Étude expérimentale de l'émission acoustique par de petits impacteurs sur un solide élastique

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Three complementary methods are presented to estimate the energy of elastic waves generated by the impact of a small object onto a thin plate. The first two methods are based on the analysis of the direct arrival. To separate this path with the reflected ones, the plate has to be sufficiently large. In contrast, the last method is well adapted for a rather small and weakly dissipative plate where the elastic field reaches a diffuse regime. These three methods have been experimentally validated with steel beads onto a glass plate. We show that all of them provides a correct estimation of the energy. These methods are well adapted for an in depth study of the elastic and inelastic energy transfer when an object bounces off a surface.

1 Introduction

When a small object bounces off a surface, a force localized in time and space acts onto the interface and an elastic wave is radiated outward from the point of application [e.g. *Lamb*, 1904]. The quantification of the total energy transmitted to the structure is an important problem in various fields such as vibroacoustics and shielding. Numerous laboratory experiments have been conducted to better understand the energy transfer processes that are involved during an impact. In case of simple normal impact, the coefficient of restitution which is the ratio of the energy after and before the impact can be precisely estimated from the rebound height [e.g. *Hunter*, 1957 ; *Falcon et al.*, 1998 ; *McLaskey and Glaser*, 2010]. The energy that is lost is due to three main processes : elastic radiation into the structure, plastic deformation of the bead and the local viscoelastic dissipation around the contact [*Falcon et al.*, 1998]. Viscoelastic dissipation is inherent to the bead and structure material [*Falcon et al.*, 1998] and plastic deformation is significant when the bead initial momentum is greater than a threshold value [*Davies*, 1949]. The interaction process between the impactor and the surface is very complex because it depends on numerous parameters (velocities, roughness, materials, ...). It is important to estimate the part of the energy that is transferred in the form of elastic waves independently of the two other processes because it represents the acoustic signature of the impact.

This elastic energy W is the time integral of the instantaneous power transmitted to the structure, i.e., the scalar product of the instantaneous values of the force $\mathbf{F}(\mathbf{r}_0, t)$ and surface vibration velocity $\mathbf{v}(\mathbf{r}_0, t)$ at the position of force application $\mathbf{r}_0$ [e.g., *Royer and Dieulesaint*, 2000] :

$$W = \int_{-\infty}^{+\infty} \mathbf{F}(r_0, t).\mathbf{v}(r_0, t)\mathrm{d}t. \tag{1}$$

Hunter [1957] thus estimated the elastic energy W radiated in a semi-infinite elastic block using a force expression derived from Hertz's theory [see *Johnson*, 1985] and predicted that the elastic energy W in these conditions is smaller than 5% of the impactor cinematic energy. *Hutchings* [1979] extended *Hunter*'s [1957] calculation to the case of plastic (inelastic) impacts of spherical beads on thick blocks using an asymmetric profile for the impact force and reported elastic energy losses of a few percent of $\frac{1}{2}mv_{impact}^2$.

Hunter's [1957] study is based on a force modeled by Hertz's theory of elastic impact. However, this assumption is not always verified for complex impact geometry (e.g., in seismology or impact science). This is why, in this paper, we estimate the elastic energy transmitted into a structure using the generated vibration, measured at a given location of the surface, without an a priori knowledge of the impact force.

We introduce three different methods to estimate the elastic energy produced in a thin plate. The two first methods require that the plate is so large that the direct wave between the source and the sensor can be clearly separated from its reflections on the structure borders. In contrast, the third method is based on a diffuse field assumption.

2 Elastic energy estimation in a thin plate

A force $\mathbf{F}(t) = -F_z\mathbf{u}_z$ is applied normally at a given position $(x, y, 0)$ over the surface ($z = 0$) of a plate (Figure 1). The emitted elastic wave propagates radially from the source location (direction $\mathbf{u_r}$, Figure 1). Because only thin plates are considered, the fundamental antisymmetric mode is mainly excited (A_0 of Lamb) [e.g. *Royer and Dieulesaint*, 2000]. This implies that the vibration is mainly normal to the plate surface (direction $\mathbf{u_z}$, Figure 1) and is constant within the plate thickness [*Royer and Dieulesaint*, 2000]. In the following, the radial displacement $u_r(r, t)$ is therefore assumed negligible and only the normal displacement $u_z(r, t)$ is considered.

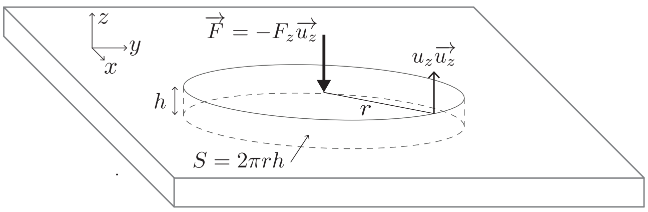

Figure 1 – Scheme of the plate of thickness h, characterized by the Cartesian coordinates x, y, z. Coordinate $z = 0$ corresponds to the free surface of the plate. When a transient normal force $-F_z\mathbf{u}_z$ excites the plate at the origin $(0, 0, 0)$, A_0 Lamb wave is emitted radially and generates a mainly normal displacement field $\mathbf{u}_z$. Surface S is a cylinder of radius r and thickness h that surrounds the impact position.

Mode A_0 is highly dispersive at low frequency, when the wavelength is much larger than the plate thickness h. In the limit $kh << 1$ (where k is the wave number), the approximate relation of dispersion is :

$$\omega = \frac{v_p}{2\sqrt{3}}k^2h. \tag{2}$$

The plate velocity v_p is defined by $v_p = \sqrt{E/(\rho(1 - \nu^2))}$, where ρ, E and ν are the density, Young's modulus and Poisson's ratio of the plate constitutive material, respectively. We can easily show that the group velocity v_g is given by

$$v_g(\omega) = \frac{v_p}{\sqrt{3}}kh. \tag{3}$$

In contrast, at higher frequencies, when the wavelength becomes smaller than the plate thickness (i.e, $kh > 1$), the relation (2) is not valid anymore and the group velocity v_g of A_0 mode tends towards the frequency independent Rayleigh waves speed [*Royer and Dieulesaint*, 2000].

2.1 Energy flux method

The first method to estimate the elastic energy is based on the energy flux conservation. Indeed, one can show that the power $\tilde{P}(\omega)$ at pulsation ω that goes across section S is equal to the energy density flux [*Royer and Dieulesaint*, 2000] :

$$\tilde{P}(\omega) = v_g(\omega) \iint_S \rho |\tilde{V}_z(r,\omega)|^2 \mathrm{d}S \tag{4}$$

According to the Parceval theorem, the elastic energy W radiated within the plate (equation (1) is equivalent to the integral of $\tilde{P}(\omega)$ over the pulsations ω over 2π :

$$W = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[v_g(\omega) \iint_S \rho |\tilde{V}_z(r,\omega)|^2 r \mathrm{d}\theta \mathrm{d}z \right] \mathrm{d}\omega. \tag{5}$$

Because of the radial symmetry, the radiated elastic energy is then simply given by :

$$W = \int_0^{+\infty} 2rh\rho v_g(\omega) |\tilde{V}_z(r,\omega)|^2 \exp(\gamma(\omega) r) \mathrm{d}\omega, \tag{6}$$

Here we have added a factor $\exp(-\gamma(\omega)r)$ to compensate the viscoelastic dissipation of energy with distance r. This correction is valid when the dissipation length $\gamma(\omega)^{-1}$ is large compared to the wavelength.

2.2 Deconvolution method

Contrary to the energy flux method, here we compute the energy from the estimation of the time dependence of the force of the impact F and the normal displacement. Indeed, when the emitted waves train is not reflected on the plate lateral sides, the power $\tilde{P}(\omega)$ transferred into a plate at the point of application of a normal force $\tilde{F}_z(\omega)$ is [*Goyder and White*, 1980] :

$$\tilde{P}(\omega) = \frac{|\tilde{F}_z(\omega)|^2}{8\sqrt{B\rho h}} \tag{7}$$

where ρ and h are respectively the plate density and thickness and B is the bending stiffness, given by $B = h^3 E/(12(1-\nu^2))$ with E and ν, the plate elastic parameters. The total energy is then :

$$W = \frac{1}{8\pi\sqrt{B\rho h}} \int_0^{+\infty} |\tilde{F}_z(\omega)|^2 \mathrm{d}\omega. \tag{8}$$

The normal force $\tilde{F}_z(\omega)$ at the impact location can be deduced from the deconvolution of $\tilde{U}_z(r,\omega) = \tilde{F}_z(\omega)\tilde{G}_{zz}(r,\omega)$ [*Aki and Richards*, 1980] where $\tilde{U}_z(r,\omega)$ is the measured normal displacement and $\tilde{G}_{zz}(r,\omega)$ is the vertical-vertical Green's function. For calculations, we used the far field asymptotic expression of $\tilde{G}_{zz}(r,\omega)$ [e.g. *Goyder and White*, 1980], which is valid for distances greater than half a wavelength from the excitation source [*Noiseux*, 1970] :

$$|\tilde{G}_{zz}(r,\omega)| = \frac{1}{8Bk^2} \sqrt{\frac{2}{\pi k r}} \exp\left(-\frac{\gamma(\omega)}{2} r\right), \tag{9}$$

where $k = v_\phi/\omega$ is the wave number with v_ϕ the phase velocity.

2.3 Diffuse method

This technique is derived from classical methods used in room acoustics [see e.g. *Weaver*, 1985 and references therein]. When the emitted wave is reflected off the boundaries many times, the elastic field become diffuse, i.e., the wave becomes homogeneously distributed over the plate and equipartioned. When the field is equipartioned, the potential and kinetic energy are equal and the total energy $E(t)$ verifies at a given time t :

$$E(t) = h \iint_S \rho v_z(r,t)^2 \mathrm{d}^2 r, \tag{10}$$

where $v_z(r,t)$ is the normal surface vibration speed. Moreover, because the field is diffuse, the average value (over several periods) of the squared field $v_z(r,t)^2$ is constant over the plate surface :

$$\overline{E(t)} \approx hS\rho\overline{v_z(t)^2}. \tag{11}$$

When the field is diffuse, the energy decreases exponentially with time due to dissipation :

$$\overline{E(t)} \approx hS\rho\overline{v_z(t_0)^2} \exp(-(t-t_0)/\tau), \tag{12}$$

where t_0 is the impact time and τ is the attenuation time. This time equals $(\gamma v_g)^{-1}$. Thus, from the exponential decrease of the squared normal vibration speed, we can estimate the total elastic energy produced by the impact at time t_0 :

$$W = \overline{E(t_0)} \approx \rho hS\overline{v_z(t_0)^2}, \tag{13}$$

3 Experimental results

3.1 Experimental setup

We have compared the three methods of energy estimation with steel bead impacts onto a thin glass plate. Piezoelectric charge shock accelerometers (type 4374 and 8309, *Brüel & Kjaer*) record the vibrations generated by impacts at various positions. The accelerometers type 4374 and 8309 have an uniform response over a wide range of frequencies from 1 Hz to 26 kHz and to 54 kHz, respectively, and resonate for 85 kHz and 180 kHz, respectively. The normal acceleration is digitalized with an acquisition rate of 0.3 MHz. The beads are made of steel (density 7800 kg m^{-3}) and their diameter ranges from 1 mm to 20 mm. The beads are dropped from various heights from 2 cm to 25 cm on a circular glass plate of radius 40 cm and thickness 9 mm. The elastic parameter of the glass plate are represented in Table 1.

For all the methods, we assume that the plate vibration is mostly normal to the surface. To check this hypothesis, we measure the radial vibration with an accelerometer on the plate border and compare it with the normal vibration measured by an other accelerometer on the plate surface and close to the first one. Figure 2 shows that the energy transported in the radial direction, that is proportional to the integral of the squared signal $\int |a_r(t)|^2 \mathrm{d}t$, is negligible because it is only about 0.2% of that on the normal direction $\int |a_z(t)|^2 \mathrm{d}t$. Interestingly, the energy radiated in the plate does not seem to depend on the angle of impact up to 20° and then decreases very slightly (Figure 2).

Table 1 – Elastic parameters of glass material used in experiments : density ρ, Young's modulus E, Poisson ratio ν. The characteristic distance $1/\gamma$ and time τ of energy attenuation and group velocity v_g, that depend on the frequency f (in Hz) is estimated for the 9mm-thick plate

kh -	ρ (kg m^{-3})	E (GPa)	ν -	$1/\gamma$ (m)	τ (s)	v_g (m s^{-1})
< 1	2500	83	0.2	~ 15	$0.8 f^{-1/2}$	$18.7 f^{1/2}$
> 1					~ 0.0048	~ 3100

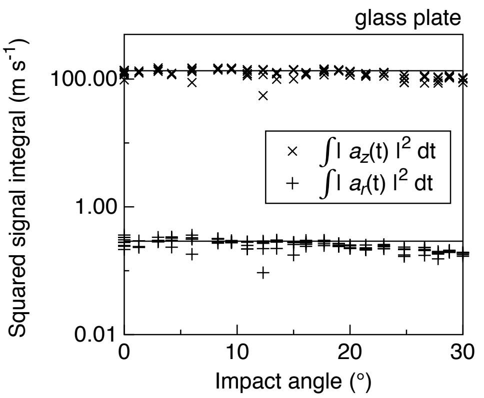

Figure 2 – A steel bead of diameter 4 mm is dropped from height 10 cm onto the surface of the glass plate. The time integral of the squared vibration acceleration on the direction normal to the plate surface $\int |a_z(t)|^2 \mathrm{d}t$ ($\times$) and on the radial direction $\int |a_r(t)|^2 \mathrm{d}t$ (+) are represented for different angles of impact of the bead with respect to the vertical. The horizontal black lines represents the mean values of $\int |a_z(t)|^2 \mathrm{d}t$ and $\int |a_r(t)|^2 \mathrm{d}t$ for impact angles smaller than 20° with respect to the vertical, that are respectively 132 m s^{-1} and 0.29 m s^{-1}.

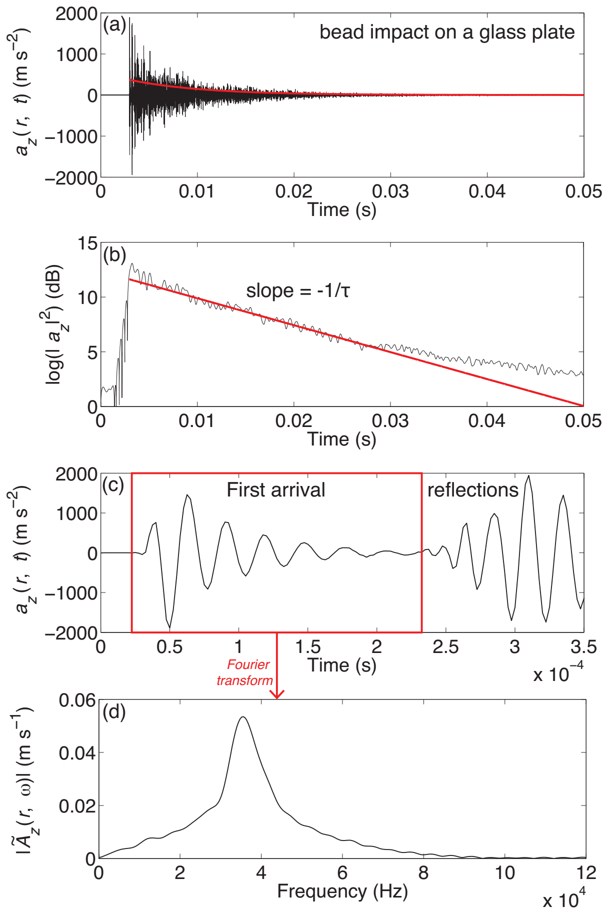

Figure 3 – (a)-(c) Trace of the normal acceleration $a_z(r, t)$ recorded at $r = 6$ cm from the impact of a 4mm-diameter steel bead onto the glass plate. (a) The acceleration with respect with time. The envelop of the acceleration is plotted in semilog scale. (c) Focus on the direct arrival (The first 0.2 ms). (d) Fourier transform of the first wave arrival.

In order to test simultaneously the three methods of energy calculation, we also have constraints on the plate size and elastic parameters. On one hand, the characteristic distance $1/\gamma$ of energy attenuation in the plate has to be significantly greater than the plate largest dimension L. This is verified for glass for which $1/\gamma \simeq 15$ m $\simeq 17.5L$ (Table 1). Practically, for the two first methods, we can neglect attenuation for the direct path. Therefore, a large number of wave reflections occurs before the extinction of the coda (Figure 3a). After about 30 side reflections, the averaged squared vibration amplitude decreases exponentially with time, as expected (Figure 3b). We could thus estimate the characteristic time of energy attenuation τ and we verify that $\tau(\omega) = (\gamma(\omega) v_g(\omega))^{-1}$ for every frequency ω. Time τ depends on frequency as $\sim 0.8 f^{-1/2}$ in the frequency range considered (Table 1). On the other hand, the plate has to be sufficiently large so that the direct wave between the impact and the probe locations can be measured entirely before the return of the first reflections (Figure 3c).

Impacts of beads excite a wide frequency range from 1 Hz to about 80 kHz and are characterized by a peak of energy whose central frequency vary from 30 kHz to 40 kHz depending on the bead diameter (Figure 3d).

3.2 Results

The energy flux and deconvolution methods to estimate the elastic energy W from the first wave arrival (equations (6) and (8)) give almost identical results. These results are also in good agreement with the energy estimated from the diffuse method. The error bars are ± 1 standard deviation of the dispersion from reproducibility tests conducted on a series of 20 experiments and are about 37% with the first method, 36% with the second and 52% with the third.

We measure the total energy lost by the beads ΔE_p from their coefficient of restitution e [e.g. *Hunter*, 1957 ; *McLaskey and Glaser*, 2010]. The proportion of energy radiated in elastic waves W with respect to ΔE_p, i.e., the elastic transfer efficiency, depends on the bead diameter (Figure 5). The elastic transfer efficiency is maximum for beads of diameter 4 to 5 mm for which nearly all the energy lost ΔE_p is converted in elastic energy W.

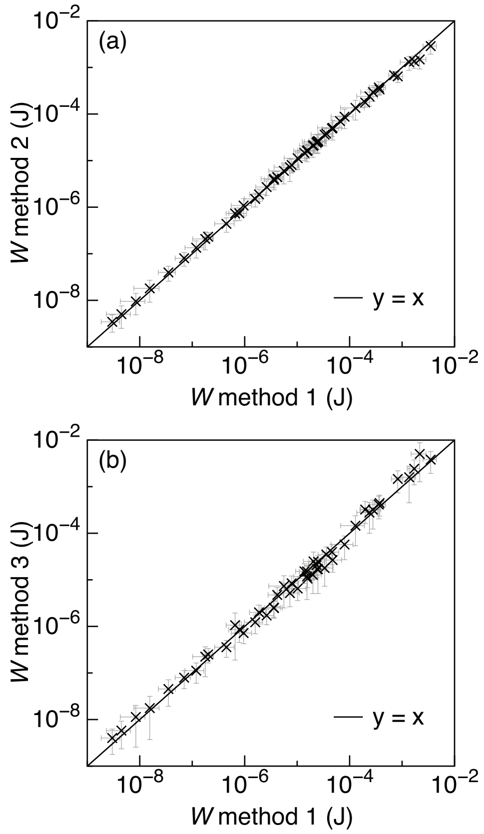

FIGURE 4 – Comparison of the elastic energy W calculated using the three methods (equations (6), (8) and (13)) for impacts of steel beads of various diameters from 1 to 20 mm dropped from various heights from 2 to 25 cm onto the glass plate. Error bars are ±1 standard deviation of the dispersion from reproducibility tests conducted on a series of 20 experiments.

4 Discussion

4.1 Discussion of the experimental results

Let us first discuss the possible source of errors in our experiments. To estimate the normal force generated at the impact for the deconvolution method, one have to deconvolve the measured signal from the Green's function. The problem of deconvolution is known to be very difficult [e.g. *McLaskey and Glaser*, 2010]. Classically, it consists in dividing, in the frequency domain, the amplitude of the vibration by the Green's function. However, the inverse Green's function diverges when ω tends towards 0 (equation (9)). Consequently, in practice we cannot deconvolve the signal and therefore estimate the energy below a cutoff frequency. To quantify the error due to this cutoff, we calculate the acceleration due to an ideal Hertzian force [e.g. *Johnson*, 1985] (Figure 6). We find out that when the low frequency cut-off equals 3 kHz, the estimated energy is less than 5% smaller than the exact elastic energy for every bead diameters investigated (Figure 6c). The result of the deconvolution of a signal measured on the glass plate with the Green's function in far field (Figure 6b) shows that the temporal profile of the obtained impact force onto a glass is similar to that of a Hertzian force (Figure 6d), in agreement with previous experimental studies [e.g. *McLaskey and Glaser*, 2010]. The small precursor in the force profile may arise from the viscoelasticity of the contact [*McLaskey and Glaser*, 2010].

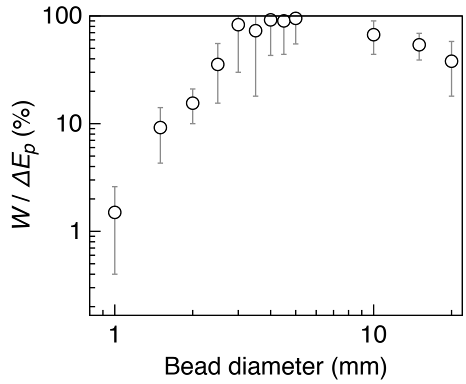

FIGURE 5 – Ratio of the energy W radiated in elastic waves over the total energy lost during an impact ΔE_p, i.e. elastic transfer efficiency, as a function of the bead diameter for the experiments on the glass plate. Error bars are ±1 standard deviation of the dispersion from reproducibility tests conducted on a series of 5 experiments.

The error bars of the diffuse method are greater than for the two first methods (Figure 4b) because it is based on statistical assumptions that induce extra-fluctuations.

The radiated elastic energy W do not seems to be affected when the impact force is not normal to the surface, at least up to an angle of impact of about 20° with respect to the vertical (Figure 2). This result implies that the mode A_0 of Lamb remain the main mode excited even if the impact force is not normal to the plate.

We observe that even when the elastic transfer efficiency is low (for bead diameter smaller than 3 mm and greater than 10 mm, see Figure 5), the presence of strong inelastic dissipation does not affect our estimation of the elastic energy radiated because the three methods of calculation give very similar results, regardless of the bead size (Figure 4a and 4b).

4.2 Limits of applicability of the methods

The energy flux and deconvolution methods are well adapted to estimate the elastic energy when the side reflexions are very attenuated or the bandwidth is sufficiently large that the first wave arrival can be discerned from side reflections. When this last assumption is not fullfilled and the number of reflections is large enough, the diffusive method becomes very efficient. Another interest of this last approach is that there is no assumption on the impact, such as isotropy. Finally, the choice of one or the other method depends on how much well we know the characteristics of the investigated structure and on the number of sensors available. The first two methods require to know the elastic parameters, the waves speeds and the attenuation coefficient within the impacted structure before any calculation of the elastic energy (equations (6) and (8)). If one can not have access to those characteristics, the third method should be preferred because only the structure density and dimensions and the instant t_0 of the source are sufficient (equation (13)). Note that the three methods are efficient with only sensor to measure the elastic energy with all of the presented methods. The precision of the energy estimation can be enhanced when several sensors are used. For the direct path methods, it can take into account an anisotropic emission. For the diffuse

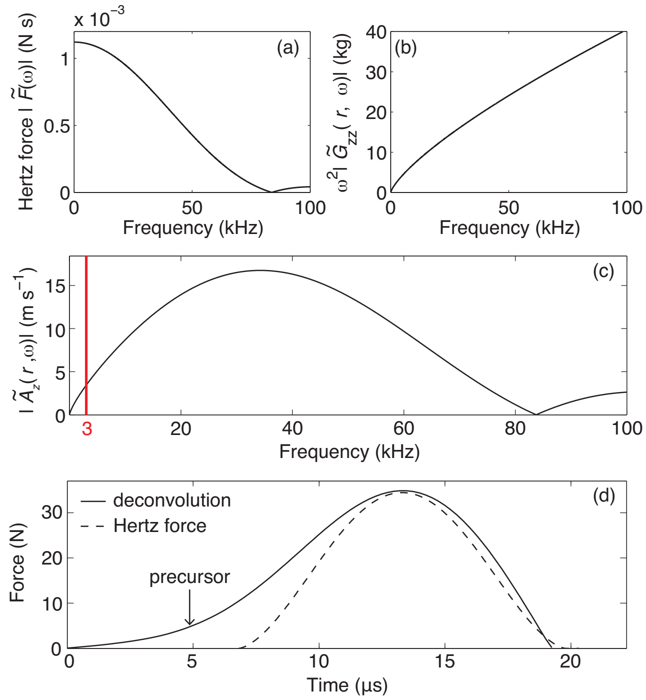

Figure 6 – (a) Fourier transform $|\tilde{F}(\omega)|$ of the ideal Hertz force of elastic impact of a steel sphere of diameter 5 mm on glass. This force is multiplied by (b) the second derivative of the Green's function at $r = 10$ cm from the force location $|\tilde{G}_{zz}(r, \omega)|$ (equation (9)) to obtain (c) the analytic time Fourier transform of the normal vibration acceleration $|\tilde{A}_z(r, \omega)|$ generated by the impact. Practically, we discard frequencies below 3 kHz (red line) to deconvolve the signal with the Green's function (see text). (d) shows the result of the deconvolution of the measured acceleration vibration generated by the impact of a steel bead of diameter 5 mm on the glass plate, i.e. the impact force $F_z(t)$ (full line) and the theoretical Hertz force of elastic impact (dashed line). The two forces are similar with the exception of a small precursor in the measured force.

method, it can compensate a not completely equipartioned field.

5 Conclusion

An object impacting a surface creates a force localized in time and space onto the structure. This time dependent force generates a wave within the elastic material. We have presented three methods to estimate the energy transferred into a thin elastic plate from the measurement of the surface normal vibration at a single position. The energy flux and deconvolution methods use the direct wave between the impact and the sensor while the third method take benefit of the diffuse coda when multiple reflections occur. The three methods have been validated experimentally with drop tests of steel beads of different diameters onto a glass plate. They give close results, even when the impact is strongly inelastic.

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