Experimental Characterisation of a small Compression Driver

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The relationship between volume velocity of a small compressor driver in a closed-box baffle and sound pressure in an acoustic space was investigated experimentally. The principal factors influencing sound radiation of a closed-box loudspeaker in a free space is the vibration pattern of the driver and diffraction effects due to the cabinet. A compression driver was tested, its radiation characteristics confirming that it can be seen as a simple volume velocity source at low frequencies. The experimental characterisation of the driver should not depend on its acoustic loading, and a load invariant metrics was established using an internal pressure microphone. The driver volume velocity was estimated using four techniques: the adiabatic gas law in a small compression chamber, laser velocimetry on the axis of the driver, free space pressure response in an anechoic room, and travelling waves in a pipe. The metrics enables experimental estimation of transfer impedances. The loudspeaker was modelled by a vibrating flat circular disk in an otherwise passive and closed box. The sound field was computed by an expansion into multiple simple sources interior to the box. Measured and computed transfer impedances are of similar magnitude and phase.

1 Introduction

When a driver’s diaphragm is small compared to the wavelength and if its entire surface moves in phase, the driver can be considered as a simple source [1]. Acoustic response to a simple source can be expressed by a transfer impedance \( Z \) of the driver in \( s \) to sound pressure \( p \) in \( f \):

\[
Z(f|s) = \frac{p(f|s)}{Q(s)}
\]  

(1)

A driver becomes inefficient when the wavelength increases as the radiation resistance decreases, whereas at higher frequencies the driver either breaks up or develops pronounced directivity and thereby ceases to be a simple source [1]. A limited frequency range of an acoustic source can be extended by subsequent measurements using increasingly smaller drivers.

Transfer impedance(s) can be computed numerically, e.g. by finite element method. An alternative method is by superposition of sound fields from substitute sources representing the original vibrating body. There are a few benefits of working with substitute sources compared to the finite element method. The discretisation of a volume is conveniently replaced by the discretisation of an enclosing surface. Using the source substitute method the reconstruction error of normal velocity on the enclosing surface is known and the solution of the wave equation is exact.

The driver characterisation and radiation model hereby described has been developed as part of research on airborne characterisation of vibrating bodies. The developed characterisation method assumes that sound radiation by a complex source can be represented by sound field superposition of a limited number of small radiators set in a rigid closed baffle of similar shape and volume. Sound in a listening position \( f \) by a source with an enclosing surface \( S \) and velocity \( v(s) \) is [2]:

\[
p(f|s) = \int_S Z(f|s) v(s) \cdot n \; dS.
\]  

(2)

Here \( n \) is the outward normal to the surface and \( s \) is a point on the surface.

2 Driver characterisation

Salava [3] has described how acoustic transfer impedances can be measured in practice. A sensing transducer proportional to volume velocity is needed. If one assumes that the volume of air inside of the driver’s back cavity is tightly closed and if its enclosure is small and rigid, the compression and expansion of the air will make the pressure \( p \) inside the enclosure become proportional to volume velocity \( Q \) of the driver \( p \propto Q \). Anthony and Elliott have compared two concepts of implementing known volume velocity sources: an implementation of Salava’s source using two identical drivers put together and an implementation using an internal pressure microphone in the driver enclosure [4]. They used laser velocimetry as a reference measurement of volume velocity.

In this study a known volume velocity source was implemented using a commodity driver and a pressure microphone embedded in the driver’s enclosure. The properties of the driver enclosure being unknown, the proportionality between internal pressure and volume velocity had to be obtained experimentally. The modified driver is shown in Figure 1.

Using internal pressure as a reference of volume velocity the transfer impedance in Equation 1 can be split into two frequency response functions; a so called source function \( \Psi \) which relates internal pressure \( p(i) \) to velocity \( Q \) and a room function \( \Omega \) which relates external pressure \( p(f) \) to internal pressure \( p(i) \). The transfer impedance is

\[
Z(f|s) = \frac{p(i|s) p(f|s)}{Q(s | p(i))} = \Psi \Omega.
\]  

(3)

The centre of the internal microphone is denoted by \( i \). The source function is measured in a space where the volume velocity can be assessed:

1. in a small, front-added, compression chamber assuming an adiabatic process,
2. in a room assuming a rigid diaphragm,
3. in a room assuming a known transfer impedance,
4. inside of a pipe assuming plane waves.

2.1 Assuming an adiabatic process inside of a compression chamber

Consider a volume of an ideal gas inside of a chamber enclosed by an impenetrable surface. Let the volume at rest \( V_e \) be compressed and expanded by an oscillating source which constitutes one part of the enclosing surface. Inside of the chamber density and pressure are related by

\[
\left( \frac{p}{p_e} \right) = \left( \frac{\rho}{\rho_e} \right)^\gamma,
\]

where the total pressure \( p = p_e + p_r \) is given by the sum of acoustic pressure \( p_e \) and static pressure \( p_r \) [1]. The density is denoted by \( \rho \), the density at rest \( \rho_e \) and the ratio of specific heats is denoted by \( \gamma \). The mass inside the chamber is conserved. The volume velocity is related to the change of volume \( \Delta V = V - V_e \) by \( Q = \frac{\rho}{\rho_e} \Delta V \). If the excess pressure inside of the chamber is considered much smaller than the equilibrium pressure the volume velocity is:

\[
Q = -j\omega \frac{V_s}{\gamma p_s} p_e.
\]

The sound pressure is proportional to volume velocity inside of the compression chamber. The imaginary unit is denoted \( j \). The speed of sound in an ideal gas is \( c^2 = \frac{\gamma p_s}{\rho_e} \) [5]. Substituting \( \gamma p_s \) with \( p_s c^2 \) yields the result of Anthony and Elliot [4]. The source function is

\[
\Psi = -\frac{1}{j\omega} \frac{\gamma p_s}{V_s} \frac{p_{ref}}{p_e}.
\]

The pressure inside of the back enclosure is denoted \( p_{ref} \). It is assumed that the ratio of specific heats \( \gamma \) is 1.4 and the equilibrium pressure \( p_s \) is 101.3 kPa. Apart from the sound pressure, the volume at rest \( V_e \) has to be measured. The upper frequency limit of the technique is reached when the size of the chamber approaches half the wavelength which causes the pressure to be unevenly spatially distributed.

2.2 Assuming a rigid diaphragm

Let the driver be flush-mounted in a flat baffle with its rigid diaphragm oscillating perpendicularly to the baffle. The vibrating surface projected onto the baffle corresponds to a circle of radius \( a \). The volume velocity is

\[
Q = \pi a^2 \nu,
\]

and the source function can be estimated from

\[
\Psi = -\frac{1}{\pi a^2} \frac{p_{ref}}{\nu}
\]

using a laser doppler vibrometer [4]. If the diaphragm is a convex dome it becomes difficult to measure the normal velocity at multiple points. The velocity was measured in the middle of the diaphragm. Such a point estimate is questionable; the vibration of a dome shaped diaphragm is complex in nature. The vibration of different zones of the diaphragm might be of different amplitudes, and once the surface breaks up they might be of opposite phase. Even below the breakup frequency the velocity at the rim is far lower than at the centre of the diaphragm. The measurement using laser velocimetry required further modifications of the driver. The diaphragm was found to be non optical reflective, and the pickup point had to be treated with a small piece of optical reflective tape. A prior measurement with reflective spray failed. The setup is shown in Figure 2.

\[ ZQ = p. \]

where the scalar volume velocity \( Q \) has to be fitted to the measured data by a least squares approach. The column vector of transfer impedances is denoted \( Z \) and the column vector of pressure response by \( p \). The volume velocity is \( Q = Z^* p \), where the superscript + denotes a pseudo-inverse.

Introducing internal pressure on both sides, the source function is

\[
\Psi = \frac{1}{Z^* y}.
\]

where \( y \) is the column vector of response terms and each term is \( y = \frac{Z^* y}{p_{ref}} \).

Choice of transfer impedance Measurements were done in an anechoic room using a finite sized flat rectangular baffle. Measurements directly in a free space would have been possible, but the back enclosure of the compression driver vibrates and radiates unwanted sound. It was considered that the sound radiation from the diaphragm and from the back enclosure can be separated by the use of a baffle. An approximation of the setup would be a rigid piston set in an infinite baffle. The pressure amplitude along the axis \( z \) for a unit volume velocity is [1]:

\[
Z = \frac{-DC}{\pi a^2} \left( e^{-jk \sqrt{\omega^2 - \omega^2}} - e^{-jk z} \right).
\]

This model does not account for specific features of the real space e.g. the convex dome shaped diaphragm or diffraction from the finite sized baffle. Ultimately, acoustic transfer
impedances accounting for additional phenomena can be found by numerical simulation. However, the radiation characterisation assumes that the driver can be approximated by a simple source, at least in the far-field, and therefore one can continue and exploit simple analytical results to deduce the volume velocity. The setup is shown in Figure 3.

2.4 Assuming travelling plane waves in a pipe

Sound propagating in a cylinder of air with a radius \( a \) and length \( l \) inside of a rigid pipe can be idealised as forth and back traveling plane waves [1]. The pressure fluctuation inside the pipe is caused by the driver at the termination \( x = l \). The pressure amplitude at a cross-section \( x \) \((0 \leq x \leq l)\) is

\[
p = A_x e^{-jkx} + A_- e^{jkx}, \tag{12}
\]

and the particle velocity amplitude is

\[
u = \frac{1}{\rho c} (A_x e^{-jkx} - A_- e^{jkx}). \tag{13}
\]

The subscript \( \pm \) denotes forth going waves, in the direction of the x-axis and taken to be towards the piston, and \( - \) back going waves, in the opposite direction. For a circular duct of radius \( a \), the plane wave assumption is valid away from discontinuities, at frequencies satisfying \( ka < 1.8 \) which is the cutoff value of higher propagation modes [5].

2.4.1 Wave separation

If the driver is modeled as a rigid piston fitting snugly into the pipe, the velocity has to be continuous \( v = u \) at the interface between the surface of the piston and the air inside of the chamber. It follows that the volume velocity is

\[
Q = -\pi a^2 u \bigg|_{x=l}. \tag{14}
\]

The unknown particle velocity amplitude cannot be measured at the interface of the piston. An approach is to estimate the wave amplitudes \( A_+ \). \( A_- \) from the pressure response inside of the pipe. The separation requires two simultaneous pressures to be measured. The estimate of wave amplitudes might be more robust if an array of more than two microphones are used. Assuming that the pressures in \( x_1, x_2, \ldots \), and \( x_N \) are known one has

\[
\begin{bmatrix}
    e^{-jx_1} & e^{jx_1} \\
    e^{-jx_2} & e^{jx_2} \\
    \vdots  & \vdots  \\
    e^{-jx_N} & e^{jx_N}
\end{bmatrix}
\begin{bmatrix}
    A_+ \\
    A_-
\end{bmatrix}
= \begin{bmatrix}
    p_1 \\
    p_2 \\
    \vdots \\
    p_N
\end{bmatrix}. \tag{15}
\]

which can be written as \( \mathbf{TA} = \mathbf{p} \). The equation can be scaled by one over the reference pressure inside the enclosure. In this case is the source term given by

\[
\Psi = -\frac{\rho c}{\pi a^2} \frac{1}{A_+ e^{jkl} - A_- e^{jkl}} \tag{16}
\]

where the wave amplitudes are estimated for a unit pressure inside of the driver enclosure.

2.4.2 Blocked pressure

Let the pipe termination at \( x = 0 \) be sealed by a rigid surface with a flush mounted microphone embedded in it. The driver remains at \( x = l \) and the assumption of a flat circular rigid piston fitting snugly into the pipe is not made. The overtones of a closed pipe are given by \( f_n = n f_0 = n \frac{c}{2l} \) where \( n \) is a positive integer. At frequencies equal to \( f_{n-0.5} \leq (n - 0.5) f_0 \), at the antiresonances, the complex relationship between volume velocity \( Q \) at the speaker and the blocked pressure \( p_b \) at the termination is simplified to

\[
Q = j(-1)^{n-1} \frac{\pi a^2}{\rho c} p_b. \tag{17}
\]

The source term for a blocked pressure is then given by

\[
\Psi(\rho_{n-0.5}) = -j(-1)^{n+1} \frac{\pi c}{\pi a^2} \frac{Pr_{\text{ref}}}{p_b}. \tag{18}
\]

2.5 Comparison of source functions

A small volume inside of a rigid enclosure, the driver’s back cavity, can be understood as an acoustic filter [1]. Since the distance between the internal microphone and the diaphragm is small compared to the wavelength, \( |s - l| \ll \lambda \), the pressure at the acoustic centre of the driver is \( p(s) \approx p(i) \).

The source function takes the form of an acoustic impedance

\[
\Psi = \frac{p(s)}{Q(s)} = R + jX. \tag{19}
\]

The estimated source function contains noise. To reduce noise the measured source function \( \tilde{\Psi} \) was fitted to a polynomial

\[
j\omega \tilde{\Psi} = \xi_0 + \xi_1 (j\omega)^1 + \xi_2 (j\omega)^2 + \ldots. \tag{20}
\]

using a least squares approach. A second order polynomial was fitted in the frequency range 100 Hz to 1000 Hz. The polynomial model was then used to obtain transfer impedances. The fitting technique is useful when using blocked pressure in a pipe, which only estimates volume velocity at a few frequencies. Estimated source functions are shown in Figure 7.

Results are not shown for wave separation based on two microphones: it is the only one technique out of the five discussed that was found not suitable. The separation is sensitive to the exact position of the source plane, which is uncertain in view of the dome shape of the diaphragm. The assumption of a rigid piston fitting snugly into the pipe is also questionable in itself.

A fair comparison between the estimation techniques would require that the experiments were done in a controlled environment. However, the series of experiments were done using different equipment, at different times, and in different geographical locations. E.g. the measurement in an anechoic
Figure 4: Estimated (a) real part and (b) imaginary part of the source function, legend: c.c. compression chamber, l.v. laser velocimetry, a.c. anechoic chamber, b.p.p. blocked pressure in a pipe

room was done in Sweden whereas the measurement using blocked pressure in a pipe was done about one year after the other ones.

Despite mentioned inconveniences, the measurements tend to agree in the imaginary part of the source function. The frequency behaviour of the imaginary part is dominated by a compliance law. However, the transformation to a lumped element filter is not straightforward. The real part which should have theoretically been zero is frequency dependent, and disperses more between the measurements. The deviation from zero real part may be a consequence of damping treatments inside of the driver enclosure. The behaviour of real part should be studied in more depth.

3 Radiation model

Let the radiating object be a box of dimensions $l_1$, $l_2$, and $l_3$. The geometric centre $x$ coincides with the origin of coordinates, thus $x = (0, 0, 0)$, and the coordinate system is aligned with the edges of the box. On the otherwise passive surface $S$ is placed at one face a vibrating disk of radius $a$ centred in a point $s$. It is assumed that the sound pressure is of small amplitude, that the vibration is sinusoidal in time, and that sound propagation takes place in a lossless homogeneous medium. Pressure amplitude $p_m$ and radial particle velocity amplitude $u_m$ at a reception point $f$ due to a simple source at a point $m$ are given by

$$p_m = j k p_c \frac{Q_m e^{-jkr}}{4\pi r}, \quad u_m = \frac{Q_m (1 + j kr)}{4\pi r^2}, \quad (21)$$

where $Q_m$ is the volume velocity and the subscript $m$ denotes a simple source [1]. It is assumed that the speed of sound $c$ is $343$ m$^{-1}$s and the density of air $\rho$ is $1.2$ kg m$^{-3}$. The wavenumber is given by $k = 2\pi/\lambda$ where $\lambda$ is the wavelength. The distance between source and reception points is denoted by $r = ||r - m||$ where $r = f - m$. Assuming that the box has a rigid surface, the particle velocity field created by the substitute sources $u$ has to reproduce the normal component of surface vibration $v$ of the original source $[5]$. This is the basic assumption of the synthesis, which together with the Helmholtz equation enables the computation of sound $[6–8]$.

Substitute sources are found by a greedy search algorithm $[7]$ where the best simple source positions are selected from a group of $K$ prescribed positions located at candidate points $m_1, \ldots, m_K$ inside of the enclosing surface. The best fit corresponds to the candidates producing the sound field which most closely matches the prescribed velocity at the control points on the body. The search of substitute sources is done by iteration where a new source is selected in each step.

The normal velocity is represented by a vector $v_\perp$ $(N \times 1)$, where each element is given by $v_{\perp m} = v \cdot n$, prescribed at control points $b_1, \ldots, b_N$ on the enclosing surface. The subscript $\perp$ indicates motion perpendicular to the surface. In order to find the volume velocity distribution $Q$ $(M \times 1)$ which gives the best matching at the end of the $M^{th}$ iteration step all selected substitute sources are tuned to best reproduce the prescribed normal velocity. This is done by finding the solution to

$$T Q = v_\perp$$

by a least squares approach. Each element in the transfer matrix $T$ $(N \times M)$ corresponds to the outward pointing normal component of particle velocity at $b_m$ contributed by a simple source of a unit volume velocity at $m_m$. An element of $T$ reads:

$$T_{nm} = \frac{1}{4\pi} (1 + j kr_{nm}) e^{-jkr_{nm}} \cos \gamma_{nm} \quad (22)$$

The radius vector is given by $r_{nm} = b_m - m_n$ and $\gamma_{nm}$ is the angle between the normal vector $n$ and the radius vector. The residual vector $\Delta u_\perp = u_\perp - v_\perp$ is the difference between prescribed and obtained $u_\perp = TT^*v_\perp$ normal component of velocity. The superscript $+$ denotes a pseudo-inverse. A velocity error is defined from the residual vector as a pseudo-inverse.

$$e_u = \sqrt{\Delta u_\perp^* \Delta u_\perp \frac{1}{v_\perp^* v_\perp}}$$

The asterisk denotes a conjugate transpose. Once suitable positions of a group of $M$ simple sources and their volume velocities are known the sound pressure amplitude at any reception point $f$ can be obtained by

$$p(f) = \sum_{m=1}^{M} j k p_c \frac{Q_m e^{-jkr_m}}{4\pi r_m} \quad (25)$$
where the radius vector is \( \mathbf{r}_m = f - \mathbf{m}_m \). The described method uses a discrete representation of the box, and prescribed and reconstructed normal velocities are defined in control points across the enclosing surface. A discrete model of a box is shown in Figure 5.

3.1 Boundary value on driver

A simple source is characterised by the volume velocity it produces. Volume velocity of a driver is obtained by integration of normal component of velocity over its entire vibrating surface \( D \) [1]:

\[
Q(s) = \int_D \mathbf{v} \cdot \mathbf{n} \, dD. \tag{26}
\]

Let the vibrating surface be a circle of radius \( a \) in radially symmetric motion. The velocity profile on the disk is \( \mathbf{v} \cdot \mathbf{n} = \zeta_1(\mathbf{r}) \) and depends only on the distance \( \mathbf{r} \) from a point \( \mathbf{b} \) on the disk to the center of the disk at \( \mathbf{s} \), \( \mathbf{b} = \mathbf{s} - \mathbf{n} \). Greenspan considered profiles on the form

\[
\zeta_1(\mathbf{r}) = \frac{1}{\pi a^2 (n + 1)} \left( 1 - \frac{\mathbf{r}^2}{a^2} \right)^n H(a - \mathbf{r}), \tag{27}
\]

which produce unit volume velocity [9]. The Heaviside step function is denoted by \( H \). The profile order \( n \) is an integer. The zeroth order corresponds to a uniform profile \( \zeta_1(\mathbf{r}) = a \) where the velocity constant is \( a = \frac{1}{\pi a^2} \). The first order corresponds to the simplest case of a simply supported disk and the second order to the simplest case of a clamped-edge disk [10].

3.2 Boundary value on cabinet

Motion of a real driver can cause vibration on the cabinet which in turn will radiate sound. This phenomenon is neglected. The cabinet is assumed to act only as an obstacle on the propagating sound. The normal velocity and pressure are continuous at an interface between an ideal fluid without viscosity and a body and for a rigid surface the normal velocity should vanish \( \mathbf{v} \cdot \mathbf{n} = 0 \) [5].

3.3 Boundary value on the floor

If the floor is assumed to be an infinite rigid plane such that the normal component of velocity vanishes \( \mathbf{v} \cdot \mathbf{n} = 0 \), then the reflection can be taken into account by considering the floor as an acoustic mirror [5]: the original boundary problem is replaced by one with the original box and its image in an infinite space. Let the box be suspended at a height \( h \geq 0 \) such that its centre is given by \( \mathbf{x} = (0, 0, h + 0.5l_z) \). Furthermore let the origin of coordinates lay on the floor. An image source is obtained if all points \( (b_1, b_2, b_3) \) on the surface of the original source are mirrored such that corresponding points are given by \( (b_1, b_2, -b_3) \). In the same manner the prescribed velocity at a point on the original surface \( (v_1, v_2, v_3) \) is changed on the mirror surface to \( (v_1, v_2, -v_3) \).

4 Transfer impedance

Measurements were done in an engine test cell at INSA de Lyon. The room is not an ideal semi-anechoic room. It contains multiple objects, such as engines and brakes, and ventilation installation which affects its performance. All frequencies below 100 Hz are omitted because the sound pressure level was too low with respect to the background noise. All frequencies above 1000 Hz are rejected because the driver is too large to act as a simple source. The setup is shown in Figure 6.

The driver was mounted at \((10, -116, 100)\) mm as seen from the centre of the box. The box of dimensions \(300 \times 232 \times 500 \) mm was suspended at a height of \(200\) mm above the floor. A transfer impedance taken \(165\) mm away on the axis of the driver is shown in Figure 7. Below \(500\) Hz the difference between the measured and computed sound pressure level is less than \(1.5\) dB, and the measured transfer impedance fluctuates around the predicted value. Around \(800\) Hz and \(1000\) Hz there are dips where the difference in level is about \(5\) dB, believed to come from the imperfect room.

Measured transfer impedances are uncertain. The uncertainty cannot be quantified easily since the true volume velocity is not easy to measure. Despite shortcomings measured and computed transfer impedances look similar.

5 Conclusions

To measure acoustic transfer impedances of a small driver, acting as a simple source, its volume velocity has to be known. Using a pressure microphone embedded in the driver the transfer impedance can be split into a source function and a room function. The source function relates internal pressure in the driver enclosure to volume
velocity. Five techniques to estimate the source function were considered, these were: a small compression chamber, laser velocimetry, response in an anechoic room, pipe wave separation, and pipe antiresonance technique.

The use of a compression chamber or a blocked pressure in a pipe are recommended being insensitive to the shape of the driver’s vibrating surface. The latter gives results only at discrete frequencies thus needing interpolation. The pipe wave separation or laser velocimetry assumes the membrane to be a rigid surface which contradicts reality. Data obtained in an anechoic room using a flat baffle were more noisy than the data obtained by other techniques.

The imaginary part of the measured source function follows a compliance law, and similar results were obtained using the different estimation techniques. The real part of the source function, theoretically equal to zero, was found to be more uncertain. Despite the uncertainty, a small commodity driver can be used for measuring acoustic transfer impedances.

The final validation of the prediction of sound radiation using the driver’s impedance, obtained by different characterisation methods, was done by modelling the sound created by a rigid box with the driver mounted in it. In spite of inevitable differences between the predicted and measured sound pressure, the matching between the two was found to be fairly consistent. The small driver was found to be suited for the measurement of acoustical transfer functions.

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