Asymmetry in lateral forces on vocal folds due to the Coanda effect

A. Hirschberg[a], J. Haas[b] et X. Pelorson[b]
[a]Technische Universiteit Eindhoven, Dep. Applied Physics, CC.3.01b, MTP Group, Postbus 513, 5600 MB Eindhoven, Pays-Bas
[b]GIPSA-lab UMR 5216, 11 rue des mathématiques, BP 46, 38402 Saint-Martin-D'Hères, France
a.hirschberg@tue.nl

A model is proposed to predict the asymmetry in lateral forces on the vocal folds due to attachment of the jet to one of the folds. The model predicting flow separation in the glottis is based on a quasi-steady boundary layer approximation. Attachment of the resulting jet flow to one of the vocal folds has to be assumed. Flow separation of this wall jet from the wall is assumed at the glottal exit. The lateral force is estimated from the lateral momentum flux of the resulting free jet by means of an integral momentum balance. The effect of friction is taken into account by means of displacement thicknesses at the wall and at the free shear layer of the wall jet. Results are confirmed by comparison with the lateral force calculated from the static wall pressure measurements and numerical simulations provided by the literature.

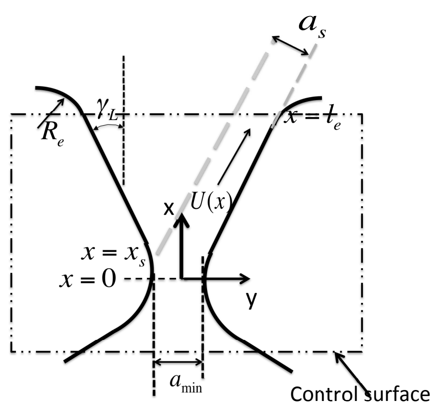

Figure 1 – Sketch of the geometry of the glottis showing an asymmetric flow due to attachment of the free jet downstream of the neck to the right wall of the diverging part of the channel.

1 Introduction

In a series of recent papers Erath et al. [1],[2],[3] and Chaigne et al. [4] consider the interesting problem of the asymmetry of the flow in the downstream part of the glottis during the closing phase of the vocal fold oscillation cycle. The glottis is then a slit shaped converging-diverging channel with a neck (width a_{min}) at the upstream side of the glottis. In first approximation the flow is two dimensional (see Fig. 1). The downstream part can be described as a diffuser of length l_e terminated by a strong widening. The diffuser part is assumed to have straight walls at an angle $|\gamma_\alpha|$ with the channel axis (x-direction), where $\alpha = L$ for left and $\alpha = R$ for right. At the beginning of the closing phase the opening angle ($|\gamma_L|+|\gamma_R|$) is so small that the flow only separates from the walls at the diffuser exit $x = l_e$ where the channel height $a(x)$ increases abruptly. A free jet is formed downstream of the glottis. The radius of curvature of the walls at $x = l_e$ is R_e. The angle of divergence($|\gamma_L| + |\gamma_R|$) of the downstream part of the glottis increases as a_{min} decreases, during the closing phase. Above a critical value of the divergence angle ($|\gamma_L|+|\gamma_R|$) $\sim \pi/18$ flow separation occurs within the diffuser. The separation points move towards the neck of the glottis with increasing angle. Under steady flow conditions in such a configuration the jet will tend to attach to one of the walls of the diffusor (Fig. 1). This is the result of entrainment of the fluid surrounding the jet by momentum transfer from the jet. A small asymmetry results into a pressure difference that bends the jet toward the closest wall. This asymmetry can be an asymmetry in the vocal folds geometry or some flow perturbation. When the jet attaches to one of the walls it remains there. We have assumed attachment to the right wall in Fig. 1 ($\alpha = R$). Alternatively, we refer to the wall along which the jet flows as the Flow-Wall. The other wall is referred to as the No-Flow-Wall. In the glottis for sufficiently low Reynolds number, the jet remains laminar so that the initialisation of the attachment is driven by viscous (molecular) momentum transfer. The molecular momentum transfer is not very efficient and therefore the laminar jet will in general separate from the wall at the glottal exit ($x = l_e$) because the wall curvature is too strong. At high Reynolds numbers the jet downstream of the glottal exit will be turbulent. In such a case the Coanda effect is stronger because of the enhanced momentum transfer from the jet to its surroundings. The turbulent jet can follow the wall downstream of the glottal exit. Also when the jet is laminar if it is very thin it will tend to follow the wall downstream of the glottal exit. This is observed from measurements by Erath et al. [2] as discussed by Hirschberg [5].

For a curved wall the pressure gradient necessary to bend the streamlines in the wall jet is established, resulting into a low pressure at the wall on which the jet is attached. Hence the jet exerts a transversal force on that wall, corresponding to the reaction of the flow to the suction force of the wall on the flow. Once established the asymmetric flow can be described, in first order approximation, as a frictionless flow. A viscous boundary layer will grow between the jet and the wall to meet the no-slip viscous boundary condition at the wall. The flow separation of the attached jet from the wall is determined by the momentum thickness θ of this boundary layer and adverse pressure gradient that occurs as the radius of curvature of the wall increases. The build-up and maintenance of this asymmetric flow is referred to as the Coanda effect [6],[7],[8].

Asymmetry in the flow due to the Coanda effect is commonly observed in static glottal model flows [9] [10]. It was argued by Hofmans et al. [11] that the time scale needed to establish such an asymmetric flow was too long to allow asymmetry in glottal flow. It is one of the merits of Erath et al. [1] that they have carried out experiments with an oscillating in-vitro model of vocal folds, demonstrating the existence of the Coanda effect during the closing phase of the oscillation cycle. The occurrence of asymmetry of the jet flow downstream of an oscillating glottal model has been confirmed by [4]. Coanda effect in actual glottal flow is difficult to observe, but can certainly not be excluded.

Once this fact established, Erath et al.[2] made an attempt to estimate the impact of such an asymmetry on the motion of the vocal folds and the sound production. They have developed a semi-empirical model, which they call Boundary Layer Estimation of Asymmetric Pressure (BLEAP) aiming at calculating the transversal force on the wall on which the jet is attached [2]. Hirschberg [5] discusses some problems in the approach of Erath et al.

[2]. In particular the Erath et al. [2] apply the quasi-steady equation of Bernoulli to relate measured velocity decrease along the wall jet center-line and the wall pressure. As this velocity decrease is due to viscosity, application of the equation of Bernoulli is hazardous. Hirschberg [5] proposes a simple frictionless model. Erath et al. [12] suggest to estimate the asymmetric force by addition of the forces estimated by their original model and the force estimated using the model of Hirschberg [5]. They suggest that this would include the effect of friction, which was neglected by Hirschberg [5].

The present paper proposes an alternative modification of the model of Hirschberg [5] in order to include the influence of friction by means of laminar boundary layer theory. We use for convenience the semi-empirical boundary layer theory of Thwaites [13], [14], [8], [15]. Any other laminar boundary layer model could be used. Obviously a boundary layer model will fail when the glottis is almost closed, however the Coanda force becomes in this limit vanishingly small. We use the same notation as Erath et al. [2] to facilitate comparison. In the next section 2 we summarise the proposed theoretical model. In the following section we apply the theory to analyse the experimental and numerical results of Scherer et al. [10] on a steady flow through a model of a divergent glottis. Experiments on vocal fold models [16] and a study of the implication of flow asymmetry on the oscillation of a two mass model of the vocal folds [17] are proposed in other papers presented at this conference.

2 Theory

We assume the flow to be incompressible and steady, the jet is attached to the right side of the diffusor, and that the wall-jet separates from the right wall at the glottal exit $x = l_e$ (see Fig. 1). The jet width corresponds in first approximation to the diffusor width $a(x) = a_s$ at the position $x = x_s$ where the flow separates from the left wall (see Fig. 1). The velocity $U_B = \sqrt{2(p_u - p_e)/\rho}$ in the jet is determined by applying Bernoulli from a position upstream of the glottis where $p = p_u$ and the flow velocity is negligible, to the free jet at the glottal exit where $p(x) \simeq p_e$, corresponding to the pressure downstream of the glottis, which we assume to be uniform. Considering an integral y-momentum balance, for the volume enclosed by the control surface sketched in Fig. 1, we see that the jet leaving the glottis with an angle γ_{out} carries a y-momentum flux per unit length of the vocal folds :

$$\Phi_y = [a_s - (\delta_s^* + \theta_s) - (\delta_e^* + \theta_e)]\rho U_B^2 \sin\gamma_{out}. \quad (1)$$

where $|\gamma_{ou}| \geq |\gamma_R|$. The equality $|\gamma_{out}| = |\gamma_R|$ prevails when the jet separates from the right wall (Flow-Wall) at $x = l_e$. When the jet follows the wall further downstream we have $|\gamma_{out}| > |\gamma_R|$. The mass displacement thicknesses δ_s^* and δ_e^* are the reductions of jet width that has to be applied when assuming a uniform frictionless core flow with velocity U_B in order to match the actual mass flux of the jet [14],[8]. The momentum thicknesses θ_s and θ_e are the additional displacement that should be added to respectively δ_s^* and δ_e^* to match the momentum flux of the actual jet. While the wall boundary layer displacement thickness $\delta_e^* + \theta_e$ grow between $x = x_s$ and $x = l_e$, the displacement thickness of the free shear layer (bounding the wall jet) remains equal to $\delta_s^* + \theta_s$. This is because the diffusion of momentum in this free shear layer does not involve an external force, so that there is no effect on the momentum flux.

The net transversal momentum flux Φ_y must be induced by a force from the walls on the flow. The reaction of the flow to this force is the net asymmetric component of the force of the flow on the wall in the y-direction. Using the notation of Erath et al. [2] we have : $G_{R,close} = -\Phi_y$. This is actually the sum of the lateral forces (y-components) acting on the right and left walls of the glottis respectively.

If the flow separates at the glottal exit $x = l_e$ the deflection of the jet occurs only around the separation point $x = x_s$, that is the position along the Flow-Wall where this force should be applied, in a dynamical model. If the flow separates from the right wall downstream of the glottal exit $x > l_e$ one has, in addition to the first force $-[a_s - (\delta_s^* + \theta_s) - (\delta_e^* + \theta_e)]\rho U_B^2 \sin\gamma_R$ at x_s, to apply a second force $[a_s - (\delta_s^* + \theta_s) - (\delta_e^* + \theta_e)]\rho U_B^2[\sin\gamma_{out} - \sin\gamma_R]$ at $x = l_e$, accounting for the additional bending of the flow at the glottal exit. Note that upstream from the separation point $x < x_s$ the pressure $p(x)$ can be estimated by using the equation of Bernoulli $p(x) + \rho U(x)^2/2 = p_e + \rho U_B^2/2$ in combination with the equation of continuity $U(x)[a(x) - 2\delta^*(x)] = U_B[a_s - 2\delta_s^*]$, where $a(x)$ is the local width of the glottal channel. We assume here a quasi-one dimensional flow (uniform velocity $U(x)$ within a cross section) corrected for the viscous displacement $\delta^*(x)$ at both walls. The symmetric lateral forces (acting on respectively left and right walls) can be obtained by integration of the pressure $p(x) - p_e$ along the wall. The y-components of these two forces ($x < x_s$) have equal magnitudes and opposite signs.

Obviously the main problem is to obtain an estimation for the boundary layer thickness and the position x_s of the separation point. A simplified semi-empirical method to obtain such an estimate is the method of Thwaites [13],[14],[8],[15]. The method of Thwaites is a very crude approximation. It is however quite robust and therefore very useful in practice.The momentum thickness θ of the viscous boundary layer is estimated by using the expression of Thwaites :

$$\theta(x)^2 U(x)^6 - \theta(0)^2 U(0)^6 = 0.45\nu \int_0^x U(x')^5 dx' \quad (2)$$

where $U(x)$ is the velocity in the frictionless core flow and ν is the kinematic viscosity of air. The mass displacement thickness δ^* is related to the momentum thickness θ by the shape function $H(\lambda)$:

$$\delta^* = H(\lambda)\theta \quad (3)$$

where the shape parameter λ is defined by :

$$\lambda = \frac{\theta^2}{\nu}\frac{dU}{dx}. \quad (4)$$

The function $H(\lambda)$ has been tabulated by Thwaites [8]. The separation point is given following Thwaites [13] by $\lambda_s = \lambda(x_s) = -0.09$ and $H(\lambda_s) = 3.55$. Alternatively Vilain [15] proposed to use $\lambda(x_s) = -0.0992$. As we are mostly interested in the behaviour at the separation point we can use for $x < x_s$ the approximation :

$$U(x)[a(x) - 2\theta(x)(1 + H(\lambda_s))] = U_B[a_s - 2\theta_s(1 + H(\lambda_s))]. \quad (5)$$

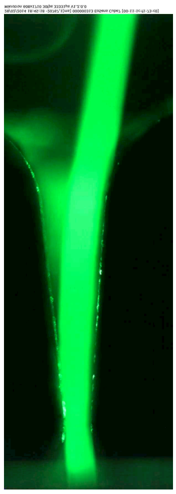

FIGURE 2 – Flow visualisation in a glottis model with opening angles $|\gamma_R| = |\gamma_L| = \pi/36$ displaying a clear Coanda effect. The jet is attached to the right wall. Corresponding wall pressure profiles measured by Scherer et al. [10] are shown in Figure 3.

The separation point has to be found by iteration. As a first step in this iteration one can use for δ^* and θ the value calculated for a flat plate in a uniform flow with velocity U_B [14] :

$$\delta^* = 1.732\sqrt{\frac{\nu(x - x_n)}{U_B}} \tag{6}$$

where x_n is the position of the neck of the glottis $a(x_n) = a_{min}$, where we neglect the boundary layer thickness. We have furthermore for a flat plate $\lambda = 0$ and $H(0) = 3.0$. Alternatively one can start the iteration procedure by assuming a uniform flow and zero boundary layer thickness at the neck, which yields the equation :

$$\lambda_s = -0.45\frac{1}{a_s}\left[\frac{da}{ds}\right]_s \int_0^{x_s}\left[\frac{a_s}{a(x')}\right]^5 dx'. \tag{7}$$

For the wall boundary layer between x_s and l_e the pressure is uniform $p(x) = p_e$ so that we can use the equation :

$$\theta_e^2 - \theta_s^2 = \frac{0.45\nu}{U_B}(l_e - x_s). \tag{8}$$

3 Comparison with experimental data

Scherer et al. [10] have carried out extensive measurements on steady flow through an upscaled symmetrical glottal model with $|\gamma_R| + |\gamma_L| = \pi/18$. As shown in Figure 2 the flow displays for such an opening angle a clear Coanda effect. Details about the flow visualisation experiment a provided in a companion paper [16]. From the pressure on the flow-wall p_R (right) and non-flow-wall p_L (left) sides we can calculate the asymmetric lateral force by integration :

$$G_{R,close} = \int_0^{l_e} [p_L(x') - p_R(x')]dx'. \tag{9}$$

Scherer et al. [10] carried out experiment for the transglottal pressures $\Delta p_g = 2.9 \times 10^2$ Pa, 4.9×10^2 Pa,

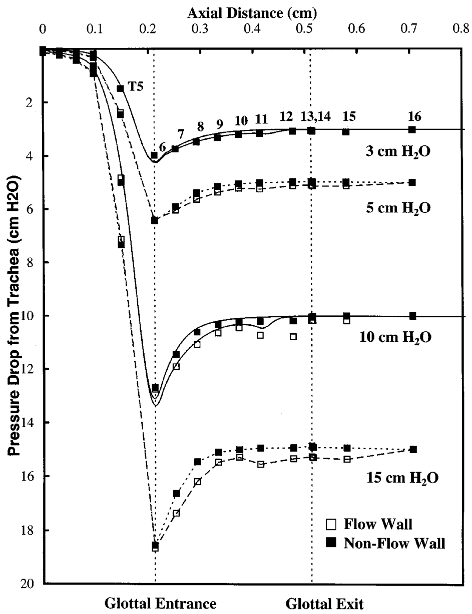

FIGURE 3 – Wall pressure measurements by Scherer et al. [10] for $|\gamma_R| = |\gamma_L| = \pi/36$. The closed symbols correspond to the non-flow wall. The open symbols correspond to the flow wall. The solid lines for $\Delta p_g = 294$ Pa and $\Delta p_g = 980$ Pa are numerical results. We observe a maximum of asymmetry in the pressure $|p_R - p_L|$ just upstream of the glottal exit ($x = l_e$) for $\Delta p_g = 980$ Pa. For $\Delta p_g = 1470$ Pa, the pressure difference $|p_R - p_L|$ does not vanish downstream of the glottal exit ($x > l_e$). Both effects indicate an additional deflection at the glottal exit, which may be related to turbulence in the jet. The numerical simulations for $\Delta p_g = 980$Pa assume a laminar flow and display an earlier separation of the jet at the glottal exit. The distance between the glottal entrance $x = 0$ and glottal exit $x = l_e$ (indicated by vertical doted lines) is 3 mm. We suppose flow separation at $x_s = 1.2$ mm. For $x_s < x < l_e$ the pressure at the non-flow wall remains constant $p(x) \simeq p_e$. Reproduced with permission from "R.C. Scherer et al. Intraglottal pressure profiles for a symmetric and oblique glottis with a divergence angle of 10 degrees, J. Acoust. Soc. Am. 109 , 1616-1630 (2001)" (http ://dx.doi.org/10.1121/1.1333420). Copyright 2001, Acoustical Society of America.

9.8×10^2 Pa and 1.47×10^3 Pa. For 2.9×10^2 Pa and 9.8×10^2 Pa they carried out numerical simulations (see Figure 3). The asymmetry in pressure is not accurately determined for the lowest pressure. For the highest pressure we observe that the asymmetry remains strong for locations further than the glottal exit $x = l_e$ indicating that the deflection angle $|\gamma_{out}|$ is probably larger than $|\gamma_R| = \pi/36$. This is confirmed by the numerical calculations results shown in figure 8.b of [10]. One observe a local minimum of pressure at the flow wall just around the flue exit at the transition between the flat wall diverging part of the channel and the rounded glottal exit. This effect seems more pronounced in the measured pressure for $\Delta p_g = 9.8 \times 10^2$ Pa than in the corresponding numerical simulation. Also the jet deflection displayed downstream of the glottis in figure 8.a of [10] is slightly larger than $|\gamma_R| = \pi/36$. The Reynolds number $U_B a_{min}/\nu$ for the flow through the glottis reaches 1300 for $\Delta p_g = 1.47 \times 10^3$ Pa. At such high Reynolds number the free jet at the flue exit will most probably be turbulent. This might explain that the continuation of the asymmetry in the pressure is most pronounced for this experiment. We now focus on the case $\Delta p_g = 9.8 \times 10^2$ Pa, for which this effect is less important and for which we have numerical simulation results.

From the measurement of the wall pressure at the non-flow wall in figure 3 we see that for $\Delta p_g \geq 980$ Pa the pressure becomes almost uniform between pressure measuring points 8 and 9 corresponding to 1.0 mm $< (x_s - x_n) < 1.4$ mm. Using the average value $(x_s - x_n) = 1.2$ mm we find $a_s = 0.6$ mm. The lateral force obtained by integrating the experimental data for $(p_L - p_R)$ yields $G_{R,close} = 0.11$ N/m. The theory using the flat plate approximation 8 yields $[a_s - (\delta_s^* + \theta_s) - (\delta_e^* + \theta_e)] = 0.47$ mm and $G_{R,close} = [a_s - (\delta_s^* + \theta_s) - (\delta_e^* + \theta_e)]\rho U_B^2 \sin \pi/36 = 0.08$ N/m. As discussed above this difference can be due to the additional bending of the flow at the glottal exit ($|\gamma_{out} > |\gamma_R|$). The frictionless approximation predicts $G_{R,close} = 0.11$ N/m, which is also close to the experimental result. In view of the uncertainty in the geometry of the flow channel and in the direction of the supposed flow asymmetry, it is not obvious that the correction for the friction is meaningful. This is certainly the case when considering simplified models such as a two-mass model [17].

While this is a very reasonable result, confirming our theory, the real challenge is to see whether the theory would have provided also a reasonable result when predicting flow separation by using the method of Thwaites. When using the method of Thwaites with $\lambda_s = -0.09$ [13] we find $x_s = 0.9$ mm while we find $x_s = 1.1$ mm for $\lambda_s = -0.0992$ as proposed by Vilain [15]. We see that the method of Thwaites is not accurate, but yields reasonable results.

4 Conclusions

A modification accounting for the effect of viscous boundary layers is proposed for the model [5] predicting the magnitude of the asymmetry in the lateral force on the vocal folds induced by the Coanda effect. Using the experimental and numerical results of Scherer et al. [10] for a glottal divergence angle $\pi/18$ we have demonstrated that the model predict within 50% the asymmetry in the forces on the vocal folds if the flow separation point is given and the geometry of the glottis specified. A method based on the theory of Thwaites [13] for the boundary layers allows a prediction when no experimental data concerning the flow separation within the glottis are available. The boundary layer model cannot be applied when the glottis is almost closed. In that case the entire flow is dominated by viscosity, however the transversal force induced by flow asymmetry is negligible in that case because the jet momentum flux becomes vanishing small. In practice using a model taking friction into account is not necessary. The frictionless model [5] should be used for application to simplified two-mass models of vocal folds vibration, because a more accurate model would be an overkill in view of the crudeness of two-mass models. The main uncertainties in such models are the geometry of the glottis and the direction of the asymmetry in the flow.

Acknowledgments

The author would like to thank Dr. R. Scherer for the information concerning his experiments.

Références

[1] B. D. Erath and M. W. Plesniak, An investigation of asymmetric flow features in a scaled-up model of the human vocal folds, *Experiments in Fluids*, **49**, 131-146 (2010).

[2] B. D. Erath and S. D. Peterson and M. Zanartu and G. R. Wodicka and M. W. Plesniak, A theoretical model of the pressure field arising from asymmetric intraglottal flows applied to a two-mass model of the vocal folds, *J. Acoust. Soc. Am.*,**130**,389-403 (2011).

[3] R. Mittal, B. R. Erath and M. W. Plesniak, Fluid Dynamics of Human Phonation and Speech, *Annu. Rev. Fluid Mech.* , **45**, 437-467 (2013).

[4] P. Sidlof, O. Doaré, O. Cadot, A. Chaigne, Measurement of flow separation in a human vocal folds model, *Experiments in Fluids*, **51**, 123-136 (2011).

[5] A. Hirschberg, Comments on "A theoretical model of the pressure field arising from asymmetric intraglottal flows applied to a two-mass model of the vocal folds"[J.Acoust.Soc.Am. 130,389-403(2011)] (L), *J. Acoust. Soc. Am.*, **134**, 9-12 (2013).

[6] H. Coanda, Device for deflecting a stream of elastic fluid projected into an elastic fluid, U.S. Patent No 2,052,869, September 1, 1936.

[7] D. J. Tritton, *Physical Fluid Dynamics*, second edition, Clarendon Press, Oxford, UK (1988).

[8] R. D. Blevins, *Applied Fluid Dynamics Handbook*, Van Nostrand Reinhold Co., NY (1984).

[9] H.M.Teager and S.M.Teager, "Active fluid dynamic voice production models, or there is a unicorn in the garden," in *Vocal Fold Physiology : Biomechanics, Acoustics and Phonatory Control*, edited by I.R. Titze and R.C. Scherer (Denver Center for the Performing Arts, Denver, 1983) pp. 387-401.

[10] R. C. Scherer, D. Shinwari, K. J. De Witt, C. Zhang, B. R. Kucinschi and A. A. Afjeh, Intraglottal pressure profiles for a symmetric and oblique glottis with a divergence angle of 10 degrees, *J.Acoust. Soc. Am.*, **109**, 1616-1630 (2001).

[11] G. C. J. Hofmans, G. Groot, M. Ranucci, G. Graziani and A. Hirschberg, Unsteady flow through in-vitro models of the glottis, *J. Acoust. Soc. Am.*, **113**, 1658-1675 (2003).

[12] B. D. Erath and S. D. Peterson and M. Zanartu and G. R. Wodicka and K. C. Stewart and M. W. Plesniak, Response to Comments on "A theoretical model of the pressure field arising from asymmetric intraglottal flows applied to a two-mass model of the vocal folds"[J.Acoust.Soc.Am. 130,389-403(2011)] (L)", *J.Acoust.Soc.Am.*, **134**, 913-916 (2013).

[13] B. Thwaites, Approximate calculation of the laminar boundary layer, *Aero. Quartely*, **I**, 245-280 (1949).

[14] H. Schlichting, *Boundary-Layer Theory*, Sixth edition, McGraw-Hill Book Company, NY (1968).

[15] C. E. Vilain, X. Pelorson, C. Fraysse, M. Deverge and A. Hirschberg, Experimental validation of a quasi-steady theory for the flow through the glottis,'*J. Sound Vib.*, **276**, 475-490 (2003).

[16] X. Pelorson, J. Haas, "Etude expérimentale de l'écoulement d'air au travers de glottes asymétriques", Congrès Français d'Acoustique 2014, Poitiers 22-25 Avril 2014.

[17] J. Haas, X. Pelorson, A.Hirschberg, "The Coanda effect and asymmetries in vocal fold models during phonation", Congrès Français d'Acoustique 2014, Poitiers 22-25 Avril 2014.