Full characterization of air-saturated porous materials using only transmitted waves

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In this contribution, we present a simple and fast method for the full characterization of rigid frame, air-saturated porous materials by measuring simultaneously the porosity, tortuosity, viscous and thermal characteristic lengths via acoustic transmission only. Usually, reflected waves are required for the full characterization, however, our approach is an improved technique by which only transmitted waves are used in the temporal domain. No relationship is assumed between the characteristic lengths such that both lengths are determined independently. This work shows that it is now possible to measure the porosity using ultrasonic transmitted waves only. In addition, the thermal characteristic length can be obtained regardless of the viscous length, without saturating the porous medium by another gas, or the use data reflected wave. Thus, the method is reliable, rapid and presents advantages over the classic techniques used to date. These results open perspectives yet to be explored for the ultrasonic characterization techniques of air-saturated porous materials.

1 Introduction

The non-acoustic parameters playing an important role in the ultrasonic propagation in air-saturated porous materials are [1, 2] : porosity, tortuosity, viscous and thermal characteristic lengths. Theses parameters describe the inertial, viscous and thermal interactions between fluid and structure in the high frequencies [2, 3]. This frequency domain corresponds to the range of frequencies such that the viscous boundary layer thickness $\delta = (2\eta/\omega\rho)^{1/2}$ is smaller than the radius r of the pores (η and ρ are respectively the viscosity and density of the saturating fluid and ω represents the pulsation frequency).

In standard methods [3, 4, 5, 6], the transmitted ultrasonic waves by an air-saturated porous materials allow to the determination of the tortuosity and viscous characteristic lengths. The thermal characteristic length is deduced from a fixed ratio with the viscous characteristic length [5, 6]. When the porous medium is subsequently saturated by two gases (air and helium), the determination of the thermal characteristic length independently of the viscous length is possible [7]. In the case of a porous material having a structure which vibrates [8, 9] the ultrasonic waves transmitted, allow measurement of the porosity, and mechanical parameters.

The reflected waves by the first interface [10, 11] of a slab of rigid porous material permit the measurement of the tortuosity and porosity. When the reflected wave by the second interface is detected experimentally, the determination of the characteristic lengths becomes possible [12]. The use of both transmitted and reflected waves simultaneously [13, 14] gives a good estimation of porosity, tortuosity, viscous and thermal characteristic lengths. Other methods [15, 16, 17, 18, 19] , not using ultrasonic waves, have been developed for the characterization of rigid porous materials, measuring some parameters mentioned above.

In this work, we solve the inverse problem in time domain using experimental data of ultrasonic waves transmitted by a slab of air-saturated porous material. An improved method is introduced for measuring the porosity, tortuosity, the viscous and thermal characteristic lengths simultaneously. The reflected experimental data are not used. No relationship is assumed between the characteristic lengths such that both lengths are determined independently. This work shows that it is now possible to measure the porosity using ultrasonic transmitted waves only. In addition, the thermal characteristic length can be obtained regardless of the viscous length, without saturating the porous medium by another gas, or the use data reflected wave. Thus, the method is reliable, rapid and presents advantages over the classic techniques used to date. These results open new perspectives for the ultrasonic characterization techniques of air-saturated porous materials.

2 Model

In the acoustics of porous materials, one distinguishes two situations according to whether the frame is moving or not. In the first case, the dynamics of the waves due to the coupling between the solid skeleton and the fluid is well described by the Biot theory [20]. In air-saturated porous media, the vibrations of the solid frame can often be neglected in absence of direct contact with the sound source, so that the waves can be considered to propagate only in fluid. This case is described by the equivalent-fluid model, which is a particular case of the Biot model, in which fluid-structure interactions are taken into account by two frequency response factors : dynamic tortuosity of the medium $\alpha(\omega)$ given by Johnson *et al* [2], and the dynamic compressibility of the air in the porous material $\beta(\omega)$ given by Allard *et al* [1]. In the frequency domain, these factors multiply the density of the fluid and its compressibility respectively and represent the deviation from the behavior of the fluid in free space as the frequency increases. Consider a homogeneous porous material that occupies the region $0 \leq x \leq L$. A sound pulse impinges normally on the medium. It generates an acoustic pressure field p and an acoustic velocity field v within the material. The acoustic fields satisfy the following equivalent-fluid macroscopic equations (along the x−axis) [1] :

$$\rho\alpha(\omega) j\omega v = \frac{\partial p}{\partial x}, \qquad \frac{\beta(\omega)}{K_a} j\omega p = \frac{\partial v}{\partial x}, \tag{1}$$

where, $j^2 = -1$, ρ the fluid density and K_a is the compressibility modulus of the fluid. In the high frequency domain, the viscous effects are concentrated in a small volume near the frame and the compression/dilatation cycle is faster than the heat transfer between the air and the structure, and it is a good approximation to consider that the compression is adiabatic. The high-frequency approximation of the responses factors $\alpha(\omega)$ and $\beta(\omega)$ when $\omega \to \infty$ are given by the relations :

$$\alpha(\omega) = \alpha_\infty \left(1 + \frac{2}{\Lambda}\sqrt{\frac{\eta}{j\omega\rho}}\right),$$
$$\beta(\omega) = 1 + \frac{2(\gamma - 1)}{\Lambda'}\sqrt{\frac{\eta}{j\omega P_r \rho}}, \tag{2}$$

where $j^2 = -1$, α_∞ is the tortuosity, Λ the viscous characteristic length, Λ' the thermal characteristic length, η the fluid viscosity, γ the adiabatic constant, P_r the Prandtl number. In the time domain, $\alpha(\omega)$ and $\beta(\omega)$ act as operators and in the high frequency approximation their expressions are given by [21] :

$$\tilde{\alpha}(t) = \alpha_\infty \left(\delta(t) + \frac{2}{\Lambda} \left(\frac{\eta}{\pi\rho} \right)^{1/2} t^{-1/2} \right),$$

$$\tilde{\beta}(t) = \left(\delta(t) + \frac{2(\gamma-1)}{\Lambda'} \left(\frac{\eta}{\pi Pr \rho} \right)^{1/2} t^{-1/2} \right), \quad (3)$$

in these equations, $\delta(t)$ is the Dirac function. In this model the time convolution of $t^{-1/2}$ with a function is interpreted as a semi derivative operator following the definition of the fractional derivative of order ν given in Samko and coll [22],

$$D^\nu[x(t)] = \frac{1}{\Gamma(-\nu)} \int_0^t (t-u)^{-\nu-1} x(u) du, \quad (4)$$

where $\Gamma(x)$ is the gamma function. Using equations (1) and (3) in the time domain, it follows the fractional propagation equation :

$$\frac{\partial^2 p}{\partial x^2} - \left(\frac{\alpha_\infty}{c_0^2} \right) \frac{\partial^2 p}{\partial t^2} - B \frac{\partial^{3/2} p}{\partial t^{3/2}} - C \frac{\partial p}{\partial t} = 0, \quad (5)$$

where the coefficients c_0, B and C are constants respectively given by ;

$$c_0 = \sqrt{\frac{K_a}{\rho}}, \quad B = \frac{2\alpha_\infty}{K_a} \sqrt{\frac{\rho\eta}{\pi}} \left(\frac{1}{\Lambda} + \frac{\gamma-1}{\sqrt{Pr}\Lambda'} \right)$$
$$C = \frac{4\alpha_\infty(\gamma-1)\eta}{K_a \Lambda \Lambda' \sqrt{Pr}}. \quad (6)$$

The transmission scattering operator T is given by [6] :

$$\tilde{T}(t) = \frac{4\phi\sqrt{\alpha_\infty}}{(\phi + \sqrt{\alpha_\infty})^2} G\left(t + \frac{L}{c}, \frac{L}{c}\right). \quad (7)$$

The expression of the transmission operator $\tilde{T}$ takes into account only the reflections at interfaces $x = 0$ and $x = L$. G is the Green function of the medium given by [23] :

$$G(t,k) = \begin{cases} 0 & \text{if} \quad 0 \le t \le k, \\ \frac{b'}{4\sqrt{\pi}} \frac{k}{(t-k)^{3/2}} \exp\left(-\frac{b'^2 k^2}{16(t-k)}\right) + \Delta \int_0^{t-k} h(t,\xi) d\xi & \text{if} \quad t \ge k, \end{cases}$$

where :
$h(\xi,\tau) = -\frac{1}{4\pi^{3/2}} \frac{1}{\sqrt{(\tau-\xi)^2-k^2}} \frac{1}{\xi^{3/2}} \int_{-1}^{1} exp\left(-\frac{\chi(\mu,\tau,\xi)}{2}\right)(\chi(\mu,\tau,\xi) - 1)\frac{\mu d\mu}{\sqrt{1-\mu^2}}$,
and :
$\chi(\mu,\tau,\xi) = \left(\Delta\mu\sqrt{(\tau-\xi)^2-k^2} + b'(\tau-\xi)\right)^2 / 8\xi$,
$b' = Bc_0^2\sqrt{\pi}, \quad c' = C.c_0^2, \quad \Delta^2 = b'^2 - 4c'$.
In frequency domain, of the transmission coefficient $T(\omega)$ of a slab of porous material is given by :

$$T(\omega) = \frac{2Y(\omega)}{2Y(\omega)\coth(k(\omega)L) + (1 + Y^2(\omega))\sinh(k(\omega)L)}, \quad (8)$$

where :

$$Y(\omega) = \phi\sqrt{\frac{\beta(\omega)}{\alpha(\omega)}}, \quad \text{and} \quad k(\omega) = \omega\sqrt{\frac{\rho\alpha(\omega)\beta(\omega)}{K_a}},$$

ϕ is the porosity of the material.

3 Inversion of experimental data

The inverse problem is to find the parameters α_∞, ϕ, Λ and Λ' which minimize numerically the discrepancy function $U(\alpha_\infty, \phi, \Lambda, \Lambda') = \sum_{i=1}^{i=N} (p^t_{exp}(x,t_i) - p^t(x,t_i))^2$, wherein $p^t_{exp}(x,t_i)_{i=1,2,...n}$ is the discrete set of values of the experimental transmitted signal and $p^t(x,t_i)_{i=1,2,...n}$ the discrete set of values of the simulated transmitted signal predicted from Eq. (7). The inverse problem is solved numerically by the least-square method. For its iterative solution, we used the simplex search method (Nedler Mead) [24] which does not require numerical or analytic gradients. Experiments are performed in air using a broadband Ultran NCT202 transducer with a central frequency of 190 kHz in air and a bandwith of 6 dB extending from 150 to 230 kHz. Pulses of 400V are provided by a 5058PR Panametrics pulser/receiver. The received signals are filtered above 1MHz to avoid high-frequency noise. Electronic interference is eliminated by 1000 acquisition averages. The experimental setup is shown in Fig. 1. Consider a sample of plastic foam F, of thicknesses $4.13 \pm 0.01 cm$. Sample F was characterized using classic methods [3, 4, 6, 10, 11, 25] and gave the following physical parameters $\phi = 0.94 \pm 0.03$, $\alpha_\infty = 1.06 \pm 0.05$, $\Lambda = (255 \pm 1)\mu m$, $\Lambda' = 3\Lambda$. The incident and transmitted experimental signals are given in Fig. 2 and their spectra in Fig. 3. After solving the inverse problem simultaneously for the porosity ϕ, tortuosity α_∞, viscous and thermal characteristic lengths Λ and Λ', we find the following optimized values : $\phi = 0.87 \pm 0.01$, $\alpha_\infty = 1.45 \pm 0.01$, $\Lambda = (32.6 \pm 0.5)\mu m$ and $\Lambda' = (60 \pm 0.5)\mu m$. The values of the inverted parameters are close to those obtained by conventional methods [3, 4, 6, 10, 11, 25]. We present in Figs. 4-7, the variation of the minimization function U with the porosity, tortuosity, viscous characteristic length, and the ratio between Λ' and Λ. In Fig. 8, we show a comparison between an experimental transmitted signal and simulated transmitted signal for the optimized values of ϕ, α_∞, Λ and Λ'. The difference between the two curves is small, which leads us to conclude that the optimized values of the physical parameters are correct.

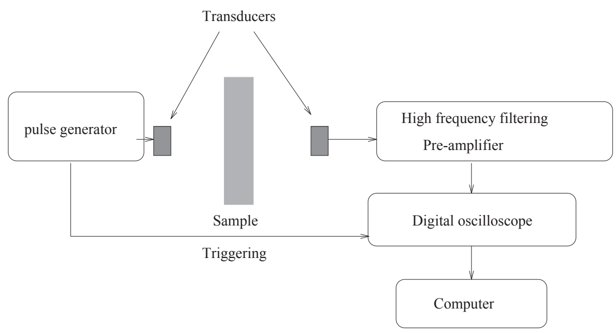

Figure 1 – Experimental set-up of the ultrasonic measurements.

This method has the advantage of being simple and easy to perform. The four non-acoustic parameters (porosity, tortuosity, thermal and viscous characteristic lengths) are simultaneously obtained with good accuracy, without using the reflected waves, or saturating the porous material with another gas (in particular for determining the thermal characteristic length). Recall that so far, the porosity was

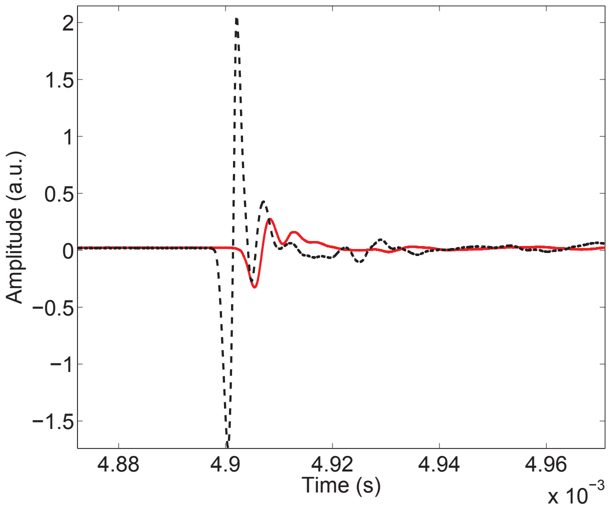

FIGURE 2 – Incident experimental signal (black dashed line) and transmitted experimental signal (red solid line).

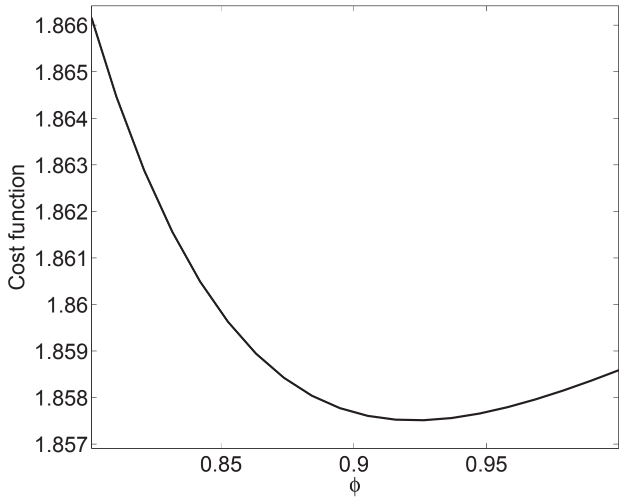

FIGURE 4 – Variation of the minimization function U with porosity.

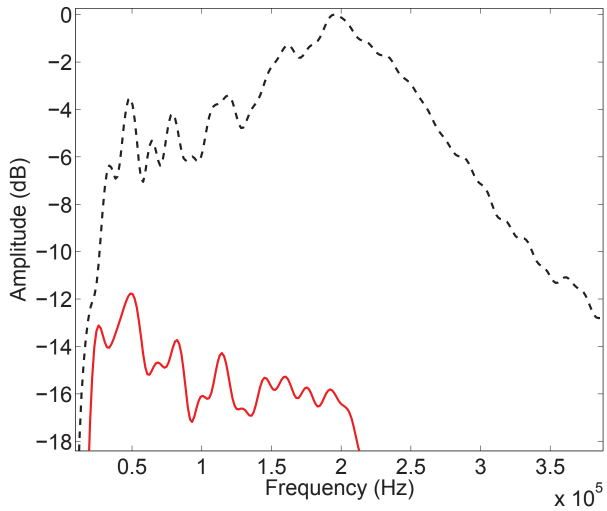

FIGURE 3 – Spectrum of the incident signal (black dashed line) and spectrum of transmitted signal (red solid line).

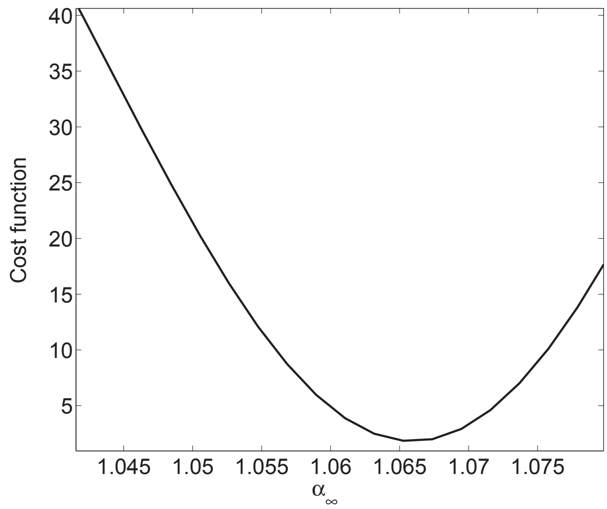

FIGURE 5 – Variation of the minimization function U with tortuosity.

measured with the waves reflected by the first interface of the porous material and the viscous and thermal characteristic lengths could not be determined without saturating the material with another gas using transmission.

4 Conclusion

The complete characterization of a porous material saturated with air is performed in the high frequency regime. The inverse problem is solved in the time domain using experimental data from waves transmitted by a porous material. One of the major results of this study was the measurement of four parameters describing the ultrasonic propagation (porosity, tortuosity, and thermal characteristics viscous lengths) using only the transmitted waves. The use of reflected waves is no longer necessary, and the saturation of the porous material with another gas for determining the thermal characteristic length is not necessary either.

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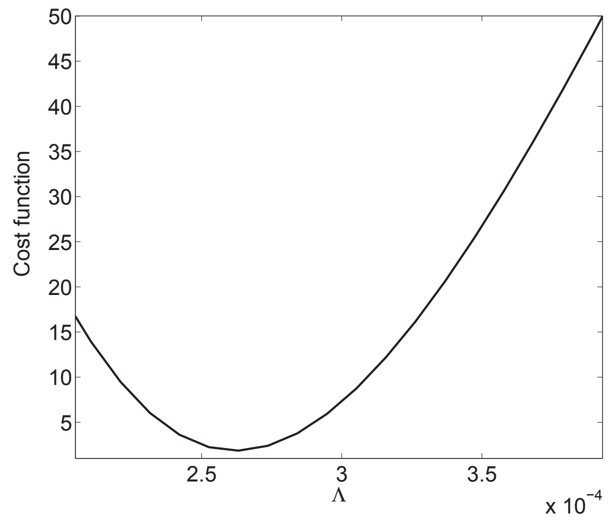

FIGURE 6 – Variation of the minimization function U with viscous characteristic length.

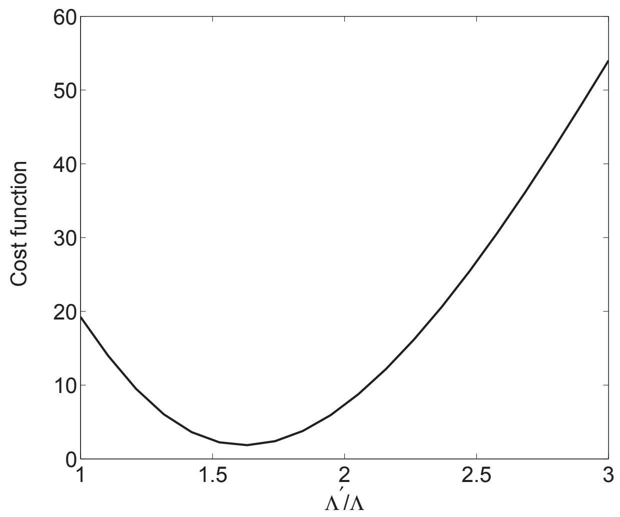

FIGURE 7 – Variation of the minimization function U with the ratio Λ'/Λ.

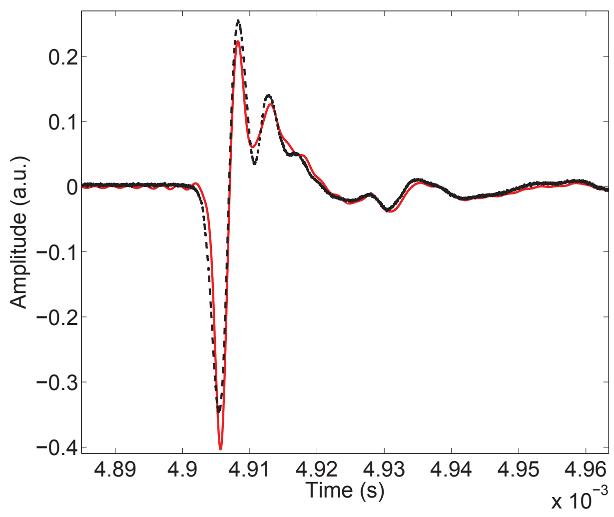

FIGURE 8 – Comparison between the experimental transmitted signal (black dashed line) and the simulated transmitted signals (red line) using the reconstructed values of ϕ, α_∞, Λ and Λ'

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