Three-dimensional time-domain simulations of the string/soundboard coupled dynamics for a twelve-string Portuguese guitar

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The Portuguese guitar is a pear-shaped instrument with twelve metal strings, descendant from the renaissance European cittern. This instrument is widely used in Portuguese traditional music, mainly in Fado, and more recently also started to play a considerable role among urban Portuguese musicians. Unlike most common guitars, this guitar has a bent soundboard (arched top) with a bridge somewhat similar, although smaller in height, to the bridge of a violin. In spite of the large amount of research aiming the understanding of guitar dynamics, few results are currently available on the Portuguese guitar. Coupling between the different vibrating sub-systems of a musical instrument is a very important feature, the reason why instruments of similar families have such different and characteristic sounds. Our recent work on this topic was somewhat restricted by several simplifications, including the assumption of planar string motions and an extremely simplified soundboard. In the present paper those restrictions are relaxed in the following manner : (a) A model for coupling the in-plane and out-of-plane string motions through the bridge kinematics is produced ; (b) A more realistic representation of the instrument dynamics is obtained through finite-element modelling of the soundboard typical of Portuguese guitars. We thus produce a set of time-domain simulations, based on coupling the unconstrained modes of the various sub-systems (12 strings and the soundboard). These computations enable, in particular, to assert the dynamical significance of the string region beyond the bridge (the so-called "dead side" of the strings). Also, these simulations enable a close tracking of the energy flow between the instrument sub-systems, in connection with sympathetic vibrations, beating phenomena and the sound identity of this instrument.

1 Introduction

Coupling the vibrations of various subsystems is an essential issue for developing physically accurate synthesis methods of any musical instrument. Actually, the sound produced by an instrument results from the mutual interaction of its components and it is the reason why instruments of similar families have such different and characteristic sounds. This is particularly significant for string instruments where the intensity of the sound radiated is conditionned by the efficiency of the energy transfers between the strings and the instrument body which pass through the bridge. Although it has a profound influence on the sounding properties of the played tone [1, 2], it seems remarkable that the bridge is often considered to simply enforce a node for the string vibration [3, 4]. One typical example is the occurence of the troublesome *wolf-note*, mostly for cellos, which emerges when the bridge ceases to act as a rigid boundary for the waves travelling on the string. The net result is the transfer of some vibrational energy of the string to both the instrument body and the *dead* part of the string, between bridge and tailpiece, which leads to a difficult control of the played note [5, 6, 7]. Other specific issue of stringed instruments closely related to the bridge action is the coupling between the vertical and horizontal polarisation of the string motion which actually occurs in two perpendicular planes. Nevertheless, other possible mechanisms may influence such coupling, namely string inhomogeneities and non-isotropic supports or the intrinsic geometrical nonlinearity of the vibrating string - see the interesting analysis by Elliott [8]. However, despite sophisticated models especially for guitar pluck [9, 10, 11], the physical mechanisms for coupling the string motions directions are still debatable. In the present paper, we will tentatively assume that the sole mechanism lays on simple kinematical relations stemming from the soundboard/bridge motion as discussed later.

Following our previous work on the dynamical modelling of the Portuguese guitar [12], we extend here significantly our physical model of a 12-string guitar coupled to a body via a bridge to include string vibration with both polarisations and propose a coupling model for the string modes, so that energy can be exchanged between the perpendicular directions of the string motion. Furthermore, to provide more realistic simulations, the model to be used involves the modelling of the top plate of a Portuguese guitar by Finite-Element Method (FEM). Using the modal parameters estimated through the FEM analysis, a modal synthesis technique was developed in order to simulate the fully coupled model of twelve strings vibrating in two perpendicular directions and interacting with the instrument soundboard via the bridge. By examining the energy transfer between the various subsystems of the model, the paper attests the relevance of the proposed approach to couple the string motions. Time-domain simulations highlight that a vertical plucked excitation may excite horizontal vibration of the string, and also that several strings may be excited by sympathetic vibrations when only one string is excited initially. Also apparent in our computations - and rarely treated in the literature - is the excitation of the *dead* part of the string which may influence, to some degree, the tone of the instrument.

2 Presentation of the model

2.1 String dynamics

The string model concerns small-amplitude vibrations and includes both polarisation of string motion, normal and parallel to the soundboard. We consider a set of $s = 1, \dots, S$ perfectly flexible strings of total length L (from the *atadilho*, a small tailpiece at the end of the body of the instrument, to the neck) and density ρ_s, stretched to an axial tension T_s. The strings are rigidily fixed at both ends and stretched over the bridge, so that their natural frequencies are lower than the sounding frequencies defined by the active length of the string, i.e the distance between the bridge and the neck. Adopting a modal framework, the transverse motions of the string in the planes parallel and normal to the soundboard, at position x and time t, denoted

$Y_s(x,t)$ and $Z_s(x,t)$ respectively, are given by :

$$Y_s(x,t) = \sum_{n=1}^{N_s} q_n^{ys}(t)\phi_n^{ys}(x), \tag{1a}$$

$$Z_s(x,t) = \sum_{n=1}^{N_s} q_n^{zs}(t)\phi_n^{zs}(x) \tag{1b}$$

where $\phi_n^{ys}(x) = \sin(n\pi x/L)$ and $\phi_n^{zs}(x) = \sin(n\pi x/L)$ are the string mode shapes in both planes of polarization, and $q_n^{ys}(t)$ and $q_n^{zs}(t)$ represent their corresponding modal amplitude responses, N_s being the size of each string modal basis. Then, the free response of string s can be formulated as a set of $2N_s$ ordinary second-order differential equations as :

$$m_n^{ys}\,\ddot{q}_n^{ys}(t) + c_n^{ys}\,\dot{q}_n^{ys}(t) + k_n^{ys} q_n^{ys}(t) = \mathcal{F}_n^{ys}(t) \tag{2a}$$

$$m_n^{zs}\,\ddot{q}_n^{zs}(t) + c_n^{zs}\,\dot{q}_n^{zs}(t) + k_n^{zs} q_n^{zs}(t) = \mathcal{F}_n^{zs}(t) \tag{2b}$$

where m_n^{ys}, k_n^{ys}, c_n^{ys}, m_n^{zs}, k_n^{zs} and c_n^{zs} are the modal masses, damping values and stiffnesses for each of the two orthogonal mode families, given by :

$$k_n^{ys} = m_n^{ys}(\omega_n^{ys})^2\,, \qquad c_n^{ys} = 2m_n^{ys}\omega_n^{ys}\zeta_n^{ys} \tag{3a}$$

$$k_n^{zs} = m_n^{zs}(\omega_n^{zs})^2\,, \qquad c_n^{zs} = 2m_n^{zs}\omega_n^{zs}\zeta_n^{zs} \tag{3b}$$

Other modal parameters are the circular modal frequencies ω_n^{ys} and ω_n^{zs} and the modal damping values ζ_n^{ys} and ζ_n^{zs} which account for both internal and acoustical dissipation. The modal forces $\mathcal{F}_n^{ys}(t)$ and $\mathcal{F}_n^{zs}(t)$ are obtained by projecting the external force $f(x,t)$ on the mode shapes of the modal basis, as :

$$\mathcal{F}_n^{ys}(t) = \int_0^L f(x,t)\phi_n^{ys}(x)dx \tag{4a}$$

$$\mathcal{F}_n^{zs}(t) = \int_0^L f(x,t)\phi_n^{zs}(x)dx \tag{4b}$$

The external force field for the strings here includes the effects of the finger/string interaction during the pluck, the body coupling via the bridge, and the stopping fret when the musician presses the string on the fingerboard. The corresponding horizontal and vertical modal forces are thus given respectively by :

$$\mathcal{F}_n^{ys}(t) = F_{Y_s}^b(t)\phi_n^{ys}(x_s^b) + F_{Y_s}^f(t)\phi_n^{ys}(x_s^f) + F_{Y_s}^e(t)\phi_n^{ys}(x_s^E) \tag{5a}$$

$$\mathcal{F}_n^{zs}(t) = F_{Z_s}^b(t)\phi_n^{zs}(x_s^b) + F_{Z_s}^f(t)\phi_n^{zs}(x_s^f) + F_{Z_s}^e(t)\phi_n^{zs}(x_s^E) \tag{5b}$$

where $F_{Y_s}^b(t)$ and $F_{Z_s}^b(t)$ are the horizontal and vertical forces between the string and the bridge, $\phi_n^{ys}(x_s^b)$ and $\phi_n^{zs}(x_s^b)$ being the string modeshapes at the bridge location, $F_{Y_s}^f(t)$ and $F_{Z_s}^f(t)$ are the horizontal and vertical forces between the string and the fret and $\phi_n^{ys}(x_s^f)$ and $\phi_n^{zs}(x_s^f)$ the string modeshapes at the fret on the fingerboard, and finally, $F_{Y_s}^e(t)$ and $F_{Z_s}^e(t)$ are the horizontal and vertical forces between the string and the excitation finger with $\phi_n^{zs}(x_s^e)$ and $\phi_n^{zs}(x_s^e)$ the string modeshapes at the finger excitation. Note that the modal frequency and damping value can be adjusted easily in our modal modelling to allow for small bending stiffness in the string as well as frequency-dependent damping effects.

2.2 Soundboard dynamics

The transverse response of the soundboard to the motion of the bridge can be represented by a simplified modal model :

$$m_n^{SB}\,\ddot{q}_n^{SB}(t) + c_n^{SB}\,\dot{q}_n^{SB}(t) + k_n^{SB} q_n^{SB}(t) = \mathcal{F}_n^{SB}(t) \tag{6}$$

where m_n^{SB}, c_n^{SB} and k_n^{SB} are the soundboard modal parameters, and $q_n^{SB}(t)$ the soundboard modal responses ($n = 1, \ldots, N_{SB}$). As for the strings, the modal forces are obtained by projecting the bridge/soundboard interaction forces on the instrument body modal basis, yielding to :

$$\mathcal{F}_n^{SB}(t) = -F_{b_1}(t)\phi_n^{SB}(x_{b_1}, y_{b_1}) - F_{b_2}(t)\phi_n^{SB}(x_{b_2}, y_{b_2}) \tag{7}$$

where $F_{b_1}(t)$ and $F_{b_2}(t)$ are the vertical forces between the bridge feet and the soundboard, and $\phi_n^{SB}(x_{b_1}, y_{b_1})$ and $\phi_n^{SB}(x_{b_2}, y_{b_2})$ are the modeshapes of the soundboard at the bridge feet contact locations. Note that the bridge motion also introduces two horizontal forces $T_{b_1}(t)$ and $T_{b_2}(t)$ due to friction at the bridge feet (see Figure 1), but that the friction interaction has no contribution to the modal forces.

2.3 Interaction forces

2.3.1 String/finger excitation

The string/finger interaction can be modeled very simply using a spring/dashpot model. The idea is to attach the finger to the string until the instant of release. The plucking action of the player during the sticking phase is therefore expressed in terms of two forces as :

$$F_{Y_s}^e(t) = -K_e\Big[Y_s(x_s^e, t) - Y_s^e(t)\Big] - C_e\Big[\dot{Y}_s(x_s^e, t) - \dot{Y}_s^e(t)\Big] \tag{8a}$$

$$F_{Z_s}^e(t) = -K_e\Big[Z_s(x_s^e, t) - Z_s^e(t)\Big] - C_e\Big[\dot{Z}_s(x_s^e, t) - \dot{Z}_s^e(t)\Big] \tag{8b}$$

where $F_{Y_s}^e(t)$ and $F_{Z_s}^e(t)$ are the forces interactions in the parallel and normal planes with respect to the soundboard, $Y_s(x_s^e, t)$ and $Z_s(x_s^e, t)$ are the string displacements in both directions at the excitation location x_s^e, $\dot{Y}_s(x_s^e, t)$ and $\dot{Z}_s(x_s^e, t)$ being the corresponding velocities, with K_e and C_e the stiffness and damping coupling coefficients between the finger and the strings. In the computations, one or several strings are pulled during 10 ms until reaching an arbitrary position $(y_{s_0}^e, z_{s_0}^e)$ and are then released to vibrate freely by assuming a null excitation forces for time $t > t_s$ where t_s is the time when the strings start slipping on the finger. As seen, the model just described does not represent the actual underlying physics of the finger-string interaction as presented in [13] for the harp. The modeling of the detailed forces occuring during the excitation is far less simple but fortunately, it does not appear necessary to represent these details accurately in order to obtain a satisfactory simulation model for the purposes of this paper.

2.3.2 String/fret interaction

In order to control the playing frequency, the musician presses the strings against the fingerboard with its left-hand fingers, which eventually prevents any string motion at the fret location, and thus shortens the active length of the vibrating string. As for the string/finger coupling, the interaction force exerted by the fret on the string at the fret location x_s^f is modelled by a penalty formulation, using two suitable coupling constants K_f and C_f and imposing a near-zero string displacement at x_s^f, according to :

$$F_{Y_s}^f(t) = -K_f Y_s(x_s^f, t) - C_f \dot{Y}_s(x_s^f, t) \tag{9a}$$

$$F_{Z_s}^f(t) = -K_f Z_s(x_s^f, t) - C_f \dot{Z}_s(x_s^f, t) \tag{9b}$$

where $F_{Y_s}^f(t)$ and $F_{Z_s}^f(t)$ are the forces interactions parallel and normal to the soundboard, $Y_s(x_s^f, t)$ and $Z_s(x_s^f, t)$ are the string displacements in both directions at the fret location, and $\dot{Y}_s(x_s^f, t)$ and $\dot{Z}_s(x_s^f, t)$ are the corresponding velocities.

2.3.3 String/bridge interaction

The coupling between the string and the body of the violin first passes through the string/bridge interaction. Following Inácio *et al.* [14], we introduce a penalty model for this interaction by connecting the string to the bridge through a very stiff spring (with a damper to minimize residual local oscillations). Then the forces exerted by the body on a given string s are given by :

$$F_{Y_s}^b(t) = -K_b\left[Y_s(x_s^b, t) - Y_s^b(t)\right] - C_b\left[\dot{Y}_s(x_s^b, t) - \dot{Y}_s^b(t)\right] \tag{10a}$$

$$F_{Z_s}^b(t) = -K_b\left[Z_s(x_s^b, t) - Z_s^b(t)\right] - C_b\left[\dot{Z}_s(x_s^b, t) - \dot{Z}_s^b(t)\right] \tag{10b}$$

where K_b is the (high) stiffness coupling coefficient between the bridge and the strings, C_b is the damping coupling coefficient, $Y_s(x_s^b, t)$, $Z_s(x_s^b, t)$ and $\dot{Y}_s(x_s^b, t)$ and $\dot{Z}_s(x_s^b, t)$ are the string displacements and velocities at the bridge location x_s^b in both directions respectively. To achieve estimates of the string/bridge interaction, one must now relate the bridge displacements $Y_s^b(t)$, $Z_s^b(t)$ and velocities $\dot{Y}_s^b(t)$, $\dot{Z}_s^b(t)$ to the displacements $Z_{b_1}(t)$ and $Z_{b_2}(t)$ and respective velocities $\dot{Z}_{b_1}(t)$ and $\dot{Z}_{b_2}(t)$ of the bridge feet. To that end, we here use simple kinematical relations instead of considering the bridge dynamics, and adopt the simplifying assumptions that the bridge is rigid, massless and contacts the instrument body at two points. From Figure 1 (lower plot), we obtain the average vertical displacement $\bar{Z}_b(t)$ of the bridge :

$$\bar{Z}_b(t) = \frac{1}{2}[Z_{b_1}(t) + Z_{b_2}(t)] \tag{11}$$

and a linear approximation for the bridge angle $\theta_b(t)$

$$\tan[\theta_b(t)]) \simeq \theta_b(t) \simeq \frac{1}{L_b}[Z_{b_1}(t) - Z_{b_2}(t)] \tag{12}$$

with $L_b = y_{b_2} - y_{b_1}$ the distance between the bridge feet centers (left-side values of y are negative). We will postulate symmetry of the bridge feet, such that $y_{b_1} = -L_b/2$ and $y_{b_2} = L_b/2$. The components of the displacement of the anchoring point of any given string s are then given as :

$$\begin{cases} Y_s^b(t) & \simeq z_s \theta_b(t) \\ Z_s^b(t) & \simeq \bar{Z}_b(t) - y_s \theta_b(t) \end{cases} \tag{13}$$

which can be rewritten in a matrix form :

$$\begin{Bmatrix} Y_s^b(t) \\ Z_s^b(t) \end{Bmatrix} = \begin{pmatrix} \dfrac{z_s}{L_b} & -\dfrac{z_s}{L_b} \\ \dfrac{1}{2} - \dfrac{y_s}{L_b} & \dfrac{1}{2} + \dfrac{y_s}{L_b} \end{pmatrix} \begin{Bmatrix} Z_{b_1}(t) \\ Z_{b_2}(t) \end{Bmatrix} \tag{14}$$

and similarly for the velocities as,

$$\begin{Bmatrix} \dot{Y}_s^b(t) \\ \dot{Z}_s^b(t) \end{Bmatrix} = \begin{pmatrix} \dfrac{z_s}{L_b} & -\dfrac{z_s}{L_b} \\ \dfrac{1}{2} - \dfrac{y_s}{L_b} & \dfrac{1}{2} + \dfrac{y_s}{L_b} \end{pmatrix} \begin{Bmatrix} \dot{Z}_{b_1}(t) \\ \dot{Z}_{b_2}(t) \end{Bmatrix} \tag{15}$$

where the bridge feet displacements and velocities are computed from the modal responses of the soundboard :

$$Z_{b_1}(t) = \sum_{n=1}^{N_{SB}} q_n^{SB}(t)\phi_n^{SB}(x_{B_1}, y_{B_1}), \tag{16a}$$

$$Z_{b_2}(t) = \sum_{n=1}^{N_{SB}} q_n^{SB}(t)\phi_n^{SB}(x_{B_2}, y_{B_2}) \tag{16b}$$

$$\dot{Z}_{b_1}(t) = \sum_{n=1}^{N_{SB}} \dot{q}_n^{SB}(t)\phi_n^{SB}(x_{B_1}, y_{B_1}) \tag{16c}$$

$$\dot{Z}_{b_2}(t) = \sum_{n=1}^{N_{SB}} \dot{q}_n^{SB}(t)\phi_n^{SB}(x_{B_2}, y_{B_2}) \tag{16d}$$

Interestingly, Equation (13) highlights the coupling between the string directions through the transverse motion of the bridge, which is parametrized by the string anchoring points' locations (y_s,z_s). Equation (13) is consistent with the fact that string vibration perpendicular to the soundboard excites only symetrical modes of top plate as one expects. It also expresses that the rockin action of the bridge induces spontaneous energy transfer between the body and the string motions in the two perpendicular polarizations.

2.3.4 Bridge/soundboard interactions

The last element of the formulation is the computation of the forces $F_{b_1}(t)$ and $F_{b_2}(t)$ at the bridge feet which excite the soundboard (see Eq.(7)). These can be obtained from a quasi-static equilibrium of the bridge, assumed rigid and massless. Figure 1 (upper plot) shows the forces exerted by the bridge on the strings and on the soundboard. Obviously, the same forces with opposite sign are exerted by the strings and soundboard on the bridge. The force balances in the y and z directions yield respectively to :

$$T_{b_1}(t) + T_{b_2}(t) - \sum_{s=1}^{S} F_{Y_s}^b(t) = 0 \tag{17a}$$

$$F_{b_1}(t) + F_{b_2}(t) - \sum_{s=1}^{S} F_{Z_s}^b(t) = 0 \tag{17b}$$

and the nil resulting torque with respect to the coordinate origin gives :

$$F_{b_1}(t)\frac{L_b}{2} - F_{b_2}(t)\frac{L_b}{2} - \sum_{s=1}^{S} F_{Y_s}^b(t) z_s + \sum_{s=1}^{S} F_{Z_s}^b(t) y_s = 0 \tag{18}$$

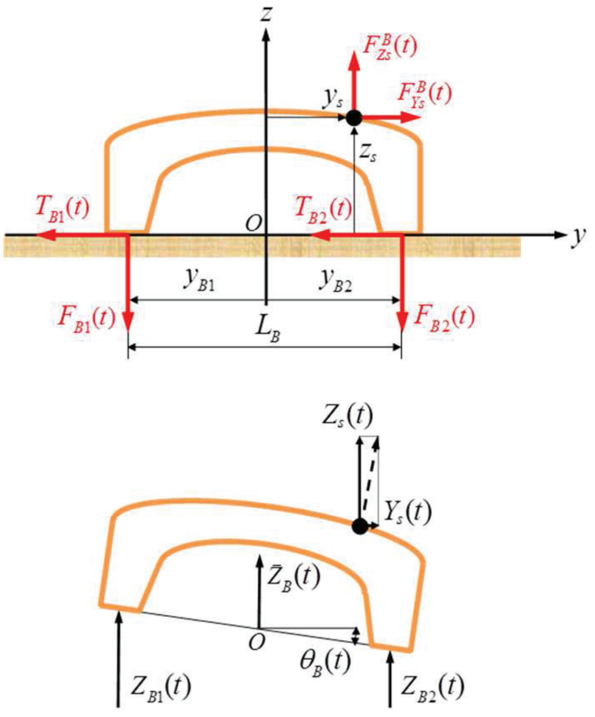

FIGURE 1 – Simplified diagram of a portuguese guitar bridge. Upper plot : coordinate system and forces extered by the bridge on the strings and on the soundboard. Lower plot : displacements of the bridge feet and anchoring points of the strings.

Finally, the bridge feet forces are obtained from (17b) and (18) which results in the following expressions :

$$F_{b_1}(t) = \frac{1}{2}\sum_{s=1}^{S} F^b_{Z_s}(t) + \frac{1}{L_b}\left[\sum_{s=1}^{S} F^b_{Y_s}(t)\, z_s - \sum_{s=1}^{S} F^b_{Z_s}(t) y_s\right] \quad (19a)$$

$$F_{b_2}(t) = \frac{1}{2}\sum_{s=1}^{S} F^b_{Z_s}(t) - \frac{1}{L_b}\left[\sum_{s=1}^{S} F^b_{Y_s}(t)\, z_s - \sum_{s=1}^{S} F^b_{Z_s}(t) y_s\right] \quad (19b)$$

3 Vibrations of the instrument body

A Finite-Element (FE) model for the soundboard of the Portuguese guitar has been carried out using the finite element package CAST3M [15]. The soundboard is a thin plate, made of wood, sometimes having a slighlty curved profile, with a round soundhole. Here, the FE model considers the soundboard as a thin flat plate, thus ignoring the influence of arching on the flexural vibrations of the soundboard, but includes its bracing as well as the orthotropic nature of the wood. The different elastic parameters towards the three principal axes (x is the direction along the

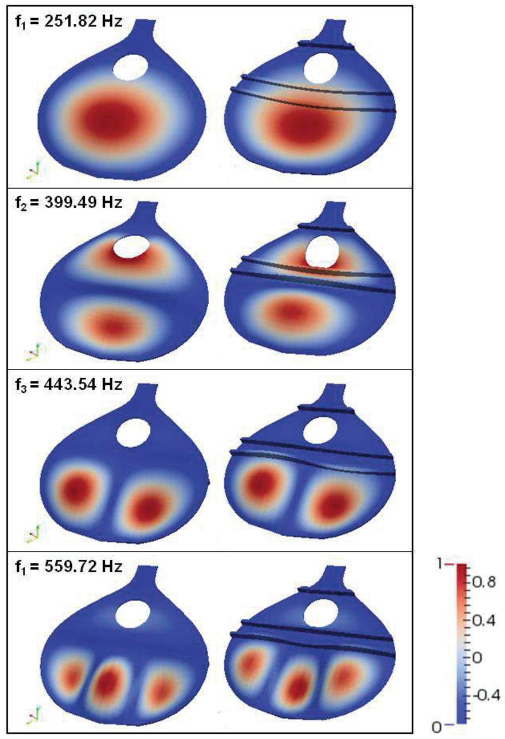

FIGURE 2 – FEM-computed modeshapes for the four lowest modes of the soundboard model.

grain) and density were chosen to correspond to typical values for spruce found in [16]. The mesh consists of 8043 nodes, for 1122 solid elements. The model assumed the soundboard clamped on the outer boundary and eigenvalue computations were performed over the frequency range of 0-5000Hz. The first four computed vibrational patterns and corresponding modal frequencies are presented in Figure 2 where both symetric and asymetric modes relative to the x-axis can be seen.

4 Time-domain simulations

4.1 Computational parameters

The simulations were performed for six courses of double strings as typically found on a Lisbon Portuguese guitar model. The total length of the strings is 0.615 m, with 0.175m from the tailpiece to the bridge. The values for their sounding frequencies once streched, their positions with respect to the origin O (see Figure 1) and corresponding linear density are given in Table 1. The natural frequencies of the string were harmonic and proportional damping is assumed for both the strings and the body, separately, with value of 0.01% and 1% for all modes of each subsystem respectively. The coupling coefficients used were $K_b = K_f = 10^6$ N/m, $K_e = 10^4$ N/m, and $C_b = C_f = 10$ Ns/m with C_e = 30 Ns/m. After recasting the modal equations into first-order form, numerical integration was performed by implementing the analytical integration method [17], an explicit approach well-suited for such problem, assuming a constant acceleration within the time-step. In order to achieve accurate simulations, the modal basis for the strings and soundboard cover the frequency range 0-5000 Hz, and time-domain responses were sampled using a convenient time step of 5.10^{-7} s. At the beginning of the simulations, all displacements and velocities are null. Numerical simulations presented here pertain to open strings, with one string being excited at one-fifth of its acoustical length from the bridge. A number of 607 modes are used.

4.2 Single string allowing for both polarisations

To check whether the model for the coupling of the transverse vibrations of the string is reliable, the first simulation pertains to the sound produced by a single string, plucked in the direction normal to the soundboard. As attested in Figure 3, the string motion trajectory does not strickly remain in the vertical plane, therefore highlighting some energy exchanges between the two perpendicular polarizations of the string via the bridge action. One significant consequence of this interaction, also displayed in Figure 3, is the double decay observed for the soundboard motion, as recorded by a microphone, which indicates the different coupling strength between the soundboard and the two perpendicular motions of the string. Also, as observed for the piano by Weinreich [1], Figure 4 shows that the vertical motion has the faster decay when considering an oblique excitation, which reveals its stronger coupling with the normal motion of the soundboard. Finally, Figure 5 reveals an aspect of string vibrations for instruments such as the violin and the guitar which is rarely addressed in the literature. It is a plot of the standard deviation of the Y and Z components of the string velocity, along the string length, which has been discretized in 60 small elements. Interestingly, it emphasizes some vibratory motion in the *dead* part of the string, between tailpiece and bridge, which surely influences the dynamical response and the tone of the instrument. Not shown here, other extensive simulations were performed to assess the well-behaviour of the synthesis method, by examining the reciprocal theorem for vibration response or the correct rockin action of the bridge when a single asymetric mode is accounted for the soundboard dynamic. In conclusion, although the coupling between the string orthogonal motion provided by the present model is somewhat marginal, the results obtained are consistent with the expected behaviour.

4.3 Simulation of the fully coupled model

For illustration, Figure 6 presents two spectrogams of the soundboard velocity obtained for a vertical pluck, considering first a single string, and then the fully 12-string/bridge/body coupled model. Note the appearance of audible beats in the sound which shows the sympathetic excitation of slightly mistuned harmonics of the different string subsystems, due to their coupling at the bridge.

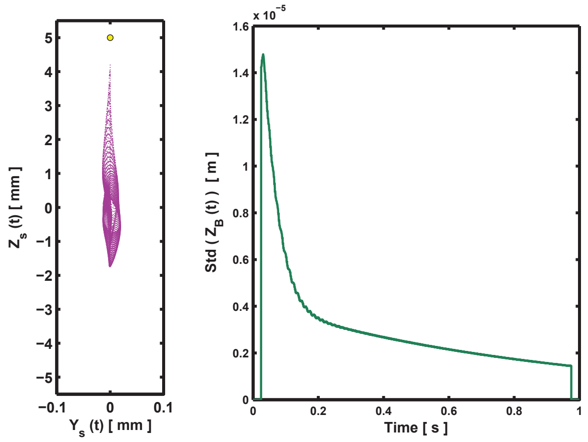

FIGURE 3 – Left : trajectory of the string motion at the excitation location in the $x - y$ plane. Right : time evolution of the standard deviation of the soundboard motion, near the bridge, for a single string.

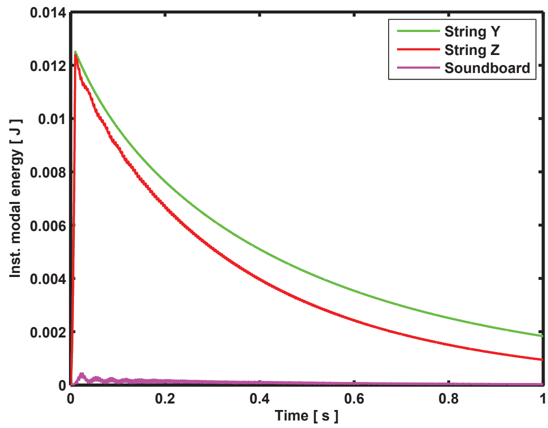

FIGURE 4 – Time evolution of the modal energy of the subsystems for a single string with oblique excitation (plucking angle of 45^o).

5 Conclusion

The paper presents a modal-based computational approach for simulating the Portuguese guitar. It involves the interaction of twelve strings, allows for both polarizations, and includes the dynamics of the top plate of a typical Portuguese guitar. The coupling of all subsystems is ensured through a simple model for the bridge action, based on geometric rationale. The present approach focused on the vibrational aspects of the instrument and still lacks the computation of the radiated sound, as well as the complex geometry of the instrument. These aspects will be treated soon by the authors and other improvement will include the intrinsic dynamics of the bridge.

Acknowledgments

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	1^{st} pair	2^{nd} pair	3^{rd} pair	4^{th} pair	5^{th} pair	6^{th} pair
f_s [Hz]	493.88	440	329.63	493.88	440	293.66
	493.88	440	329.63	246.94	220	143.83
ρ_s [10^{-4} Kg/m]	3.78	3.94	6.20	3.78	3.94	11.30
	3.78	3.94	6.20	14.48	21.22	35.36
(y_s,z_s) [mm]	(-28.6,16.0)	(-19.0,16.0)	(-9.3,16.0)	(0.0,16.0)	(9.5,16.0)	(19.8,16.0)
	(-25.5,16.0)	(-15.6,16.0)	(-6.2,16.0)	(3.1,16.0)	(12.8,16.0)	(23.3,16.0)
N_s	14	15	21	14/28	15/31	23/47

TABLEAU 1 – Sounding frequency, linear density, position and size of the modal basis used for the twelve strings.

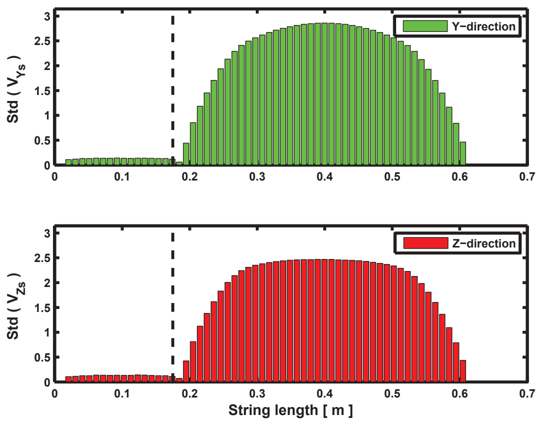

FIGURE 5 – Standard deviation of the string planar (up) and normal (bottom) velocities as a function of the string length for a single string (plucking angle of 45^o). The dot stands for the bridge location.

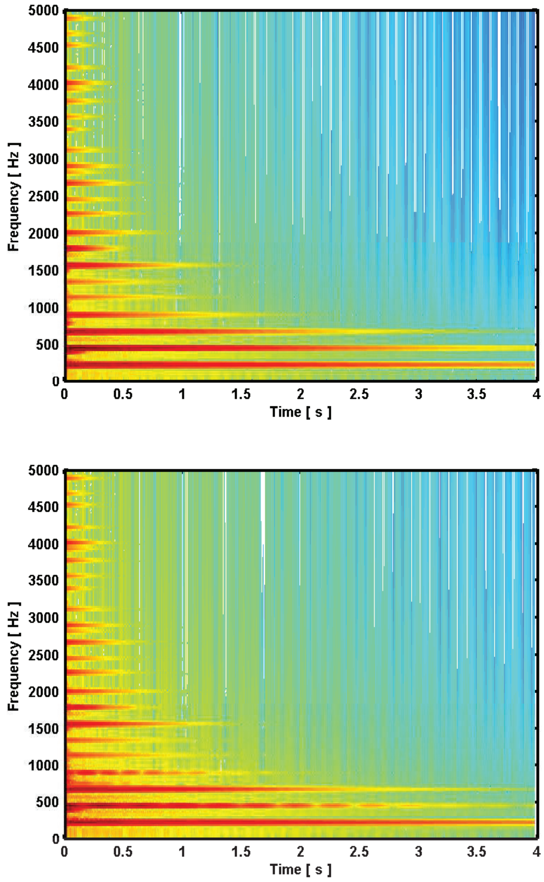

FIGURE 6 – Spectrograms of the soundboard velocity. Up : one string simulation. Bottom : fully coupled model simulation.

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