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Virtual SEA Analysis of a Warship Classification

Gérard Borello

InterAC, 10 impasse Borde-Basse, ZA La Violette, F-31240 L'Union, gerard.borello@interac.fr

In the mid-frequency (MF) range, the number of modes of warships is very large. Predicting acoustic power radiated by the underwater part of the shell becomes difficult with a classical modal synthesis. Statistical methods such as SEA (Statistical Energy Analysis) are also limited in their capability of modelling the structural complexity. Virtual SEA method is introduced to get benefit of both deterministic modal synthesis and SEA approaches to build a fully predictive model in the MF range. Virtual SEA analysis works on an original in-vacuo finite element model (FEM) of the warship. The structural model is identified in the MF range to a numerical SEA model of which parameters are extracted from FEM. First the dynamics of the warship structure is reduced to a statistical FRF (Frequency response function) matrix (between sets of observation nodes). The partitioning into subsystems is computed from this FRF matrix by an attractive algorithm which re-orders nodes to obtain weak coupling between subsystems. Then the FRF matrix computed between all nodes is compressed following the portioning scheme. Finally, using inverse method, the various SEA parameters of the subsystems (modal density, mass and coupling loss factors) are identified from previous compressed FRF. The parametric SEA model provided by this process encapsulates the FEM dynamics in the MF range. This SEA model is then analytically coupled to external fluid to predict the power radiated in water. At the end the virtual SEA model can predict power radiated in fluid under mechanical loading in band-frequency format.

1 Introduction

Due to the large size of warships, the prediction of their dynamic behaviour involves to handle huge number of structural modes. For predicting the noise radiated by the ship in water, the structural modal database has to be coupled with the adjacent fluid involving a stronger computational effort. To cover the mid-frequency range, typically, 200-500 Hz and predicting noise radiation, it may be more efficient to create reduced statistical models of the warship dynamics. The technique of virtual SEA allows compressing a Finite Element model (FEM) into a small Statistical Energy Analysis model (SEA) of which degrees of freedom are the mean frequency-band averaged quadratic energies of subsystems. This technique is applied here to the modelling of the acoustic radiation of a warship in sea water.

2 The SEA method

Virtual SEA (VSEA) has been initially developed for automotive and aerospace applications [1, 2]. The aim of VSEA is to build a valid SEA model in the mid-frequency range made of a few numbers of weakly coupled subsystems. In a SEA model [3], the unknowns are mean modal energies per subsystem. The subsystems energies are cross-coupled by the power-balanced equations which are traducing the energy conservation for each subsystem in a targeted frequency band of analysis of central radian frequency ω_c.

Direct SEA equations

Given a partition, Ω of the system into subsystems, $\Delta\Omega$ indexed by (i), and a vector set of injected powers P^i_{in} due to action of external forces on each subsystem, the power balanced equations may be written as :

$$\frac{P^i_{in}}{\omega_c} = \eta_i E_i + \sum_{j/j \text{ coupled to } i} \left[\eta_{ij} E_i - \eta_{ji} E_j \right] \tag{1}$$

These equations relate the energy of (i)-subsystem to the coupled subsystem energies through coupling loss factors η_{ij} (CLF) and damping loss factors η_i, (DLF). The energy is defined as the product of subsystem mass by its related mean quadratic velocity over both space (the domain of the subsystem) and frequency (the width of the analysis frequency band : $E_i = m_i \left\langle v_i^2(\omega_c) \right\rangle_{\Delta\omega,\Delta\Omega}$

Introducing the modal density $N_i(\omega_c)$, as the subsystem number of resonant modes in a band, and the reciprocity of coupling loss factors, given by (5), (1) may be rewritten as :

$$\frac{P^i_{in}}{\omega_c} = \eta_i N_i \varepsilon_i + \sum_{j/j \text{ coupled to } i} \eta_{ij} N_i \left[\varepsilon_i - \varepsilon_j \right] \tag{2}$$

with $\varepsilon_i = E_i / N_i$

In a matrix-form, (2) expresses as:

$$\frac{\mathrm{P_{in}}}{\omega_c} = \mathbf{L} \cdot \mathrm{N} \cdot \mathbf{I} \cdot \varepsilon \tag{3}$$

$\mathbf{L}$ is the Loss matrix. This matrix is real-valued and its size is given by the number of subsystems.

Its general form is given by:

$$\mathbf{L} = \begin{bmatrix} \eta_1 + \sum_{j_1} \eta_{1j_1} & \cdots & -\eta_{i1} & \cdots & -\eta_{N1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\eta_{1i} & \cdots & \eta_i + \sum_{j_2} \eta_{ij_2} & \cdots & -\eta_{Ni} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\eta_{1N} & \cdots & -\eta_{1i} & \cdots & \eta_N + \sum_{j_N} \eta_{Nj_N} \end{bmatrix} \tag{4}$$

In traditional SEA method, all off-diagonal $\mathbf{L}$-coefficients are obtained using analytical modelling of subsystems and junctions. Due to underlying weak coupling assumption between subsystems, analytical CLF are only provided for adjacent subsystems. Reciprocity states that

$$\eta_{ij} N_i = \eta_{ji} N_j \qquad (5)$$

and then the matrix $\mathbf{L} \cdot \mathrm{N} \cdot \mathbf{I}$ is symmetric.

Inverse SEA equations

(1) or (2) may be also used to infer DLF and CLF from the energies. By exciting at a turn the various subsystems by a known injected power and measuring the transfer energies E_{ji} for each load case, one can build a set of N x N linear equations to identify L

$$\frac{\mathrm{P_{in}}}{\omega_c}\mathbf{I} = \mathbf{L} \cdot \mathbf{E} \Rightarrow \mathbf{L} = \mathbf{E}^{-1} \cdot \mathbf{I}\frac{\mathrm{P_{in}}}{\omega_c} \qquad (6)$$

In practice (6) may be reorganized taking into account the assumption there is no dissipation in junctions which leads to write that the sum of power dissipated in all subsystems is simply equal to the injected power in the source subsystem.

$$\frac{\mathrm{P_{in}}}{\omega_c} = \mathbf{E} \cdot \eta \text{ where } \eta \text{ is the DLF vector} \qquad (7)$$

Subtracting (7) from (6) leads to a set of linear equations relating only coupling loss factors with both energies and injected power, system easier to solve than (6).

This inverse problem process, called experimental SEA or ESEA, has been first introduced in 1980 [6] and improved by N. Lalor [4] and later on by G. Borello and M. Rosen [7] who make ESEA available for industrial applications through SEA-XP software linking fast acquisition system to automated ESEA post-processing.

Due to analytical limitations and to all assumptions embedded in the direct SEA modelling method, ESEA has proven to be a very efficient tool for tuning theoretical SEA models and handling more structural complexity. It also showed practical limits of the analytical SEA modelling scheme for some class of structures [5].

First limit is the partition into subsystems and the weak coupling assumption. How to verify it in practice prior to build a model as it conditions the inverse problem?

Second is the presence of long-distance CLF in the MF range which couple indirectly connected subsystems and may drive the transfers between far away subsystems.

Long distance (indirect) CLF are obtained by ESEA and their addition to SEA network description improves the transmission path analysis.

Nevertheless without any prototype for performing ESEA analysis, indirect CLF cannot be obtained theoretically by analytical SEA. Thus a pure theoretical analytical SEA prediction may not be very reliable especially when working on structure-borne-noise.

So ESEA technique has then to be used in the long term for building progressive expertise and deducing rules for tuning newly created design with the help of analytical SEA.

3 The Virtual SEA method

At design stage, ESEA cannot be applied due to the lack of prototype, then to get expertise in SEA modelling without real physical test available, a straightforward way is to simulate $\mathbf{E}$ using FEM, i.e. to use ESEA virtual testing in place of real ones.

FEM description of system dynamics is carrying much more information than an analytical model. The problem is thus to reduce this amount of information in the SEA sense: to derive consistent relationships between injected power and space and frequency-averaged quadratic velocities in a set of weakly coupled subsystems.

Virtual SEA requires:

1. a way of generating transfer velocities from FEM,
2. a partition into subsystems,
3. a way of identifying SEA parameters.

Generating virtual transfer functions

As MF range is addressed, one needs a fast way to extract MF information from the FEM. This is performed by computing all FEM eigenvalues within a frequency domain compatible with mesh size and exporting modal amplitude at a grid of observation nodes defined by indexes k and l. The complex velocity tensor between a node k and a node l is then obtained by modal summation by independently applying a unit-force excitation at node location in each direction $\{x, y, z\}$.

$$\left\{-\mathbf{M}\omega^2 + \mathbf{K}\right\}\mathrm{X} = \lambda\,\mathrm{X} \Rightarrow \mathbf{V_{kl}} = \sum_i \frac{i\omega \mathrm{X}_i(x_k)\mathrm{X}_i^T(x_l)}{\lambda_i - \omega^2 + i\eta\omega\sqrt{\lambda_i}}$$

As the number of required nodes may be large, typically around 1000, several millions of transfer functions have to be generated under assumption of constant uniform damping $\eta(\omega_c)$, requiring a dedicated fast modal synthesizer solver.

The resulting transfer matrix $\mathbf{V^2}$ between observation nodes is then frequency-band averaged and projected in the direction of maximal excitation and maximal response to allocate to each pair of nodes a single real value in a band.

If n is the number of observation nodes, the final computed transfer matrix, $\mathbf{V_{kl}^{2\,\mathrm{MAX}}}$ has n x n components. The injected power vector Y_l is computed as $real\{\mathrm{V}_{ll}\}$ and also projected in the direction of maximal excitation.

Auto-partitioning of the transfer velocity matrix (finding subsystems)

To compute the spaced-averaged velocity, nodes needs to be partitioned into weakly coupled groups (subsystems). This is achieved using the attractive algorithm developed by G. Borello [1] which applies to $\mathbf{V_{kl}^{2\,\mathrm{MAX}}}$.

The ratio $\mathbf{V_{kl}^{2\,\mathrm{MAX}}} / \left\|\mathbf{V_{kl}^{2\,\mathrm{MAX}}}\right\|$ defines an attractive force between two nodes from which the attraction of a node by a group can be calculated. From an initial partition into subsystems, the attraction force will iteratively move nodes to most attractive subsystems to end up with a final partition. Final groups are qualified by an entropy function indicating the degree of weak coupling or more exactly relating it to the energy gap observed between two distinct subsystems.

Then matrix is compressed on subsystem partition over all k-nodes to get the rectangular transfer velocity matrix $\mathbf{V}_{\mathbf{ijl}}^{2}=\left\langle \mathbf{V}_{\mathbf{lk}}^{2\,\mathbf{MAX}}\right\rangle_{\mathbf{k}\in\Delta\Omega_i,\mathbf{l}\in\Delta\Omega_j}$ (size #subsystems x #nodes) and gives the statistical transfer between the source and the receiver subsystems.

In practice the reduced matrix $\boldsymbol{\varepsilon}_{\mathbf{r}}$ is used instead of $\mathbf{V}_{\mathbf{ijl}}^{2}$ of which components are obtained by performing SEA compression on the local modal energy matrix, $\boldsymbol{\varepsilon}_{\mathbf{kl}}$ of which components are calculated from $\mathbf{V}_{\mathbf{kl}}^{2}$ as $\varepsilon_{kl}=v_{kl}^{2}/4y_k y_l$

$\boldsymbol{\varepsilon}_{\mathbf{r}}$ is proportional to modal energy and is less variant than $\mathbf{V}_{\mathbf{ijl}}^{2}$ and also less sensitive to node choice. We call it the reduced velocity matrix.

Identifying SEA parameters

The identification of SEA parameters is then performed using the modified power balanced equations in two steps:

- Solving for modal density: $\mathrm{N}=\boldsymbol{\varepsilon}_{\mathbf{r}}^{-1*}\dfrac{1}{\omega_c}$
- Solving for CLF: $\eta_{ji}=\dfrac{1}{\omega_c}\mathbf{C}^{-1*}\varepsilon_{ri}/c_{ii}$ where $\mathbf{C}$ is computed from $\boldsymbol{\varepsilon}_{\mathbf{r}}$ [4]

VSEA and ESEA differ in two ways: first the DLF are unknowns in ESEA and input data in VSEA, and second the partitioning process is automatically performed in VSEA while it is a user's guess in ESEA. This is mainly because all nodes in VSEA are both excitation and response nodes from which the attractive algorithm may sort nodes into subsystems. In practice, ESEA driving points are limited in number because they need to be physically acquired, which should be a long process when dealing with millions of transfers.

VSEA is thus improving $\boldsymbol{\varepsilon}_{\mathbf{r}}$ conditioning and it facilitates extraction of real-positive values of modal densities and CLF. The **L**-matrix is then fully identified by the VSEA process in an easier way than in ESEA.

Providing a wavenumber to VSEA subsystems

For 2D subsystems, an equivalent wavenumber is derived from modal density through the knowledge of the surface area of the subsystem:

$$k=\sqrt{\frac{2\omega N}{S}}$$

The virtual wavenumber is a key parameter as it provides the way to couple any VSEA subsystem with any analytical SEA subsystem.

Linking ESEA and VSEA methods

ESEA test and VSEA can also be chained by simulating with VSEA the ESEA test. The simulated VSEA partition is then imported in the ESEA test prior to perform the measurement. ESEA can then take benefit of $\boldsymbol{\varepsilon}_{\mathbf{r}}$ conditioning and has proven to deliver better results in this way, especially when DLF identification is required.

4 VSEA warship application

Extracting eigenvalues

The VSEA analysis has several objectives:

- the prediction of frequency band-averaged structural power flows
- the prediction of radiated noise of submerged warship outer shells

The targeted frequency band is ranging from 100 to 500 Hz.

The FEM represents 1/4 of a warship with shell and decks as seen in figure 1.

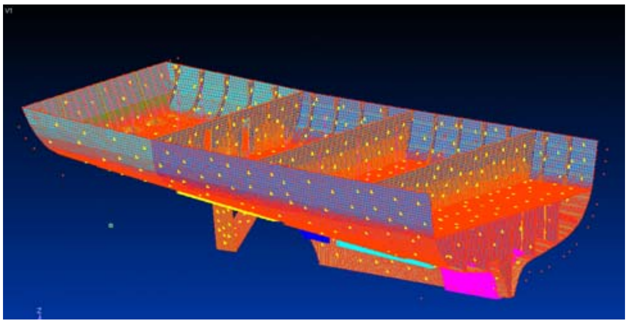

Figure 1: Warship FEM and the grid of observation nodes

Due to its large size, the modal density of the structure is around 1000 modes per Hertz.

In practice about 50000 eigenvalues were extracted from the FEM to cover the 500 Hz frequency band (up to 550 Hz) using Lanczos algorithm in NASTRAN.

Identifying SEA loss matrix using SEAVirt Processor

$\boldsymbol{\varepsilon}_{\mathbf{kl}}$ is synthesized between 1360 observation nodes using SEAVirt processor which is the software automation of VSEA process [1]. $\boldsymbol{\varepsilon}_{\mathbf{kl}}$ is shown in Figure 2 in the 400 Hz 1/3 octave band in normalized dB-amplitude (0 dB is the max value). The matrix is symmetrical and real-valued. The overall dynamic is 131 dB.

Applying the attraction algorithm to this matrix provides 20 subsystems in the bands 400-500 Hz as shown in Figure 3. Identification of the Loss matrix is then performed using the compressed $\boldsymbol{\varepsilon}_{\mathbf{r}}$ as input and returns the modal densities and the CLF between the various subsystems. The L-matrix is very small (20 x 20) and despite this fact the original $\boldsymbol{\varepsilon}_{\mathbf{r}}$ matrix can be reconstructed within less than 2 dB of uncertainty. Several quality reconstruction descriptors are available in SEAVirt to estimate the loss of information due to SEA compression. The easiest is the mean matrix error which gives the related error in dB between each term of the original $\boldsymbol{\varepsilon}_{\mathbf{r}}$ and the reconstructed $\widetilde{\boldsymbol{\varepsilon}_{\mathbf{r}}}$

$\widetilde{\boldsymbol{\varepsilon}_{\mathbf{r}}}=\dfrac{1}{\omega_c}\mathbf{L}^{-1}\cdot\mathbf{I}$ and $Err=10\log\left(\dfrac{\boldsymbol{\varepsilon}_{\mathbf{r}}-\tilde{\boldsymbol{\varepsilon}}_{\mathbf{r}}}{\boldsymbol{\varepsilon}_{\mathbf{r}}}\right)$. Using Err descriptor allows to control the information depredation among several inverse solutions. It may be seen in Figure 4 that the mode number is decreasing above 400 Hz which means the FEM mesh was too coarse in the 500 Hz bandwidth, which is traduced by an apparent increase of stiffness with frequency (i.e. decrease in modal density).

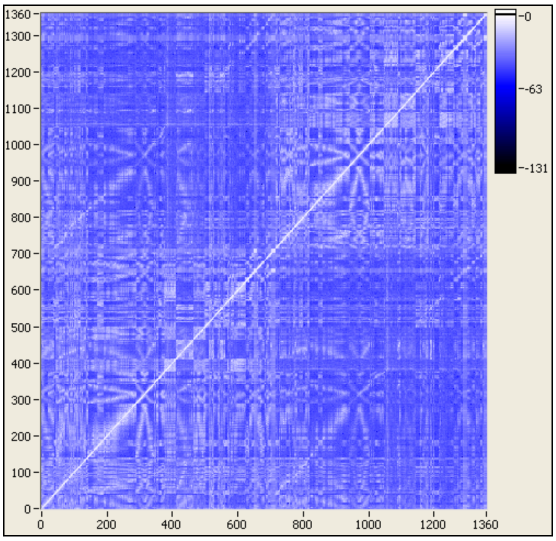

Figure 2: $\boldsymbol{\varepsilon}_r$ matrix between nodes in the 400 Hz 1/3 octave band (x: excitation nodes, y: observation nodes)

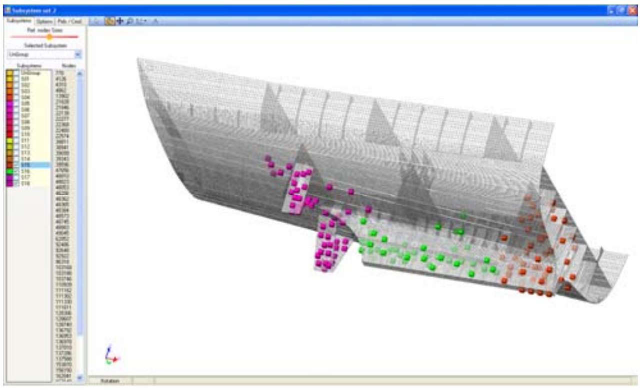

Figure 3: View of 3 different subsystems after auto partition of nodes (20 subsystems identified in 400-500 Hz band)

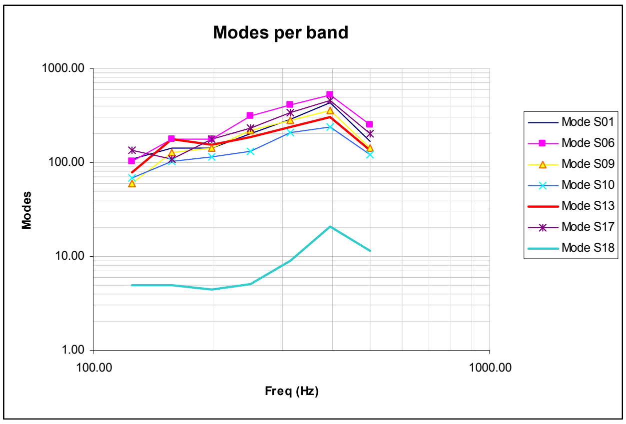

Figure 4: Number of local modes per 1/3rd octave band identified on a set of warship subsystems

Predicting noise radiated by the warship shell

The resulting VSEA model is then run to perform prediction of warship operating conditions. Within the SEA 3D graphical GUI of SEA+, the VSEA "wet" outer shell subsystems are coupled with a large sea water acoustic cavity to simulate infinite medium radiation and loaded with point forces to simulate operating conditions.

The complex radiation efficiency (real and imaginary parts) is computed in polar coordinates [8], assuming the structural vibratory field may be decomposed into infinite plane waves windowed by the shell domain and coupled to an adjacent heavy fluid medium.

The active radiation CLF and the fluid added mass are thus given by:

$$\eta_{rad} = \frac{\mathrm{Re}\{\sigma\}\,\rho c S}{\omega m} \text{ and } m_{rad} = \frac{\mathrm{Im}\{\sigma\}}{\omega}\rho c S \text{ with}$$

$$\sigma(\omega) = \frac{S}{2\pi^3}\int_0^{2\pi}\int_0^{\frac{\omega}{c}}\int_0^{2\pi} H^2(k_x,k_y)\,\frac{\frac{\omega}{c}k_r}{\sqrt{\frac{\omega^2}{c^2}-k_r^2}}\,d\phi\,dk_r\,d\psi$$

with $k_x = k_r\cos\phi - k(\omega)\cos\psi \qquad k_y = k_r\sin\phi - k(\omega)\sin\psi$

Example of radiation efficiency prediction is given in Figure 6.

Some additional mass corrections of structural VSEA CLF are also performed automatically by SEA+ when submerging structural subsystems because of the increase in their modal density modifying both wet and wet-to-dry structural CLF spectra.

The SEA+/VSEA model coupled with the external sea water is shown in Figure 5.

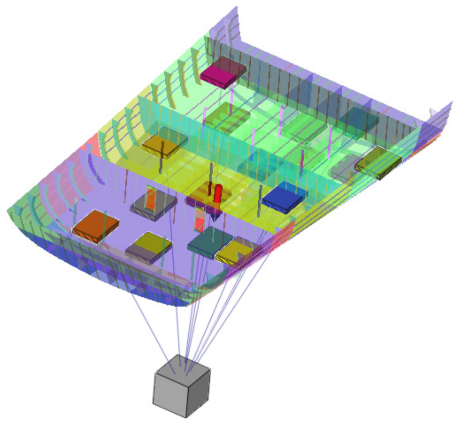

Figure 5: View of VSEA subsystems coupled with heavy fluid

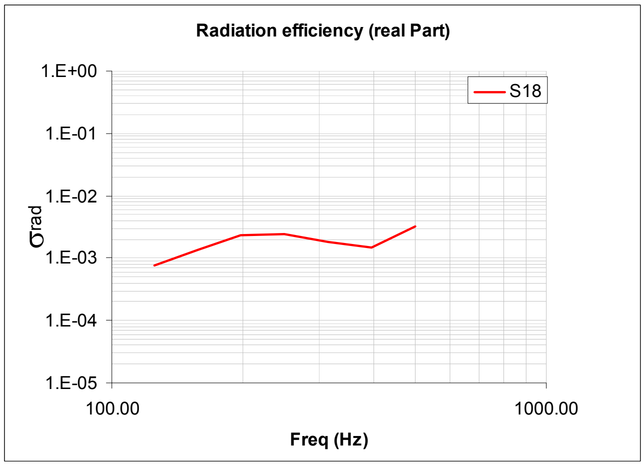

Figure 6: Radiation efficiency of a particular "wet" subsystem

Knowing location of forces due to warship operating conditions, the VSEA radiated power by the various subsystems in the external sea water cavity is predicted applying these forces to the observation nodes at physical driving points [see input mobility plot in Figure 7]. Thanks to the saving of input mobility vector of observation nodes in the SEAVirt database, the point-to-point structural transfer between two nodes k and l part of respectively i and j subsystems can be predicted following the hereafter expansion formula:

$v_{ijlk}^2 = 4 y_l y_k \varepsilon_{ij}$ with ε_{ij} is equal to the mean statistical transfer of modal energy predicted by the SEA model under the given load case.

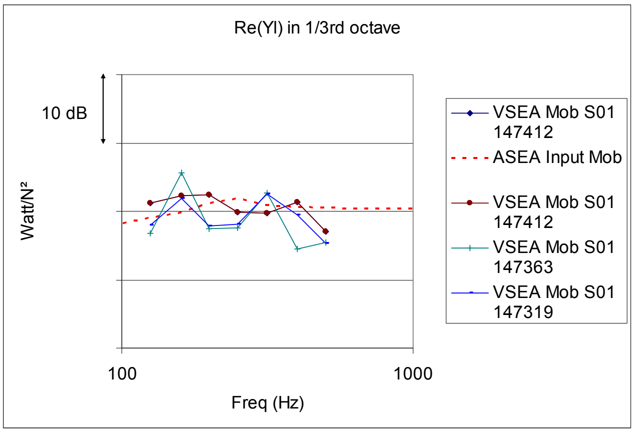

Figure 7: Real part of input mobility at various nodes of subsystem S01 and comparison with analytical equivalent mobility

Then despite the chosen SEA framework, SEA+/VSEA models can take into account local heterogeneity effects due to mobility change at nodes location (Figure 8). This is useful when predicting operating conditions as forces are generally located at reinforced points of the structure with lower input mobility than the mean input mobility of the subsystem which includes them. Testing various force locations and computing the radiated power in the external cavity bring to the fore the most effective driving points. Localization of high radiative risk zones is thus possible due to power flow output from the SEA model as shown in Figure 9.

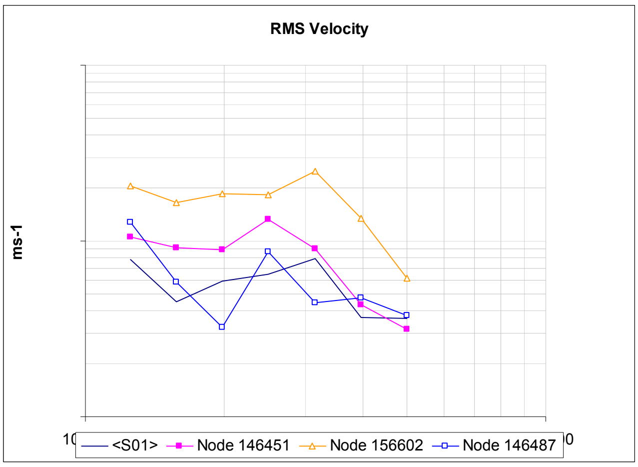

Figure 8: Prediction of structural velocity at various node locations in a given subsystem with the VSEA model and comparison with the mean spaced-averaged subsystem velocity

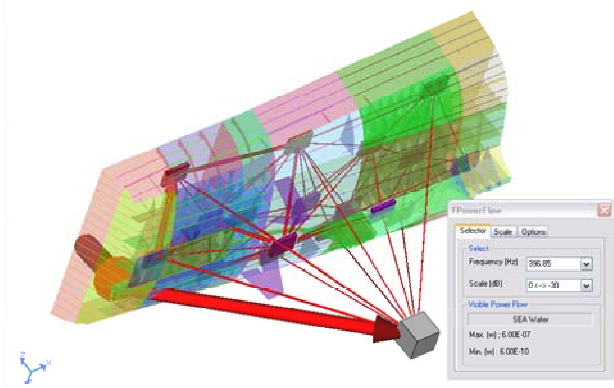

Figure 9: Power flow generated by the outer shell in the sea water cavity

5. Conclusion

VSEA technique as implemented in the SEAVirt processor can transform a warship FEM into a SEA network of cross-coupled subsystems. The benefit of this transformation is to have at disposal a reduced model of the original dynamics (compression rate of about 1/1000) of which components are compatible with standard analytical SEA subsystems. VSEA structural models may be then used to predict acoustic radiated power by linking them to analytical models of acoustic cavities. As VSEA subsystems take into account the real 3D-dynamics of the FEM, they are more accurate than their analytical counter parts.

Specificity of MF dynamics can then be brought to the fore: presence of numerous indirect CLF, evolution of subsystem mass with frequency in parallel to the substructuration which is also frequency-dependant. Thus despite the very high modal density, the auto-partitioning algorithm is showing an amazing low number of subsystems and a strong link between the interior of the ship and the outer shell. VSEA parameters may also be extrapolated to high frequency using internal-to-subsystems analytical representation of both modal density and coupling loss factors. The VSEA process has also been fully automated freeing the engineers of low-level tasks.

By reducing the dynamics a few key parameters VSEA modelling offers a new way to look at structural dynamics which already have helped us in increasing our mid-frequency understanding.

Notations

In an equation involving vector and matrices, matrices are in uppercase and bold characters, while vectors are in uppercase and plain characters.

- Y_l vector, real part of input mobility at all node l
- y_l component of Y_l at a given l-node
- $\mathbf{V}_{kl}^2$ velocity matrix under unit-force field, projected in the direction of maximal response at node k and excited in the direction of max input mobility at node l
- 1 is the excitation power vector used for reconstruction of data. This vector has a unit amplitude on all component. Dimension is equal to number of nodes within a subsystem.
- **I** is the identity matrix

A^{-1*} the star states for a pseudo-inversion of A matrix using singular value decomposition

H^2 is the spatial Fourier's transform of the rectangular window delimited by the subsystem size

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