while for a multilayer approach is necessary to know all the parameters of the N_x layers.

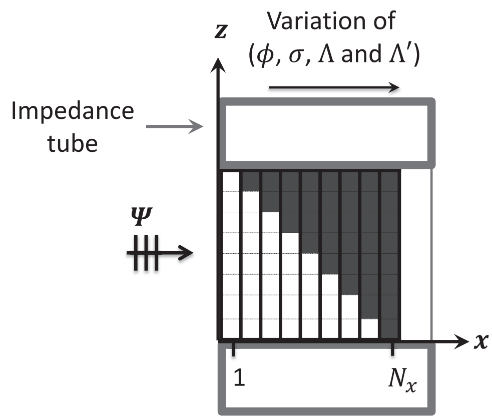

Figure 5: Parallel transfer matrix stack which the withe square elements represent the transfer matrix own of D cone diameter and the dark square elements represent the transfer matrix own of D_{N_x} cone diameter.

Figure 5 represent the transfer matrix distribution used in the simulation of compressed cone, using $N_x = 9$ series layers (as used in section 2.4.1 on multilayer approach) and $N_y = 8$ parallels layers. Note that a short convergence analysis can help to determine N_x and N_y. The transfer matrix of a series layer i (T_i) is calculated from the parallel transfer matrix stack (T_j) using the P-TMM approach proposed by Verdière et al. [8]. The global transfer matrix (T) is obtained by the transfer matrix stack (T_i) series TMM. The P-TMM approach is shown in the Figure 5.

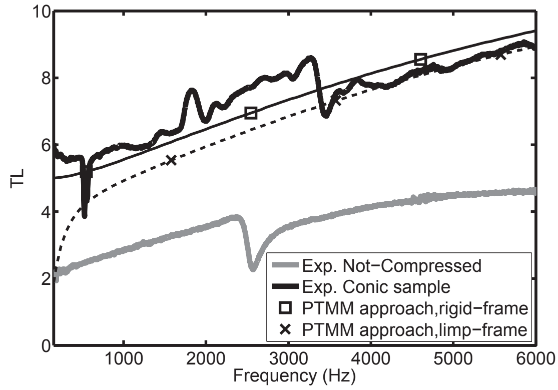

Figure 6: Transmission loss measurement and simulation of truncated conic and not compressed homogenous melamine foam (color line). The measurement (solid line), P-TMM approach rigid-frame (□ line) and limp-frame (× line).

Figure 6 depicts the transmission loss of this gradient properties material as a function of frequency. The P-TMM approach behavior is validated by the good agreement with experiment measurements and series TMM.

3 Inhomogeneous porous aluminum foam

3.1 Aluminum foam fabrication

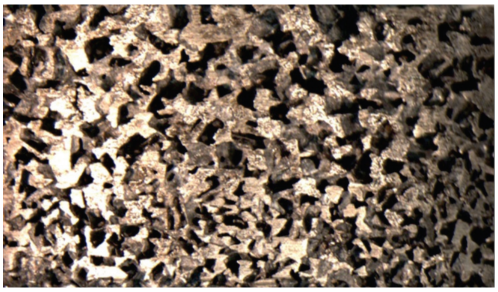

Figure 7: Cut of sample

Porous aluminum foams can be obtained from dissolving salt grains embedded in a solid metal matrix with the help of water. The solid matrix is obtained after the metal in liquid form has invaded the granular material formed by the salt particles at negative pressure and high temperature, and after cooling and solidification of the metal as is done by Gong et al. [9]. A layer with a grain size distribution can be obtained in the following way, the grain size distribution can be controlled by successive sieving of the salt grains. After the metal has cooled down, the sample is cut plunged in the water to dissolve the sodium chloride. Then the sample is dried, air replaces the space formerly occupied by the solid grains and the layer porous metal is created. To make a bi-layer aluminum foam is necessary only to alternate successively a layer of different size of salt grain.

From a fabrication method using two sizes of salt grains, an inhomogeneous sample of aluminum foam was created. This sample is made with two different homogenous layers and an inhomogeneous interface composed by the two cell and inter-cell types, from each homogenous layer, progressively mixed. The two homogeneous layers have been modeled by a transfer matrix. Their macroscopic parameters have been defined like equivalent monolayers samples. The JCA parameters of two homogenous layers are used for modeling the mixture region by the P-TMM approach proposed by Verdière et al. [8].

3.2 Parallel TMM approach

A bi-layer inhomogeneous foam can be made by using two different sizes of salt grains. The analysis of microscope pictures of longitudinal cross-sections of samples revealed that this metallic foam is composed of three layers; two different homogeneous layers and at a third layer which is localized at the interface of the two homogeneous layers. This interface layer is inhomogeneous, composed of a progressive mix of the two cells of the two other layers. Based on this observation, the bi-layer inhomogeneous foam is identified as a multilayer composed of three independent materials stacked in series. The two homogeneous layers have been modeled by a transfer matrix in which the coefficients are given by the JCA equivalent fluid approach. For the inhomogeneous layer, the "mixture" of two materials with varying volume proportions from one material to another is simply identified as a porous material with a porosity profile versus the thickness sample. The transfer matrix of the interface layer is given by the P-TMM approach (Verdière et al. [8]). Indeed this approach permits the modeling of a porosity profile. The JCA parameters of two homogenous layers are used in the P-TMM approach. The global transfer matrix of the bi-layer inhomogeneous foam is given by the product of transfer matrix of the three layers (Eq. 8). The metallic porous frame is considered to be rigid.

The bi-layer metallic foam presented in this study has been made with two kinds of salt grain sizes (the grain equivalent diameters are between 0,9 mm and 1,12 mm for group 1, and between 1,4 mm and 1,8 mm for group 2) (see Figure 7). The two homogenous layers correspond to the two metallic foams MF1 and MF2 for which the JCA parameters are given by using the classical method (measurements and indirect method) and they are given in Table 3. The global sample thickness is h=14,8 mm. With the help of the analysis of microscope pictures (see Figure 7) and an optimization approach (algorithm of simulated annealing), the thickness of each specific layer is defined. The first homogenous layer / interface layer / second homogenous thicknesses are given as 4/2,5/8,3 mm. The interface P-TMM parameters have been chosen as Nx =9 layers and Ny = 8 columns. Figure 8 presents the comparison between the experimental results and the model predictions of the transmission loss of the bi-layers aluminum foam MF1-MF2 sample. One can note that the present approach compares very well to the experiments. Based on the P-TMM and TMM approaches and with help of the JCA parameters of the two homogenous layers, the present approach seems well adapted to model the acoustic of the bi-layer metallic foam with a mixed-cell interface layer.

Table 3: JCA parameters of homogenous aluminum foam sample

	MF1	MF2
$h\,(mm)$	14,9	14,4
ϕ	0,64±0,01	0,66±0,01
$\sigma\,(Pa \cdot s/m^2)$	4316±212	3190±212
α_∞	1,6±0,2	1,7±0,2
$\Lambda\,(\mu m)$	170±13	223±9
$\Lambda'\,(\mu m)$	196±29	544±24

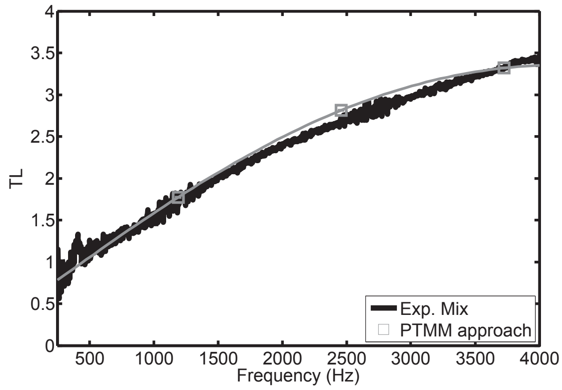

Figure 8: Transmission loss comparison measurement (black solid line) mix of MF1 and MF2 size cell sample, the (□ line) P-TMM approach.

4 Conclusions

In the present paper, the general study of rigid frame macroscopically inhomogeneous porous media saturated by air was greatly simplified as only a porosity profile is required, thanks to the microstructural geometrical configuration maintained throughout the thickness of the material, the use of homothetic relationships in a conformal transformation and the "mixture" approach of two materials with varying volume proportions from one material to another. The parameter profiles on the flow resistivity, of a circular cross-section flexible plastic foam (melamine) with tapered shape forced into a rigid constant circular cross-section hollow cylinder (the measurement impedance tube), was determined with the help of the Kozeny-Carman relationship, where the shape factor is maintained constant. The simulation of high gradient variation shows the difference between the approaches.

Acknowledgment

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