Les silencieux à baffles parallèles : Un modèle basé sur la double porosité

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Les silencieux à baffles parallèles sont construits en alternant des couches de matériaux poreux et des voies d'air. Ils sont très fréquemment utilisés dans les systèmes de ventilation et de climatisation. La modélisation de ces systèmes est bien connue mais requiert généralement une résolution numérique basée sur la méthode des éléments finis ou le racordement modal.
Ici, une approche analytique basée, sur le modèle de double porosité (DP) [Olny *et al.*, JASA 2003] est proposée dans le but de prédire et de mieux comprendre leur comportement. Grâce au procédé d'homogénéisation, ce milieux stratifié est décrit comme un milieu homogène effectif.
Bien que cette analyse soit limitée au premier mode, elle fournie un cadre théorique original, fait ressortir une séparation claire des différents régimes du silencieux.
De plus, l'existance d'une formule explicite pour le nombre d'onde axial dans le silencieux à permis d'obtenir une expression pour choisir la résistance au passage de l'air qui maximise l'atténuation du silencieux pour une géométrie donnée.

1 Introduction

Baffle/splitter-type silencers (see Fig. 1) are widely used in air conditioning systems in buildings to reduce noise emanating from air-moving devices such as fans. They consist of a periodic succession of parallel baffles, made of porous material (mineral wool) and airways. The analysis of sound propagation through parallel baffle-type silencers is well known and generally carried out with numerical methods such as finite element or mode matching methods [4, 3, 7, 5] (see the references therein).

In this work, an analytical model based on double porosity (DP) media [9] is proposed in order to predict and understand the behavior of finite length parallel baffle silencers. The *meso* scale porosity is due to the air between the baffle and the *micro* porosity is related to the porous material of the baffle.

This approach is grounded on the periodic structures homogenization process. Thanks to this approach, the succession of the airways and the baffles may be considered as an homogeneous effective medium. The size of the porous material micropore must be small in comparison with the baffle width, and this later must remain small enough in comparison with the wavelength.

After a short presentation of the DP model and a comparison with numerical results, a discussion on some design parameters of the parallel baffle silencer is proposed.

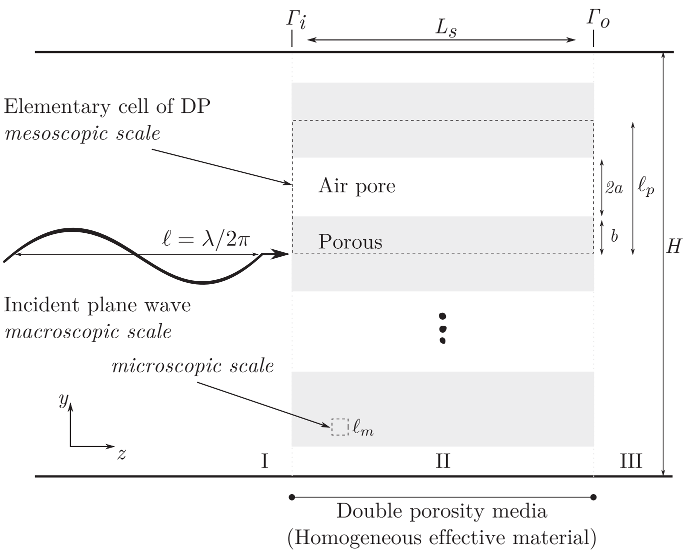


FIGURE 1 – Geometry of the silencer.

2 Double porosity material

The acoustics of double porosity materials has been introduced by the precursory work of Auriault and Boutin *et al.* [2] using a periodic structures homogenization method. The approach has been extended by Olny *et al.* [9] in order to highlight the influence, at the macroscale, of the permeabilities contrast between the macro-pores and the micro-pores arising in sound absorbing materials. The DP models have been used to enhance the normal incidence sound absorption of porous material by Sgard *et al.* [10]. In the following the double porosity material model is briefly recalled in the case of high permeability contrast using the formalism of Refs. [9, 10, 1] for slit perforation. The interested reader may refer to the explanations and the references therein.

Three characteristic lengths are required to describe each scale of the DP material. At macroscopic scale, the characteristic length is governed by the wavelength λ such as $\ell \sim \lambda/2\pi$, at mesoscopic scale ℓ_p is the size of the elementary cell, finally, at the microscopic scale $\ell_m \sim \sqrt{\frac{8\eta}{\sigma\phi}}$ is of the same order as the pores of the porous material.

To ensure the separation of scale and to apply the periodic structures homogenization method, it is required that $\ell_m/\ell_p \ll 1$, $\ell_p/\ell \ll 1$. The validity of the DP model depends on the frequency, the material and the geometry. The subscripts p, m and dp are used for the pores, for the micropores and for the double porosity medium, respectively.

An additional assumption is that the length of the silencer L_s must be larger than the mesoscopic length scale ℓ_p so that the influence of surface Γ_i and Γ_o does not participate in the diffusion mechanism. In this case, no end corrections are needed due to the finite thickness of the material as in Ref. [10, in Eq. (18)].

In this case, Olny et al. [9] have shown that the macroscopic behavior of the acoustic waves is given by the Helmholtz equation (2), with effective wavenumber

$$k_{dp} = \omega\sqrt{\rho_{dp}/K_{dp}}, \qquad (1)$$

involving an effective density ρ_{dp} and an effective bulk modulus K_{dp}. The details of such parameters are listed for air slits of width $2a$ between two layers of porous material of width $2b$ in Appendix B. Results are also available in Ref. [9] for circular holes. The meso scale porosity, corresponds to the open area ratio ϕ_p.

3 Application to baffle silencers

3.1 Transfert matrix method

The bidimensional silencer considered here and sketched in Fig. 1, consists of periodic succession of parallel baffles, made of porous material, and airways. Each side of the silencer is terminated by a semi-infinite duct. It is supposed all the regions have the same cross section H with rigid walls. Let N be the number of baffles, $\phi_p = a/(a+b)$ the open area ratio and its complement $\overline{\phi}_p = 1 - \phi_p$.

In each region, only the plane wave mode is taken into account, thus the acoustic pressure field p_i fulfills the 1D Helmholtz equation ($e^{i\omega t}$) along the z direction

$$\partial_z^2 p_i + k_i^2 p_i = 0, \tag{2}$$

with the wavenumber k_i (i = I, II, III). Note all the complexity present in region II is included through the DP model. The region II is now considered as an homogeneous media, fully defined by its effective density (8) and its wavenumber (1).

At the interface Γ_i and Γ_o the continuity of the pressure and of the velocity must be satisfied. For a simple duct element, the pressure p and the axial velocity v at the input (i) of the silencer is related to the pressure p and the axial velocity v at the output (o) with the transfer matrix formalism [8, Chap. 2]

$$\begin{pmatrix} p \\ v \end{pmatrix}_{\mathrm{i}} = \underbrace{\begin{bmatrix} \cos k_i L_s & \mathrm{i} Z_i \sin k_i L_s \\ (\mathrm{i}/Z_i) \sin k_i L_s & \cos k_i L_s \end{bmatrix}}_{\mathbf{T}_e} \begin{pmatrix} p \\ v \end{pmatrix}_{\mathrm{o}}, \tag{3}$$

where $Z_i = \rho_i c_i$, ρ_i and c_i are the characteristic impedance, the density and the celerity in region i, respectively. If the silencer contains more than one section, the global transfer matrix $\mathbf{T}$ of the silencer is obtained by multiplying all the elementary transfer matrix $\mathbf{T}_e$. For the sake of simplicity, only direct coupling between each region is presented, but more realistic coupling can be obtained, such as expansion, rigid fairing or perforated plate, for the suitable discontinuity or interface matrix must be added at each interface [8, Chap. 2].

For a plane wave excitation in region I, the transmission coefficient and the backward reflexion coefficient are given by $T = 1/M_{11}$ and $R = M_{21}/M_{11}$, respectively, and the transmission loss reads $\mathrm{TL} = -20\log_{10}|T|$. Here the matrix $\mathbf{M} = \mathbf{B}^{-1}\mathbf{T}\,\mathbf{B}$, with

$$\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 1/Z_0 & -1/Z_0 \end{bmatrix}, \tag{4}$$

links the reflected and the incident waves on both side of the silencer.

3.2 Results

The DP model results, presented in Fig. 2, are compared with the quasi-exact solution calculated with a mode matching method (MMM) as described in [4, 3] for a plane wave excitation. The dimension of the silencers are $L = 0.3$ m, $H = 0.2$ m, $\phi_p = 0.5$, $2a = 0.1$ m with 1, 3 and 5 splitters arranged symmetrically with respect to the duct axis.

The comparisons are performed for 2 different wools (see Table 1). The wool GW1, is classically used in HVAC systems and the second is a high resistivity rockwool that was previously used by Sgard *et al.* in Ref. [10].

Table 1 – Material properties used in numerical tests. With the porosity ϕ_m, flow resistivity σ, the tortuosity α_∞, the viscous and thermal characteristic lengths Λ and Λ'.

Material	ϕ_m	σ	α_∞	Λ	Λ'	Ref.
	-	[Nm^{-4}s]	-	[μm]	[μm]	-
GW1	0.954	14 066	1.0	91.2	182.4	[3]
GW2	0.94	135 000	2.1	49	166	[10]

Table 2 – Characteristic frequencies for 1, 3 and 5 baffles with $\phi_p = 0.5$ and $H = 0.2$.

Material	f_b (Hz)	f_d (Hz)		
		1	3	5
GW1	1 760	1 442	12 979	36 053
GW2	8 000	152	1 372	3 812

The high permeability contrast DP model requires that $\omega_d \ll \omega_b$, with the Biot frequency $\omega_b = \sigma\phi_m/(\rho_a\alpha_\infty)$. This assumption is satisfied for the GW2 material with $N \leq 7$, but for the GW1 porous material, the diffusion frequency is of the same order or even bigger than the Biot's frequency (see Table 2). The good agreement between DP and the reference solution, shows that the low permeability contrast DP model can be recovered with high permeability contrast DP model because $F_d \to 1$ when ω_d becomes large.

For both materials, when the number of baffles N increases, the accuracy of the DP model increases. This can be explained because the ratio ℓ_p/ℓ, which limits in practice the convergence of the DP model becomes smaller. When $N = 5$, the differences between the DP and the MMM are indiscernible for both materials.

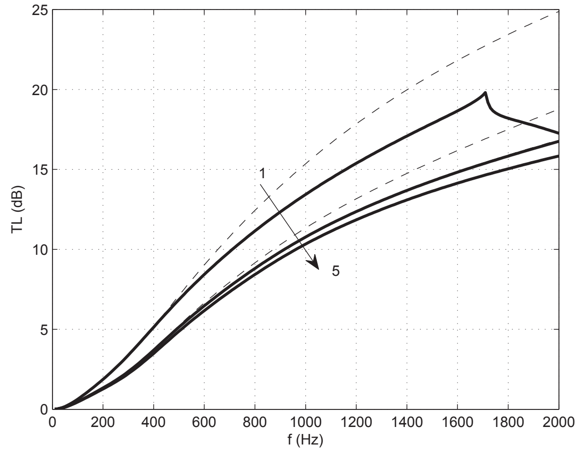
The differences can be slightly stronger around 1700 Hz in the one-baffle case but this can be explained by the cut-off frequency of the first even mode in the rigid duct. For periodic and symmetric baffle position, Mechel [7] has shown that the transmitted modes satisfy the selection rule

$$n = n^{inc} + 2qN. \tag{5}$$

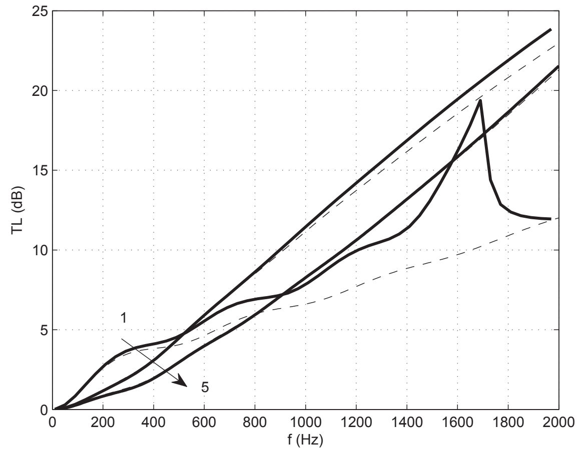
Here, n^{inc} is the order of the incident mode (which is zero here as only the plane wave mode is considered), N the number of baffles and q a relative integer. It follows that, when $N = 1$ only even modes are allowed to propagate. If $N = 3$, only modes of order $n = 6q$ are allowed, the other modes being forbidden. This explain why cut-off are not visible with $N > 1$ on Fig. 2. The selection rules indicates that the plane wave regime can be larger than expected, and that the DP model can also be valid (with this assumptions) until the $2N$ cut-off frequency.

4 Conclusion

This paper illustrates the interest for the homogenization approach to get an effective medium representation of parallel baffles silencers. The effective density and the effective wavenumber of the silencer can be used to get the four-pole parameters associated with the transfer matrix. This approach yields explicit solution for acoustic waves



(a) GW1 wool (σ = 14099 Nm^{-4}s)



(b) GW2 wool (σ = 135000 Nm^{-4}s)

Figure 2 – Comparison between TL prediction given by a reference method [3] (——) and the DP model (– –) for 1, 3 and 5 baffles. a) with the GW1 wool, b) with the GW2 wool with $H = 0.20$ m, $\phi_p = 0.50$ and $L_s = 0.30$ m.

propagation in such systems. This can be useful to derive some explicits expression for the design parameters.

The paper focus on baffle silencers but the extension to expansion chamber filled with a porous material, is possible in the plane wave regime.

A Rigid frame model

Porous materials with rigid skeleton (and quite regular pore shape), such as the porous material involved in this study, are well described by the Johnson-Champoux-Allard (JCA) [1, Chap. 5] equivalent fluid model. This equivalent fluid has the equivalent density and the bulk modulus, ($e^{+i\omega t}$).

$$\rho_m = \frac{\alpha_\infty \rho_a}{\phi}\left[1 - \mathrm{i}\frac{\sigma\phi}{\omega\rho_a\alpha_\infty}G_J(\omega)\right], \tag{6}$$

$$K_m = \frac{\gamma P_0/\phi_m}{\gamma - (\gamma-1)\left[1 - \mathrm{i}\frac{8\eta}{\Lambda'^2 \Pr \omega\rho_a}\left(1 + \mathrm{i}\rho_a\frac{\omega \Pr \Lambda'^2}{16\eta}\right)^{1/2}\right]^{-1}}. \tag{7}$$

Here, $G_J(\omega) = \sqrt{1 + \frac{4\mathrm{i}\alpha_\infty^2 \eta\rho^0\omega}{\sigma^2\Lambda^2\phi_m^2}}$, ϕ_m is the porosity, σ is the flow resistivity, Λ is the viscous length, Λ' is the thermal length, α_∞ is the tortuosity. Moreover, γ is the air specific heat ratio and P_0 is the atmospheric pressure, Pr is the Prandtl number and η is the dynamic viscosity.

B Details of the DP model

The effective density,

$$\rho_{dp} = \frac{\eta}{\mathrm{i}\omega\Pi_{dp}}, \tag{8}$$

is related to the dynamic permeability in the normal direction $\Pi_{dp} = (1-\phi_p)\Pi_m + \Pi_p$, where Π_m can be deduced from (8) and (6) by changing the subscript $dp \rightleftarrows m$, and with dynamic permeability in the meso pore $\Pi_p = \frac{\phi_p}{\mathrm{i}}\delta_v^2 F(\mu_v)$, with the function

$$F(\mu) = \left(1 - \frac{\tanh \mu\sqrt{\mathrm{i}}}{\mu\sqrt{\mathrm{i}}}\right), \tag{9}$$

the viscous boundary layer thickness $\delta_v = \sqrt{\eta/(\rho_0\omega)}$ and the ratio $\mu_v = a/\delta_v$ between the air gap and the viscous boundary layer.

The bulk modulus of the DP material is a combination of the bulk modulus of the porous media K_m (see (7)) and the bulk modulus of the air gapK_p given by the simplified Lafarge's model [6] :

$$K_p = \frac{\gamma P_0/\phi_p}{\gamma - \mathrm{i}(\gamma-1)\frac{\Theta_p}{\delta_t^2\phi_p}}, \tag{10}$$

where

$$\Theta_p = \frac{\phi_p}{\mathrm{i}}\delta_t^2 F(\mu_t) \tag{11}$$

with, $\delta_t = \sqrt{\kappa/(\rho_0 C_p\omega)}$, the thermal boundary layer thickness and the ratio $\mu_t = a/\delta_t$. This yields

$$K_{dp} = \left[\frac{1}{K_p} + (1-\phi_p)\frac{F_d\left(\omega\frac{P_0}{\phi_m K_m}\right)}{K_m}\right]^{-1} \tag{12}$$

with

$$F_d(\omega) = 1 - \mathrm{i}\frac{\omega}{\omega_d}\frac{D(\omega)}{D(0)}, \tag{13}$$

the diffusion frequency $\omega_d = \frac{\bar{\phi}_p P_0}{\sigma\phi D_0}$. For slits, Olny [9] gives $D(0) = (1-\phi_p)b^2/3$ and

$$D(\omega) = -\mathrm{i}(1-\phi_p)\delta_d^2 F(\mu_d). \tag{14}$$

Where the pressure diffusion skin depth $\delta_d = \sqrt{\frac{P_0}{\sigma\phi_m\omega}}$, gives an estimation of the boundary layer where take place strong fluctuation of the micro pore pressure and $\mu_d = b/\delta_d$. In this case the diffusion frequency reads

$$\omega_d = \frac{3P_0}{b^2\sigma\phi_m}. \tag{15}$$

The function F_d relates the mean pressure in the micro pore to the average diffused pressure in the air gap. In the case of slit, the expression of F_d can be simplified into

$$F_d(\omega) = \frac{\tanh \mu_d\sqrt{\mathrm{i}}}{\mu_d\sqrt{\mathrm{i}}}, \tag{16}$$

$\delta_d = b\sqrt{\omega_d/(3\omega)}$.

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