Prediction of the acoustic Form Function of immersed tubes by Adaptive Neuro-Fuzzy Inference System and artificial Neural Network

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Abstract— Computational Intelligence techniques have been proposed as an efficient tool for modeling and forecasting in recent years in various applications. In this work An Adaptative Neuro-Fuzzy Inference System (ANFIS) and artificial neural networks (ANN) techniques are used to predict the acoustic form function (FF) for an infinite length cylindrical shell of various radius ratio b/a (a: outer radius and b: inner radius). If this tube is excited by a plane acoustic perpendicularly to its axis, circumferential waves are generated in the shell and water/shell interface. These circumferential waves are observed in the spectrum of the FF. The Wigner-Ville distribution (WVD) is used as comparison tool between the calculated FF by the analytical method and that predicted by the neuro-fuzzy and the artificial neural networks techniques for a copper tube. During the application of these techniques, several configurations are evaluated for various radius ratio b/a (a: outer radius, b: inner radius of tube). This study shows that the neuro-fuzzy technique is able to predict the FF with a mean relative error (MRE) about 1.7% and that predicted by the neural network is about 3.9%.

Keywords-—Fuzzy logic; ANFIS; ANN; acoustic scattering, cylindrical shells; Wigner-Ville distribution.

1 Introduction

Different methods have been proposed for analysis of the circumferential waves propagating around a cylindrical shell which includes frequency and temporal analysis [1, 2, 11]. The aim of this paper is to compare the form function predicted by ANFIS and ANN techniques with that obtained analytically. The Wigner-Ville time-frequency distribution was used as a comparison tool to check the validity of the use of the ANFIS network and the ANN models to predict the form function [1]. The time-frequency distribution takes into account both the time parameter and the frequency parameter leading to synthetic images that allow us to follow the evolution of the frequential content of a wave echo as a time function [2, 3]. Major benefits using ANFIS and ANN networks are excellent management of uncertainties, noisy data and non-linear relationships. In this study, the ANFIS and ANN models predict the form functions of tubes without the use of the analytical method for various radius ratio b/a. These model is able to predict the FF for copper tube of various radius ratio b/a between 0.9 and 0.99.

2 Neuro-Fuzzy Model

Fuzzy logic is based on fuzzy set theory; it is an extension of Boolean logic which allows us to use the values between "true" and "false". In this approach the classical theory of binary membership in a set, is modified to incorporate the memberships between “0” and “1”. A fuzzy inference system using fuzzy IF–THEN rules is a way of modeling the qualitative aspects of human knowledge and reasoning processes without employing precise qualitative analysis. Basically, a fuzzy inference system is composed of five conceptual blocs, shown in Fig.1, as follows [4]:

(i) A rule base containing a number of fuzzy IF–THEN rules,
(ii) A database, which defines the membership functions of the fuzzy sets used in the fuzzy rules.
(iii) A decision-making unit, which performs the inference operations on the rules.
(iv) A fuzzification inference, which transforms the crisp inputs into degree of match with linguistic values.
(v) A defuzzification inference, which transforms the fuzzy results of the inference into a crisp output.

These functional blocks are shown in Figure 1, FIS implements a nonlinear mapping from its input space to the output space.

An adaptive network is a network structure whose overall input-output behavior is determined by a collection of modifiable parameters. Readers are referred to [5] for more details on adaptive networks. Jang [6] introduced a novel architecture and learning procedure for the FIS that uses a neural network learning algorithm for constructing a set of fuzzy if-then rules with appropriate MFs from the stipulated input–output pairs. This procedure of developing a FIS using the framework of adaptive neural networks is called an Adaptive Neuro-Fuzzy Inference System (ANFIS).

2.1 ANFIS Architecture

The proposed architecture of the ANFIS is depicted in Fig. 2(b). Selection of the FIS is the major concern when designing an ANFIS to model a specific target system. The present study uses the Sugeno fuzzy model to generate fuzzy rules from a set of input-output data pairs [7]; [8] since the conclusion of a fuzzy rule of this FIS is constituted by a weighted linear combination, and the parameters may be estimated by combination of the gradient descent method and the least squares estimate (LSE). To simplify the explanations, the fuzzy inference system under consideration is assumed to have two inputs (x and y) and one output *f*. For a first order of Sugeno fuzzy model, a typical rule set with base fuzzy if–then rules can be expressed as:

Rule 1: If x is A_1 and y is B_1, then $f_1=p_1 x+q_1 y+r_1$ *(1)*
Rule 2: If x is A_2 and y is B_2, then $f_2=p_2 x+q_2 y+r_2$ *(2)*

Where A_1; A_2 and B_1; B_2 are the MFs for inputs x and y; respectively; p_1; q_1; r_1 and p_2; q_2; r_2 are the parameters of the output function. Fig. 2(a) illustrates the fuzzy reasoning mechanism for this Sugeno model to derive an output function (*f*) from a given input vector [x, y].

The corresponding equivalent ANFIS architecture is presented in Figure 2 (b), where nodes of the same layer have similar functions. The functioning of the ANFIS is as follows:

- **Layer 1**: Each node in this layer generates membership grades of an input variable. The node output OP_1^i is defined by:

$$OP_1^i = \mu_{A_i}(x) \text{ for i=1,2 or } OP_1^i = \mu_{B_{i-2}}(x) \text{for i=3,4} \qquad (3)$$

Where x (or y) is the input to the node. A_i (or B_{i-2}) is a fuzzy set associated with this node, characterized by the shape of the MFs in this node and can be any appropriate functions that are continuous and piecewise differentiable such as generalized bell shaped functions.

Assuming a generalized bell function as the MF, the output OP_1^i can be computed as:

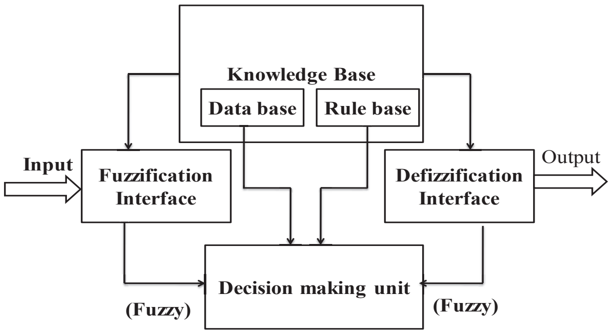


FIGURE 1- Fuzzy Inference System with crisp output.

$$OP_1^i = \mu_{A_i}(x) = \frac{1}{1+(\frac{x-c_i}{a_i})^{2b_i}} \tag{4}$$

Where $\{a_i, b_i, c_i\}$ is the parameter set that changes the shapes of the MF with maximum equal to 1 and minimum equal to 0.

Layer 2: Every node in this layer multiplies the incoming signals, denoted as π , and the output OP_i^2 that represents the firing strength of a rule is computed as:

$$OP_i^2 = \omega_i = \mu_{A_i}(x)\mu_{B_i}(y), i = 1,2. \tag{5}$$

Layer 3: The i[th] node of this layer, labeled as N; computes the normalized firing strengths as:

$$OP_i^3 = \varpi_i = \frac{\omega_i}{\omega_1 + \omega_2}, i = 1,2. \tag{6}$$

Layer 4: Node i in this layer computes the contribution of the i[th] rule towards the model output, with the following node function:

$$OP_i^4 = \varpi_i f_i = \varpi_i(p_i x + q_i x + r_i) \tag{7}$$

Where ϖ is the output of layer 3 and $\{p_i, q_i, r_i\}$ is the parameter set.

Layer 5: The single node in this layer computes the overall output of the ANFIS as:

$$OP^5{}_1 = Overall\ output = \sum_i \varpi_i f_i = \frac{\sum_i \omega_i f_i}{\sum_i \omega_i} \tag{8}$$

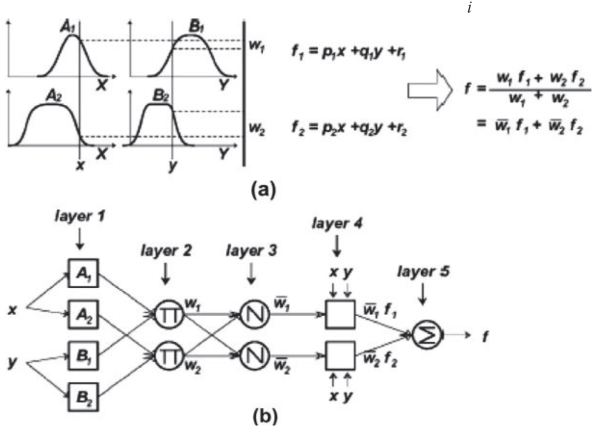


FIGURE 2**:** (a) First-order Sugeno fuzzy model (b) ANFIS architecture

2.2 Architecture

Fig. 3 shows a typical neural network consisted of input, sum function, sigmoid activation function and output. This typical neural network is reinforced with an advanced training algorithm named as Levenberg–Marquardt back-propagation. The input values to a neuron are obtained by multiplying the output of the connected neuron by the synaptic strength of the connection between them. The weighted sums of the input components x_i are calculated by using the following equation [4, 9]:

Where $(net)_j$ is the output from neuron, w_{ij} is connection's weight, b is bias weight, n is the number of neurons or processing elements (PE) in each layer and f is the activation function.

$$(net)_j = f(\sum_{i=1}^{l} w_{ij}x_i + b) \tag{9}$$

In this paper, the log-sigmoid used as activation function as follows [4, 9]:

$$O_j = f(net)_j = \frac{1}{1+\exp(-(net)_j)} \tag{10}$$

After processing all of the layers, the activated result of the output layer O_j, compared with the target value y_t, and the resulted error will be propagated backward the network's weight to minimize the overall error. This process is usually performed by a method known as error back-propagation (BP) method. In this paper, Levenberg–Marquardt back-propagation (LMBP) algorithm is utilized as training algorithm instead of commonly used standard BP method for its robustness in computing process [10].

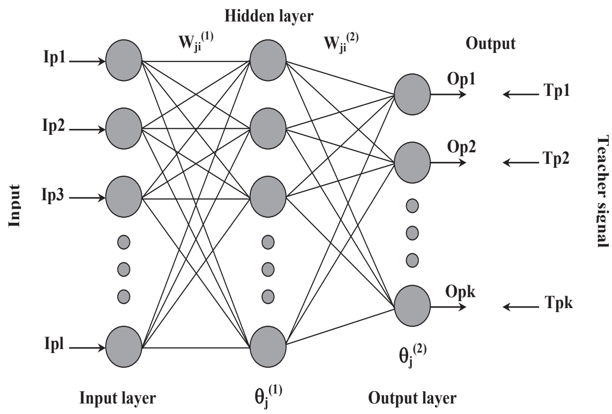


FIGURE-3 Layout of three-layer back propagation neural network.

3 Collection of data base

In this study, an air-filled tube immersed in water is excited by a plane acoustic wave perpendicularly to its axis (Fig.4). The complex pressure P_{scat}^{∞} backscattered by the tube in a far field is the sum of the normal modes which takes into account the effects of the incident wave, the reflective wave {1}, surface waves in the shell {2} (whispering gallery waves, Rayleigh wave: equivalent to Lamb waves if the tube wall is thin) and an antisymmetrical interface Scholte A wave labelled also A_0 wave {3} connected to the geometry of the object (Fig.5) [11]. Waves {2} and {3} are the circumferential waves. In our case, the

flexural waves A, A_1 and A_2 and the compression waves S_0, S_1 and S_2 are observed.
The general analytical form of the backscattered pressure P^{∞}_{scat} in far field at normal incidence can be expressed as:

$$P^{\infty}_{scat}(\omega) = P_0 \frac{1-i}{\sqrt{\pi k_1 r}} \exp i(k_1 r - \omega t) \sum_{n=0}^{\infty} \varepsilon_n \frac{D^1_n(\omega)}{D_n(\omega)} \cos(n\theta). \quad (11)$$

TABLE I. Physical parameters

	Density ρ(kg/m³)	**Longitudinal velocity C_L (m/s)**	**Transversal velocity C_T (m/s)**
copper	8930	4760	2325
Water	1000	1470	-
Air	1.29	334	-

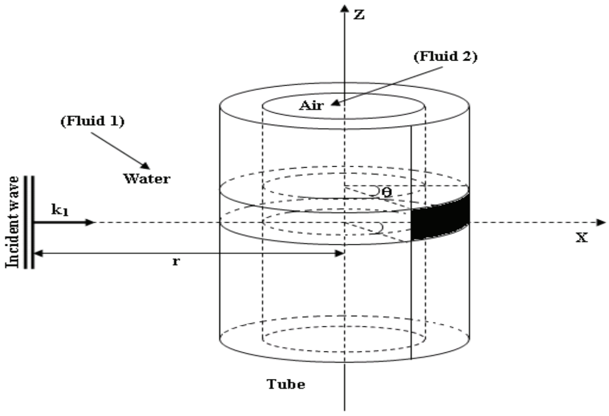


FIGURE 4- The geometry used to calculate the form function for an elastic tube.

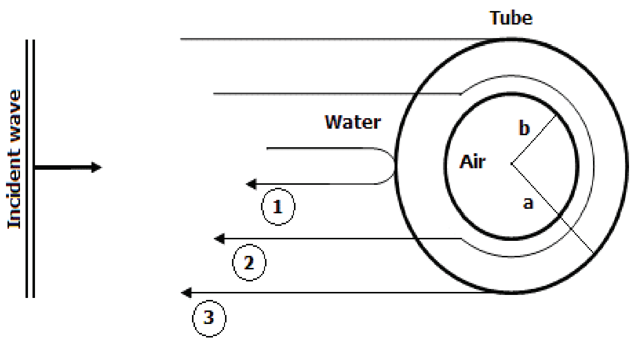


FIGURE 5- Mechanisms of the echo formation ({1}: specular reflection, {2}: circumferential shell

where ω is the angular frequency, k_1 is the wave-number with respect to the wave velocity in the external fluid, P_0 is the amplitude of the incident plane wave, r is the distance of pressure measurement, Θ is the azimuthal angle (Θ = 180° for the backscattering spectrum), $D^1_n(\omega)$ and $D_n(\omega)$ are determinants computed from the boundary conditions of the problem (continuity of stress and displacement of both interfaces), the terms of these determinants are given in ref 12 (in the present study, the normal incidence is only considered) and ε_n is the Neumann coefficient (ε_n =1 if n=0 and ε_n =2 if n≠0).

The physical parameters used in the calculation of the backscattered complex pressure are illustrated in Table 1.

Usually this backscattered pressure is presented as the form function FF defined by the relation (2) [13]:

$$\left| \frac{P^{\infty}_{scat}}{P_0} \right| = \sqrt{\frac{a}{2r}}\, FF \quad (12)$$

This form function is function of the reduced frequency $x_1 = k_1 a$ given by:

$$k_1 a = \frac{2\pi \nu a}{c_1} \quad (13)$$

where $\nu = \dfrac{\omega}{2\pi}$ is the frequency of a wave in Hz.

4 wigner-ville time frequency distribution

The Wigner-Ville Distribution (WVD) takes into account both the time parameter and the frequency parameter leading to an image that allows us to follow the evolution of the frequency content of acoustic echoes as a function of time [14].

Among the time-frequency techniques, WVD is used for its interesting properties in terms of acoustic applications [1]. Other time-frequency methods (wavelets for example) would be used in our application but we had good results with the WVD [2]. The WVD is applied to obtain the dispersion curves of the group and phase velocities of circumferential waves propagating around a tube with different radius ratio [1]. The WVD can be applied to the backscattered time signal obtained from the computation and/or the experiment. This allows us to determine the reduced cut-off frequencies [1].

The WVD of time complex signal *u(t)* displays the energy distribution of this signal in the time-frequency plane (5).

$$W_u(t,\nu) = \int_{-\infty}^{+\infty} u\left(t+\frac{\tau}{2}\right) u^*\left(t-\frac{\tau}{2}\right) \exp(-i\,2\,\pi\,\nu\,\tau)\, d\tau \quad (14)$$

with $\nu = \dfrac{\omega}{2\pi}$ the frequency and *t* the time of the signal, *u*(t)* is the conjugated complex time signal of *u(t)*.

5 Training Phase and application of the ANFIS and ANN networks to predict FF

ANFIS and ANN networks method requires for its training a set of form functions calculated by the analytical method or obtained by experiments.

In this work, the dataset is constituted from the form functions calculated by the analytical method. This dataset is divided into two sets. The first training data set was used for training the ANFIS and ANN while the remaining checking data set were used for validating the identified model. The desired and predicted values for both training data and checking data are essentially the same in fig. 8 and 11.

6 Selection of the Optimal Model

The performance of *ANFIS and ANN* models for training and testing data sets were evaluated according to statistical criteria such as, coefficient of correlation *R*, *MAE*, *MRE*. The selection of different models is done comparing the errors of the ANFIS and ANN configuration, calculating the *MAE*, the *MRE* and the *R* of the FF. The coefficient of correlation of the linear regression is used like performance measures of the model between the predicated and the desired output. The different error measures and the coefficient of correlation are given by the following relations:

$$MRE=\frac{1}{N}\cdot\sum_{i=1}^{N}\frac{|C_i-O_i|}{C_i}\times 100, \tag{15}$$

$$MAE=\frac{1}{N}\sum_{i=1}^{N}|C_i-O_i| \tag{16}$$

$$R=1-\frac{\sum_{i=1}^{N}(C_i-O_i)^2}{\sum_{i=1}^{N}(C_i-O_m)^2}, \tag{17}$$

where N is the number of data, C_i and O_i are respectively the desired and the predicted values. O_m is the mean of the predicted values.

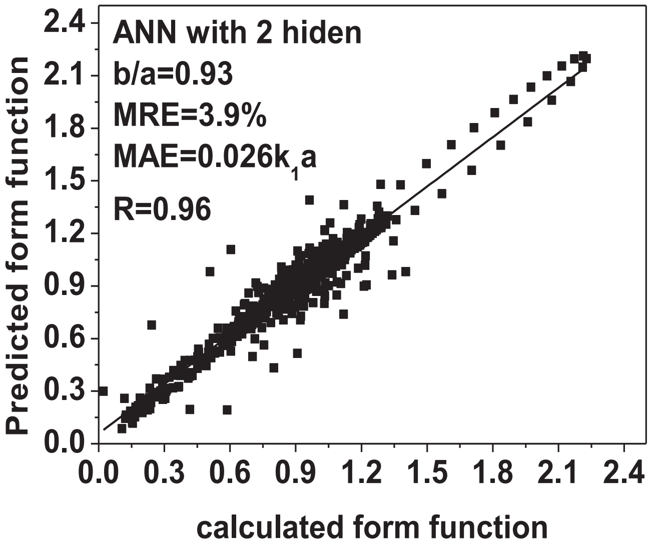
7 results and discussion

The ANFIS is trained by using randomly 50% of the dataset. While the remaining dataset 50% is reserved as a test base. The neural network is trained by using randomly 2/3 of the dataset. While the remaining dataset 1/3 is reserved as a test base. The analysis is repeated several times. Indeed, the values of errors are measured for ANFIS and ANN networks architectures.

The best configuration is found for the ANFIS network with the number of membership function is fixed to 2 MF, so the rule number is 16. The MRE and the MAE for the optimal model are respectively 2% and 2% k1a for b/a=0.93 (Fig.6 and 7). Figures 8, 10 show the comparison between the form functions predicted by the *ANFIS* model on a test database and that calculated by the analytical method for the radius ratios *b/a* = 0.93 and 0.9. (Figures 8 and 10) shows the good agreement between the predicted and the calculated values of the form function.

The best configuration is found for the ANN network with two hidden layer. The *MRE* and the *MAE* for the optimal model are respectively *3.9%* and *2.6%* k_1a *for b/a=0.93 see Fig.6 and 7.*

However, the time-frequency representation of Wigner-Ville is used as a tool of comparison to check the validity of the ANFIS network model because the studied signals are not stationary. Using a time-frequency representation can allow us to follow the evolution of the frequency content of acoustic echoes as a function of time. Figure 10 shows the good agreement between Wigner-Ville image of the predicted and the calculated form function for a copper tube of radius ratio b/a=0.93, 0.96. Moreover, starting from the Wigner-Ville image of the predicted signal, the reduced cut-off frequency ($[k_1a]_c$) of the tube corresponding to the A_1 wave can be determined. Generally, the ANFIS and ANN networks methods can contribute to the comprehension and the



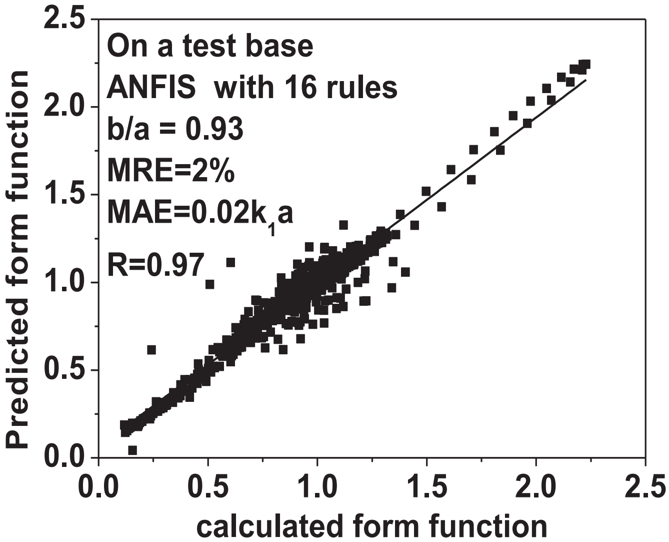


FIGURE 6- Correlation between calculated and predicted values of form function on a test dataset for radius ratio (a) b/a=0.93.

interpretation of the backscattered acoustic pressure by submerged tubes. In inverse operating mode, the network model can go up to the radius ratio b/a starting from an unknown form function for a given material and this by using the parameter of membership function and synapse weight of the optimals models.

The circumferential antisymmetric A_1 waves propagates around the circumference of the tube only for frequencies superior to the reduced cut-off frequency [15]. The reduced cut-off frequency values ($[k_1a])_c$) obtained from the Wigner-Ville image of the test database (b/a=0.93, 0.96) and analytically calculated are presented in Table II.

Starting from the operating mode direction of this optimal models we can make several things. In direct operating mode of the models optimals, one can predict the form functions for tubes of various radius ratio b/a and for various materials.

TABLE II. : Results of the different error measures and the coefficient of correlation (MRE, MAE, AND R) of ANN and ANFIS for a cooper tube b/a=0.93 (for exemple)

Error mesures	MRE	MAE	R
ANN	3.9%	0.026	0.96
ANFIS	2%	0.02	0.97

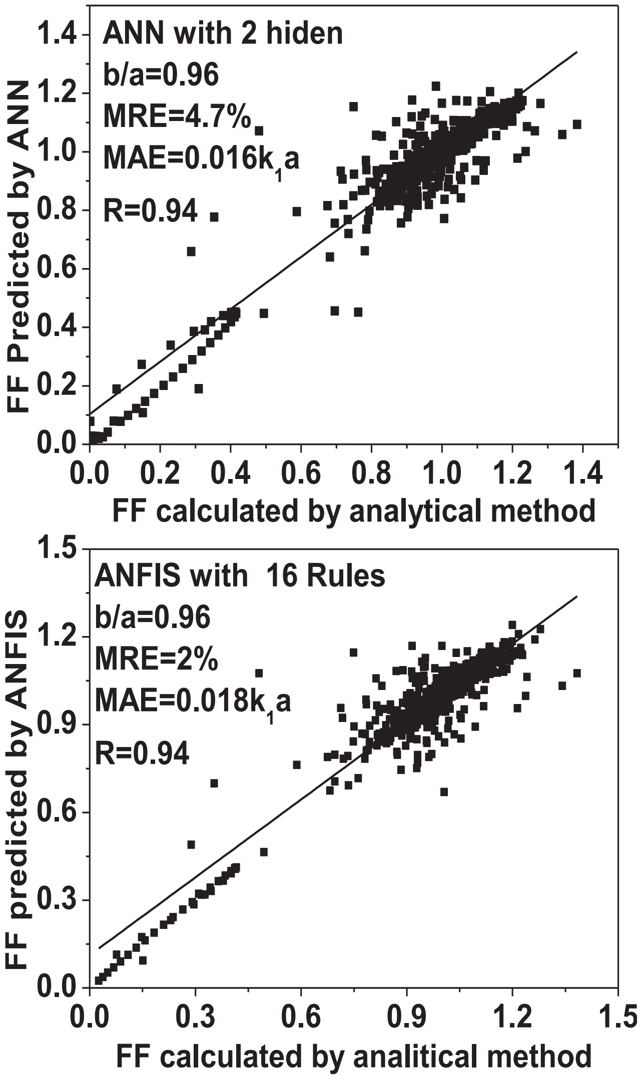


FIGURE 7-Correlation between calculated and predicted values of form function on a test dataset for radius ratio (a) b/a=0.96.

TABLE III. comparison between the reduced cut-off frequencies computed by eigen modes method and starting from the Wigner-Ville image

Circumferential wave A_1	$(k_1a)_c$ Estimated by PMT	$(k_1a)_c$ Estimated by ANFIS	$(k_1a)_c$ Estimated by ANN
B/A=0.93	70.94±0.3	70.48±0.3	70.40±0.3
b/a=0.95	99.32±0.3	99.94.0±0.3	99.98±0.3
b/a=0.97	165.54±0.2	165.00±0.2	164.90±0.3

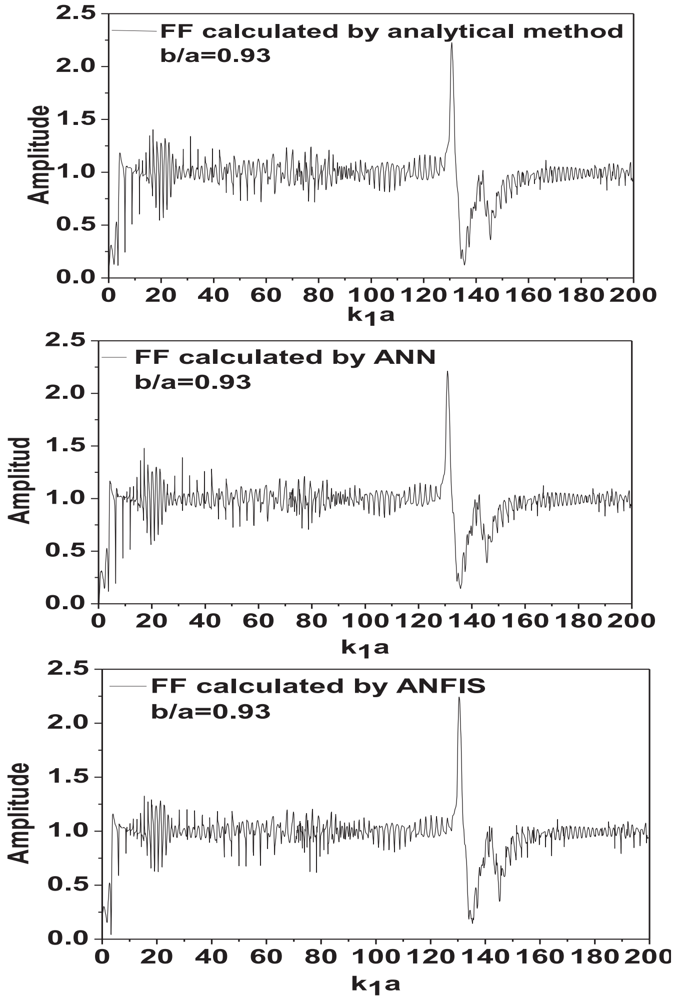


FIGURE 8- Form Function calculated by analytical method and form Function predicted by ANN and ANFIS for b/a=0.93

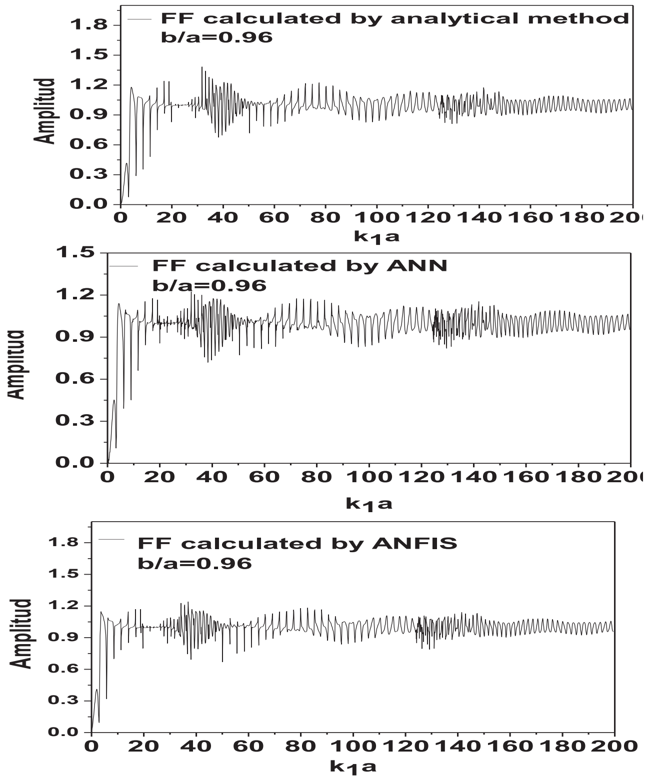


FIGURE 9- Form Function calculated by analytical method and form Function predicted by ANN and ANFIS for b/a=0.96

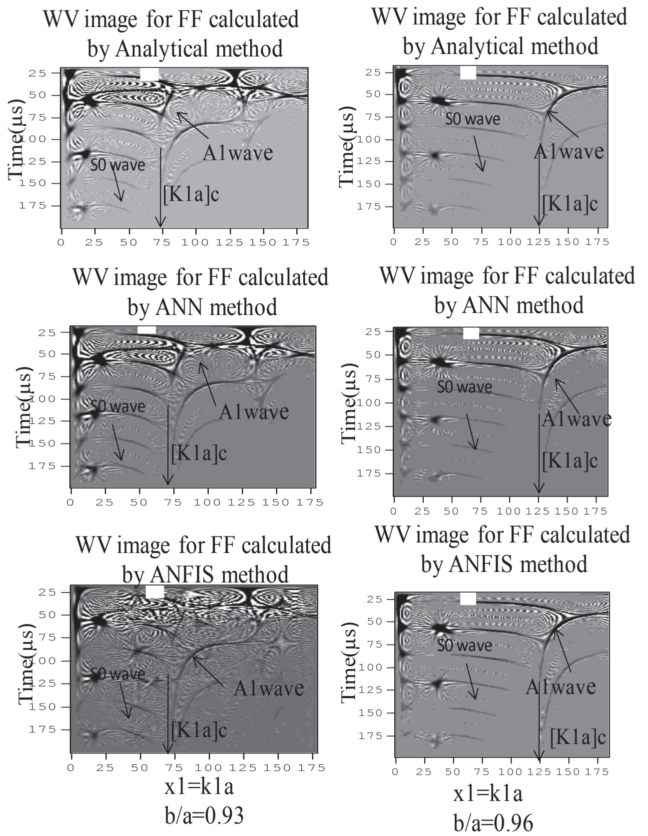


FIGURE 10- Comparison between the WV images for the form function calculated by the analytical method and predicted by the *ANFIS* and ANN methods (b/a=0.93, b/a=0.96)

8 Conclusion

The ANFIS and the ANN approachs can be used as new tools for non-destructive characterization of a thin elastic tube. To check the credibility of these approachs, the distribution of Wigner-Ville was used like a tool of comparison between the form function FF calculated by the analytical method and that predicted by the ANFIS and the ANN techniques.

The reduced cut-off frequency of the anti-symetric wave A_1 estimated from the WV images corresponding to the predicted form functions by the *ANFIS* and by the ANN methods shows that the ANFIS method are in good concordance with that computed by the eigen modes method. This comparison indicates that the *ANFIS* method is suitable to predict the backscattered pressure by a tube. Based on this comparison, we note that the adaptive neuro-fuzzy model can provide an effective description in the analysis of acoustic signal backscattered by a tube immersed in water.

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