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Numerical unsteady model for thermoacoustic devices

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The thermoacoustic devices process various nonlinear effects in high noise levels originating from the interaction between the acoustic oscillations in the fluid and thermal and mechanical conditions on the solid walls. A numerical model is presented to describe first the acoustic field in thermoviscous fluid and second the induced phenomena at large time-scale such as acoustic streaming and heat transfer. The equations of the model are derived from the instantaneous mass, momentum and energy conservation equations. The formulation for heat transfer and streaming flow is presented as a standard form of weak compressible flow based on the separation of time scales, using the velocity of mass transport vector as variable. The steady streaming flow and heat transfer is presented in various acoustic devices, showing the ability of this formulation to compute numerically the slow phenomena induced by acoustics and illustrating the physics in annular or resonant thermoacoustic devices.

1 Introduction

Thermoacoustic process results from the thermal interaction between an oscillating fluid and a solid surface. This interaction is responsible for acoustic work generation and/or hydrodynamic heat transfer. Thermoacoustic refrigerators use and amplify this effect in a stack of solid plates located in an acoustic resonator in order to transfer heat from a cold heat exchanger to a hot heat exchanger. This acoustically induced slow phenomenon can be disturbed by another slow phenomenon called "acoustic streaming". This phenomenon corresponds to a net mean flow generated by non linear processes associated with the propagation of a high level acoustic wave.

Analytical models allowing the description of such acoustical slow phenomena (thermoacoustic heat transfer and acoustic streaming) are usually based on simple geometries and idealized conditions. Real acoustic devices, for instance the resonant cavities used in thermoacoustic engines, have more complex geometries and require numerical approaches. The most accurate is the Direct Numerical Simulation (DNS) of the full-coupled Navier-Stokes equations in the time domain [1, 2], considering all the scales for time and space. However, it is often not suited to real systems because of the high computational cost required. Less expensive methods are available for fluid flow modeling (Large Eddy Simulation LED [3], Reynolds Averaged Navier-Stokes RANS [4]), which are based on simplifying assumptions and approximations, neglecting for instance temperature dependence of physical parameters of the fluid, or time and/or spatial variation of mean fluid density [5]. However, considering space or time averaged variables, these simplified methods cannot precisely describe the nonlinear effects and the localized forces, inside the boundary layers, exciting the averaged streaming flow.

In this paper, we present a formulation for modeling the nonlinear phenomena induced by acoustics based only on time scale splitting between the fast acoustical phenomena (acoustic oscillation), and slow acoustical phenomena (acoustic streaming and heat transfer). This formulation is suited for any arbitrary geometry and considers the full-coupled equations. This formulation can be used in analytical models, but can easily be implemented for numerical simulation and leads to short computational time when compared to the direct numerical simulation models. It can be processed using standard computational fluid dynamic tools and, thus, does not need any development of specific numerical method; the formulation is based on the mass transport velocity as variable instead the streaming velocity. Using this new formulation, the full nonlinear effects can be considered in the streaming excitation forces.

2 Basic equations and time splitting

The coupled fluid motion and thermal transfer are described by the conservation equations for mass, momentum and energy. Considering Newtonian fluid and the Fourier's law for heat transfer, these equations have the following form [6]:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{v}) = 0 \tag{1.1}$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla.(\rho \mathbf{v} \otimes \mathbf{v}) = \nabla.\left[- p\underline{\underline{\mathbf{I}}} + \mu\left(\nabla \mathbf{v} + (\nabla \mathbf{v})^T\right) + \underline{\underline{\mathbf{I}}}\eta \nabla.\mathbf{v}\right] \tag{1.2}$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + \mathbf{v}.\nabla T\right) + \rho h\left(\frac{\partial p}{\partial t} + \mathbf{v}.\nabla p\right) = \nabla.(\lambda \nabla T) + \mathrm{E}_{dis} \tag{1.3}$$

where ρ is the density, t is the time, p is the pressure, μ is the shear dynamic viscosity, $\eta = \xi - (2\mu/3)$ is the second viscosity with ξ the bulk dynamic viscosity, C_p is the heat capacity at constant pressure, T is the temperature, $h = -\left(C_p - C_v\right)/\hat{\beta}$ is a calorimetric coefficient with C_v the heat capacity at constant volume and $\hat{\beta} = (\partial p/\partial T)_\rho$ is the increase in pressure per unit increase in temperature at constant density, λ is the thermal conductivity of the fluid and E_{dis} is the dissipation of kinetic energy by viscosity effect. The vectors $\mathbf{v}$ and $\underline{\underline{\mathbf{I}}}$ are the particle velocity and the unit tensor, respectively. The equations (1) are completed by a state law linking the thermodynamical variables of pressure, density, and temperature of the form $f(p,\rho,T)=0$.

To study the acoustic movement (fast phenomenon at the acoustic period time scale) and the secondary induced fluid motion or heat transfer (slow phenomena), each variable or physical property φ is assumed to be the addition of two components, where one is quasi-static φ_m and the other $\tilde{\varphi}$ is an acoustic perturbation [7]. This involves two time scales which have sufficiently different magnitude to describe the fast acoustic phenomena with the ''short'' time t_a, while the acoustic streaming and heat transfer are described in using the "long" time t_s:

$$\varphi = \varphi_m(t_s) + \tilde{\varphi}(t_a) \qquad (2)$$

where the time averaged component φ_m over the acoustic period $\tau = 2\pi/\omega$ is used for slow phenomena, and $\tilde{\varphi}$ denotes the small acoustic perturbation whose time averaged value over the acoustic period vanishes $\langle\tilde{\varphi}\rangle = \frac{1}{\tau}\int_{t_s}^{t_s+\tau}\tilde{\varphi}dt_a = 0$.

3 Linear acoustic formulation

The harmonic solution for the acoustic field is usually obtained by solving the Helmholtz equation for the acoustic pressure variable, but such formulation is not suited to model the vortical and entropic motions which cause dissipation of acoustic energy, the shear viscosity stress and the associated momentum transfers exciting the streaming. In order to account for the acoustical, viscous and thermal effects, and their respective or combined effects particularly in the boundary layers, the acoustic modelling in a thermoviscous fluid is based on acoustic particle velocity and temperature variation [8]. The linear formulation is obtained from the basic equations (1), where only the "fast" variations $\tilde{\varphi}$. The other variables can be expressed as a function of the both variables $\tilde{\mathbf{v}}$ and $\tilde{T}$; For instance the pressure and density variations related to the acoustic perturbation are $\tilde{p} = \hat{\beta}\tilde{T} - \left(i\rho_m c_0^2/\gamma\omega\right)\nabla.\tilde{\mathbf{v}}$ and $\tilde{\rho} = (i\rho_m/\omega)\nabla.\tilde{\mathbf{v}}$, where c_0 is the adiabatic speed sound, and γ is the ratio of heat capacities per unit mass. The oscillating field is numerically computed using this formulation, with the finite element method as presented in reference [9].

4 Thermoacoustic formulation

The time splitting decomposition is next applied to the fundamental equation (1), and time-averaging is applied over the acoustic period to consider only the slow phenomena developed by acoustic at large time-scale t_s.

4.1 Velocity of mass transport

Taking both the averaged and perturbation components for density $\rho = \rho_m + \tilde{\rho}$ and velocity $\mathbf{v} = \mathbf{v}_m + \tilde{\mathbf{v}}$ in the continuity equation (1.1), and in time-averaging over the acoustic period τ, we obtain $\frac{\partial\rho_m}{\partial t} + \nabla.\mathbf{M} = 0$, where the vector $\mathbf{M}$ sets the total average mass flux. The ratio of this vector to the average density ρ_m is a velocity, called the velocity of mass transport and denoted $\mathbf{U}$. It is composed of the average velocity of acoustic wind and a ratio between an acoustic quadratic quantity and the average density of fluid:

$$\mathbf{U} = \mathbf{v}_m + \left(\langle\tilde{\rho}\tilde{\mathbf{v}}\rangle/\rho_m\right) \qquad (3)$$

This velocity is a fundamental variable for the acoustic streaming modeling, because it considers the net mass flow in combining both the fluid dynamics and the acoustic contribution to the average mass transport. Rudenko and al. [10] defined $\mathbf{U}$ with the initial state density of fluid ρ_0 (incompressible flow) instead of ρ_m which takes into account time and spatial variations.

4.2 Governing equations

Using "long" and "short" time components for all variables and properties of fluid in the basic equations (1) and averaging over the acoustic period and using the velocity of mass transport (4) yields to the following expressions for the conservation equations expressed at the large time scale $t = t_s$:

$$\frac{\partial\rho_m}{\partial t} + \nabla.(\rho_m\mathbf{U}) = 0 \qquad (4.1)$$

$$\frac{\partial(\rho_m\mathbf{U})}{\partial t} + \nabla.(\rho_m\mathbf{U}\otimes\mathbf{U}) = \nabla.-p_m\underline{\underline{\mathbf{I}}} + \mu_m\left(\nabla\mathbf{U} + (\nabla\mathbf{U})^T\right) + \eta_m(\nabla.\mathbf{U})\underline{\underline{\mathbf{I}}} + \mathbf{F} \qquad (4.2)$$

$$(\rho C_p)'\left(\frac{DT_m}{Dt}\right) + (\rho h)'\left(\frac{Dp_m}{Dt}\right) = \nabla.(\lambda_m\nabla T_m) + \langle E_{dis}\rangle + q \qquad (4.3)$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{U}.\nabla$ is the particle derivative, and the terms $(\rho C_p)' = \rho_m C_{pm} + \langle\tilde{\rho}\tilde{C}_p\rangle$ and $(\rho h)' = \rho_m h_m + \langle\tilde{\rho}\tilde{h}\rangle$ denote the effective heat capacities including average fluctuant components. We note that these governing equations keep the usual basic form for fluid mechanics modeling because the velocity of mass transport is used as variable. The acoustic nonlinear contributions are accounted by source terms in the right hand side: $\mathbf{F}$ in the momentum conservation equation and q in the energy conservation equation. Then, the velocity of mass transport (4) is well suited for modeling streaming in acoustic devices because it takes into account variations of density due to temperature and pressure gradients, and giving desired streaming velocityfrom Eq. (4) $\mathbf{v}_m = \mathbf{U} - \left(\langle\tilde{\rho}\tilde{\mathbf{v}}\rangle/\rho_m\right)$. The excitation force for the momentum contains three nonlinear components,

$$\mathbf{F}_1 = -\nabla.\langle \rho_m \tilde{\mathbf{v}} \otimes \tilde{\mathbf{v}} \rangle \quad (5.1)$$

$$\mathbf{F}_2 = \nabla.\left\langle \tilde{\mu}\left(\nabla\tilde{\mathbf{v}} + (\nabla\tilde{\mathbf{v}})^T\right) + \mathbf{I}\tilde{\eta}(\nabla.\tilde{\mathbf{v}})\right\rangle \quad (5.2)$$

$$\mathbf{F}_3 = -\nabla.\left(\mu_m \left(\nabla\frac{\langle\tilde{\rho}\tilde{\mathbf{v}}\rangle}{\rho_m} + \left(\nabla\frac{\langle\tilde{\rho}\tilde{\mathbf{v}}\rangle}{\rho_m}\right)^T \right) + \mathbf{I}\eta_m \left(\nabla.\frac{\langle\tilde{\rho}\tilde{\mathbf{v}}\rangle}{\rho_m} \right) \right) \quad (5.3)$$

and the term of the heat source q has five nonlinear components,

$$q_a = \left\langle \left(\rho_m \tilde{C}_p + C_{pm}\tilde{\rho}\right)\left(\frac{d\tilde{T}}{dt} + \tilde{\mathbf{v}}.\nabla T_m\right) \right\rangle + \left(\rho_m C_{pm}\right)\left\langle \tilde{\mathbf{v}}.\nabla\tilde{T} \right\rangle \quad (6.1)$$

$$q_b = \left\langle \left(\rho_m \tilde{h} + h_m\tilde{\rho}\right)\left(\frac{d\tilde{p}}{dt} + \tilde{\mathbf{v}}.\nabla p_m\right) \right\rangle + \left(\rho_m h_m\right)\left\langle \tilde{\mathbf{v}}.\nabla\tilde{p} \right\rangle \quad (6.2)$$

$$q_c = \nabla.\left\langle \tilde{\lambda}\nabla\tilde{T} \right\rangle \quad (6.3)$$

$$q_d = C_{pm}\langle\tilde{\rho}\tilde{\mathbf{v}}\rangle.\nabla T_m \quad (6.4)$$

$$q_e = h_m\langle\tilde{\rho}\tilde{\mathbf{v}}\rangle.\nabla p_m \quad (6.5)$$

The average term of the dissipation contains a nonlinear component representing the average heating in dissipating kinetic energy by the viscosity effect,

$$E_1 = \mu_m\left\langle\left(\nabla\tilde{\mathbf{v}} + (\nabla\tilde{\mathbf{v}})^T\right):(\nabla\tilde{\mathbf{v}})\right\rangle + \mathbf{I}\eta_m\left\langle(\nabla.\tilde{\mathbf{v}}):(\nabla\tilde{\mathbf{v}})\right\rangle \quad (7)$$

5 Applications

To investigate the acoustic streaming and heat transfer using a standing sound wave, a rectangular enclosure (L=0.1m, H=0.01m) filled with air is considered as illustrated in Figure 1. The left wall vibrates as a piston, at a frequency such that $L = \lambda/2$ where λ is the wavelength. The harmonic velocity of the vibrating wall is given by $v_w = \omega X_{\max}$ where $X_{\max}$ is the maximum displacement of the wall, and $\omega = 2\pi f$ is the angular frequency. When considering progressive waves, the general formulation is used to describe the behavior of an acoustitron [11]: a schematic representation of this resonator is shown in Fig. 2a: it consists of an annular resonator with a radius of curvature R_0 and a waveguide diameter D. A set of driven elements regularly spaced along the duct acts on the acoustitron walls. The velocity of each driving element is normal to the walls. The frequency is set in such a way that the length of developed resonator fits an integer number n of the wavelength of the perturbation profile of the acoustitron walls ($L = n\lambda_a$ with $\lambda_a = 2\pi/k_a$ where k_a is the acoustitron wave number). Tuning adequately the relative phase shift between the velocities of the acting elements allows the generation of a given kind of acoustic wave inside the annular resonator. In particular, a travelling acoustic wave can be generated. For a diameter much less than the acoustic wavelength ($D \ll \lambda$), the channel curvature curvature can be neglected on the propagation of sound waves and on the hydrodynamic flow [12, 13]. In this case, the annular geometry of the resonator can be considered as a linear one, that is a straight duct whose length fits the circumference perimeter, as presented in Fig. 2b. Consequently, all functions ϕ and their derivatives describing the acoustic field and fluid flow satisfy the periodicity condition $\phi(x=0) = \phi(x=L)$. Recently, the transient streaming in acoustitron has been studied by Amari et al. for quasi-adiabatic and quasi-isothermal wave/wall interaction regimes [14].

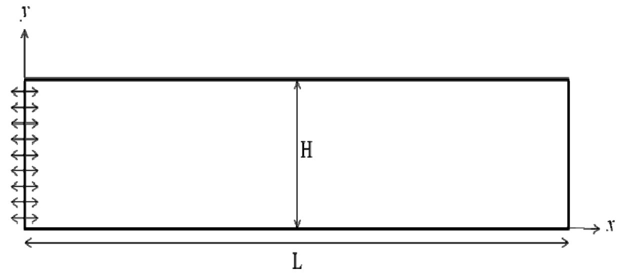

Figure 1: Bidimensional resonator for standing waves.

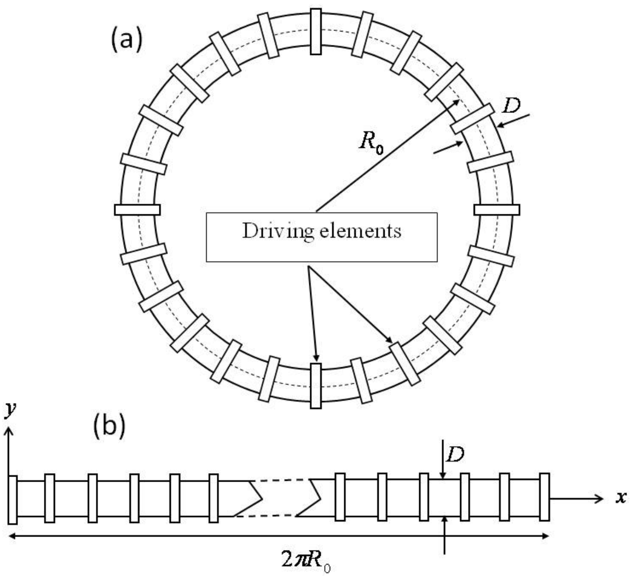

Figure 2: The acoustitron with travelling wave, a- annular geometry, b- developed geometry.

The governing equations are discretised using the finite element method, where non-slip and zero variation temperature are used for acoustic field, non-slip for the streaming flow, and heat flux for thermal conduction, were used for all the solid walls. Since the interaction between the wave field and the viscous boundary creates the acoustic streaming, and because the interaction with the thermal boundary creates a temperature gradient especially in the ends of stack, modeling the acoustic boundary layers thickness is a required condition to accurately compute the acoustic flow and temperature gradient. The used mesh has a non uniform distribution for the elements, highly refined in the vicinity of the solid walls. The numerical computations were made with constant mean static pressure (101325 Pa), temperature (20°C) and density (1.2 kg/m^3). We considered only the steady-state analysis ($\partial/\partial t = 0$).

In the following, we investigate numerically the acoustic streaming and heat transfer resulting from standing or travelling acoustic wave in two-dimensional devices. First, the acoustic streaming structures from standing wave in a resonator empty (Fig. 3), and in a resonator with a stack (Fig. 4), then from travelling wave in an acoustitron (Fig. 5). Second, the temperature gradient (heat conduction) resulting from standing acoustic wave in a resonator with a stack (Fig. 6-7).

5.1 Acoustic streaming

When the Mach number of the flow is small ($M < 0.3$), the streaming flow can be considered as quasi-incompressible because the variation of density caused by the variation of velocity is negligible. Assuming that there is no mean heat transfer (no stack or heat exchanger where the temperature field is homogeneous $T_m = T_0$), the acoustic wave propagation generates a streaming flow with a constant average density ρ_m and a constant shear dynamic viscosity μ_m; the bulk dynamic viscosity η is neglected here (Stokes assumption). As mentioned above, the energy equation is not considered, because it is uncoupled to the momentum and the continuity equations for isothermal and incompressible streaming flow. In this case, the incompressible acoustic streaming results from momentum transfer in the fluid, which is caused by the nonlinear acoustic phenomena expressed by the motion forces $\mathbf{F}$ in Eq. (6), mainly localized inside the boundary layers. It is governed by the formulation composed of two-dimensional simplified equations (5.1) and (5.2):

$$\nabla.\mathbf{U} = 0 \qquad (8.1)$$

$$\rho_m(\mathbf{U}.\nabla\mathbf{U}) = \nabla.\left(-p_m\underline{\underline{\mathbf{I}}} + \mu_m\left(\nabla\mathbf{U} + (\nabla\mathbf{U})^T\right)\right) + \mathbf{F} \qquad (8.2)$$

For the standing wave resonator, the computations were done with 1µm amplitude for the vibration of the wall. The acoustic streaming flow field is shown in Fig. 3. This flow is based on the average mass transport velocity. Four large size cells are observed inside the resonator where two ones (up right & down left) are clockwise and two ones counter-clockwise. The horizontal length of the vertical structures is a quarter-wavelength ($\lambda/4$). The steady movement of air is directed from the maximum of acoustic velocity to the two nodes along the walls of the tube, and in the opposite direction at the tube axis.

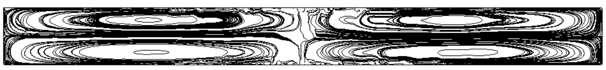

Figure 3: Streamlines of the steady-state acoustic streaming flow in the standing wave resonator of figure 1.

In a second modell, the streaming flow is computed around a stack placed inserted inside the resonator. The length l of the stack is much smaller than the resonator (l=H is considered here). The effect of the stack on acoustic streaming is presented in Fig. 4a. The thickness of stack plates is neglected and the distance between two plates is $\Delta y = 2mm$. In this case, there are only two main cells near the ends of stack, as presented in Fig. 4b. These circulations result of the localized exciting forces located in a small zone near the stack (Fig. 4c). Due to the shear movement, the field shows vortices in the ends of stack (Fig. 4d).

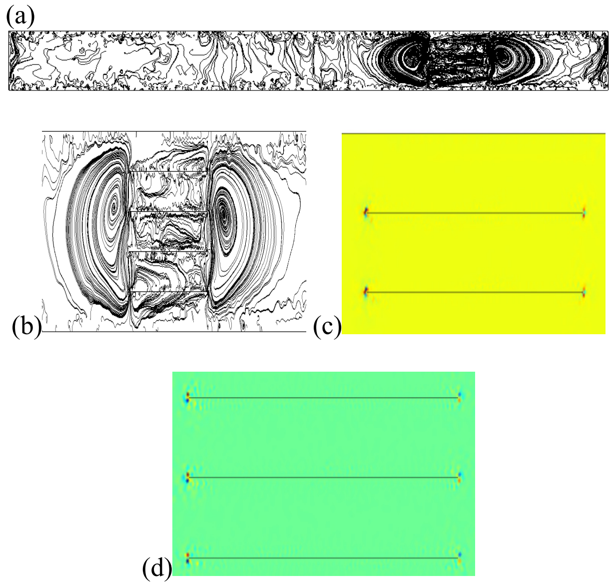

Figure 4: a- Streamlines of the acoustic streaming flow in a resonator equipped with a stack, b- Detailed streamlines near the Stack, c- x component of the volume exciting forces for the flow, localised near the ends of the stack (blue -550 N/m^3, red +365 N/m^3), d- Excited vortices near the ends of the stack (blue -600 s-1, red +560 s-1).

In a third modell, the formulation presented in this paper is used to describe the acoustic streaming generation in a travelling wave resonator (developed acoustitron) filled with air at atmospheric static pressure and ambient temperature. The travelling wave is excited by the initial velocity of the vibrating wall $v_w = 1$ µm/s; its wavelength λ is equal to the developed length of the resonator where the frequency is $f = c_0/2\pi R_0$. We studied the acoustitron with $R_0 = 0.34$ m and $D = 0.075$ m. The stationary solution of the velocity of mass transport field, on a half section of the acoustitron, is illustrated in Fig. 5. The momentum diffuses from the wall to the bulk of the fluid, due to the viscosity effects developed inside the viscous boundary layer.

The velocity field is symmetric, and the maximum velocity is obtained inside the viscous boundary layer (Fig. 5a). The velocity of mass transport is always positive and represents the direction of mass flow. In the acoustitron there is no recirculating closed cell for the streaming flow in the waveguide cross-section, but the fluid motion is guided by the annular resonator. When a stack is inserted inside the acoustitron, the general shape of the acoustic streaming flow is changed: the exciting forces for the mean flow are mainly localized inside and near the stack, and far from the stack, the velocity field takes the form of "Poiseuille" flow (Fig. 5b); there are no vortices, except very small ones just near the ends of the stack, as previously observed in the case of a standing acoustic wave. The effect of the viscous boundary layers in the walls of resonator is still present, but it is much less intensive because the sack takes the role of an engine exciting streaming.

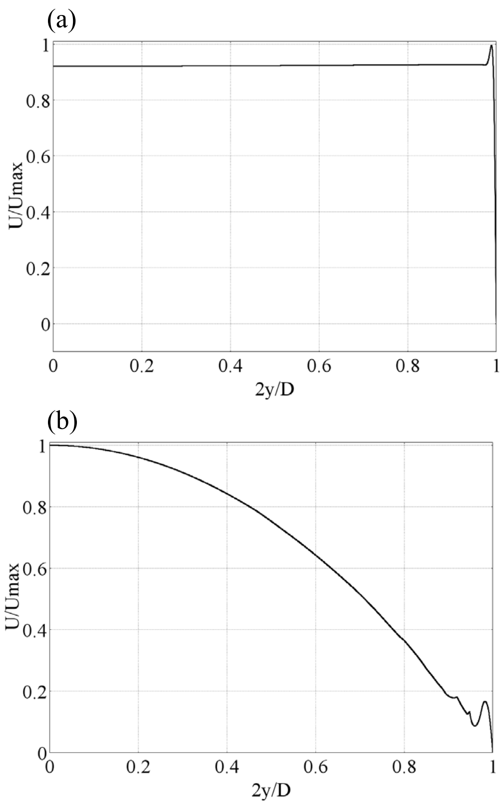

Figure 5: Dimensionless velocity field in half section of developed acoustitron, a- without stack, b- with stack.

5.2 Heat transfer

We consider here heat transfer due to the only heat conduction: the secondary flow in the enclosure is now neglected $(\mathbf{U}=\mathbf{0})$, such that there is no convective heat transfer. The governing equation for heat transfer reduces to:

$$\nabla.(-\lambda_m \nabla T_m) = \langle \mathrm{E}_{dis} \rangle + q \qquad (9)$$

The heat source q caused by nonlinear effects distribution is presented in figure 6: it is localised just near the stack. This source is very small everywhere, except the ends of the stack. This heat source is positive in the side of the moving wall, and negative in the other. The steady-state temperature field for conduction is presented in Fig. 7a, showing a small temperature difference (Fig. 7b) between the two end of stack.

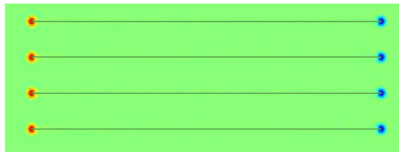

Figure 6: Heat source q inside the stack (bleu -2.37e4 W/m^3, red 2.282e4 w/m^3).

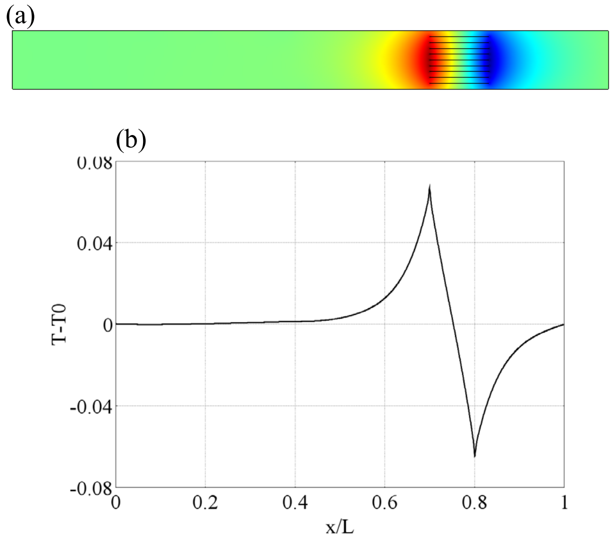

Figure 7: a- Numerical steady-state temperature field inside the resonator (Bleu 293.085 °K, Red 293.217 °K), b- Temperature variation inside the resonator.

6 Conclusion

A general formulation based on the variable of mass transport velocity is presented to compute slow acoustic phenomena. The used variable is the mass transport velocity $\mathbf{U}$. It represents the net mass flow and contains both the average velocity of acoustic streaming and the average mass transfer related to the acoustic perturbation in compressible fluid. Using the velocity of mass transfer, a very convenient form is obtained for the governing equations of fluid dynamics and heat transfer, where the nonlinear exciting forces for acoustic streaming and the term source for heat transfer, are accounted for by new source terms. The formulation is adapted to a resolution by usual computational fluid dynamic tools with the usual numerical methods. The application of this formulation to compute the acoustic streaming generated by a standing or travelling wave shows the simplicity of its numerical resolution. This formulation is well suited to study the acoustic phenomena in various acoustic devices and can be used for diverse applications, for example in micro-fluidics.

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