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Nonlinear Analysis and Modeling of Electrodynamic Loudspeakers

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Many real-world systems exhibit nonlinear behavior that must be taken into account when modeling such systems. In practise, especially in the field of electro-acoustics, nonlinearities appear with increasing input level. These nonlinearities are usually modeled by adding nonlinear parameters to the linear model, but the determination of nonlinear physical parameters is rather difficult problem.
The authors present a simple method for analysis and identification of nonlinear systems, based on swept-sine signal excitation. The method is based on nonlinear convolution, firstly proposed by Angelo Farina [1], which serves for analysis of nonlinear amplitude characteristics. The result of the nonlinear convolution method is the set of harmonic distortion products (higher order nonlinear impulse responses) that can be expressed either in the form of the separated impulse responses or in the form of frequency dependent components. The original method [1] was improved in order to analyze also the phase characteristics of all the nonlinear parts. Therefore, the excitation sweep signal from the improved method [2] has to be setup very properly. The precisely measured nonlinear amplitude and phase characteristics can be used for analysis of the nonlinear system under test, or for identification of a nonlinear model (i.e. generalized Hammerstein model). The method is presented here when studying an electrodynamic loudspeaker from the nonlinear point of view. More precisely the electrodynamic loudspeaker under test is characterized by its electrical impedance leading to Thiele/Small parameters in a nonlinear framework.

1 Introduction

The electrical impedance measurement of a loudspeaker is an important diagnostic tool. It is indeed possible to find out a lot about how a loudspeaker behaves from the knowledge of its electrical impedance.

Since the impedance of a loudspeaker is a frequency-dependent parameter, it must be measured all over the working frequency range of the loudspeaker. Moreover, when exceeding a certain value of input level, the loudspeaker behaves as a nonlinear system with input level dependence and thus it must be also measured for multiple input levels [3].

In this paper we propose a method for nonlinear electrical impedance measurement based on Synchronized Swept-Sine method developed for the analysis and the identification of nonlinear systems [2].

2 Linear impedance measurement

To estimate the impedance of the loudspeaker at any particular frequency ω, we need to measure simultaneously the values of both the current I through the loudspeaker and the voltage U_{hp} across it. The impedance of the loudspeaker Z is then given by

$$Z(\omega) = \frac{U_{hp}}{I}, \qquad (1)$$

where U_{hp} and I are complex-valued.

As almost all the acquisition devices measure the voltage, an additional resistor R, called shunt, is generally used. Since the shunt is in series with the loudspeaker, the current I flowing through it is the same than through the loudspeaker. Then, two values are needed to be measured: the voltage across the loudspeaker U_{hp} and the voltage across the shunt U_r. The impedance Z is then given by

$$Z = R\frac{U_{hp}}{U_r}. \qquad (2)$$

The block diagram of the loudspeaker impedance measurement is depicted in Figure 1.

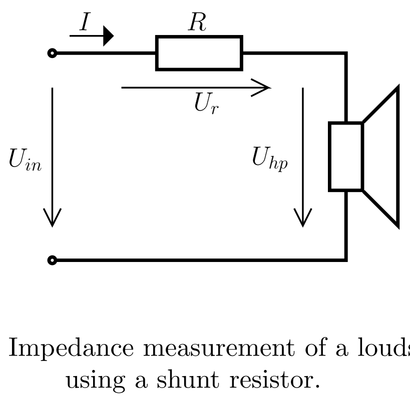

Figure 1: Impedance measurement of a loudspeaker using a shunt resistor.

3 Nonlinear modeling of impedance and admittance

For high levels of input current (or voltage), the loudspeaker can not anymore be considered as a linear device. Then, the relation between current through the

Figure 2: Nonlinear relation between current $i(t)$ and voltage $u(t)$ expressed as a general nonlinear system.

loudspeaker, $i(t)$, and voltage across it, $u(t)$, is a nonlinear relation. As a consequence a harmonic current $i(t) = I_{max}\cos(\omega t)$ creates a distorted voltage $u(t)$ including higher harmonics

$$u(t) = \sum_{n=1}^{\infty} U_n cos(n\omega_0 t + \varphi_n). \quad (3)$$

In the case of a linear system we have $U_{n>1} = 0$, $\Phi_{n>1} = 0$ and we usually define a complex linear impedance as

$$Z_{lin}(\omega) = \frac{U_1}{I_{max}} \exp(j\varphi_1). \quad (4)$$

For a nonlinear system, the definition of linear impedance (voltage and current ratio) falls down, but the generation of higher harmonics allows to model the nonlinear impedance Z_{nonlin} as a a general nonlinear system with input $i(t)$ and output $u(t)$ (Fig. 2).

The nonlinear systems are usually modeled by Volterra series [4], or by structures such as Hammerstein or Wiener models [5]. Considering the nonlinear impedance as a nonlinear system, we can model it by generalized polynomial Hammerstein structure (Fig. 3) and we can write

$$u(t) = \sum_{n=1}^{N} i^n(t) * z_n(t), \quad (5)$$

where $z_n(t)$ is inverse fourier transform of $Z_n(\omega)$ and N is the number of nonlinear terms chosen by user. The higher the value of N the higher the accuracy of the identification.

In the same way, we can define a nonlinear admittance when replacing current by voltage and vice-versa (5). This case can be more suitable for standard measurements, because it is simpler to keep the voltage sinusoidal and measure the distorted current. In that case, we model the admittance Y_{nonlin}.

If we consider only the real parts of the nonlinear impedance $R_n = \Re\{Z_n\}$, we can write

$$u(t) = \sum_{n=1}^{N} R_n i^n(t) = R_1 i(t) + R_2 i^2(t) + R_3 i^3(t) + \ldots. \quad (6)$$

As a consequence the impedance (or its real part - resistance) measured in linear way varies with current $i(t)$ in relation $u_R(i) = R(i)i$ as,

$$R = R_1 + R_2 i(t) + R_3 i^2(t) + \ldots = \sum_{n=1}^{N} R_n i^{n-1}(t). \quad (7)$$

This definition of nonlinear impedance (or admittance) is sometimes used [6] to derive the nonlinear Thiele/Small parameters $R_e(i)$ (electrical resistance of

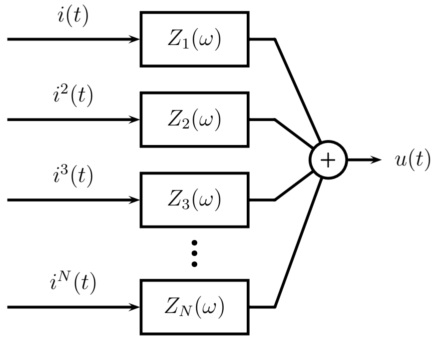

Figure 3: Nonlinear relation between current $i(t)$ and voltage $u(t)$ expressed in a form of generalized polynomial Hammerstein model.

voice coil), $Bl(i)$ (electrodynamic driving parameter), R_{ms} (mechanical damping parameter), $k(i)$ (mechanical stiffness), $M_{ms}(i)$ (equivalent mass of moving coil) and $L_e(i)$ (inductance of voice coil).

4 First harmonic measurement

One of the methods used to estimate the nonlinear impedance coefficients Z_1, Z_2, Z_3, … is to measure the nonlinear impedance in a "linear way" for different amplitudes [6]. This measurement is usually performed using a gain-phase analyzer or an impedance analyzer that filters out the higher harmonics products and calculates the impedance as defined in (4). Unfortunately, this measurements gives just the first harmonic dependency but not the nonlinear dependency as a whole, nor the nonlinear dependency of a linear part, as checked below.

For the sake of clarity, we consider here the real part of the nonlinear impedance $R = \Re\{Z\}$ and a nonlinear system with coefficients $Z_{n>3} = 0$. In that case, we can write

$$u_R(i) = R_1 i + R_2 i^2 + R_3 i^3. \quad (8)$$

When providing a harmonic current $i = I_{max}\cos(\omega_0 t)$ at the input of the (nonlinear) system, the nonlinear device produces a nonlinear voltage

$$\begin{aligned} u_R &= R_1 I_{max}\cos(\omega_0 t) \\ &\quad + R_2 I_{max}^2 \cos^2(\omega_0 t) \\ &\quad + R_3 I_{max}^3 \cos^3(\omega_0 t). \end{aligned}$$

Using trigonometric power formulas [7] we can write

$$\begin{aligned} u_R &= \frac{1}{2} R_2 I_{max}^2 \\ &\quad + \left(R_1 I_{max} + \frac{3}{4} R_3 I_{max}^3\right)\cos(\omega_0 t) \\ &\quad + \frac{1}{2} R_2 I_{max}^2 \cos(2\omega_0 t) \\ &\quad + \frac{1}{4} R_3 I_{max}^3 \cos(3\omega_0 t). \end{aligned}$$

The first harmonic of the voltage is then expressed as

$$u_{1R} = R_1 I_{max} + \frac{3}{4} R_3 I_{max}^3, \quad (9)$$

and it is obvious that, if only first harmonic is considered, then the influence of the third order R_3 is also taken into account in u_{1R}. Consequently, all odd orders will influence the first harmonic. In contrary, all even orders will not. In other words, odd and even orders are mutually uncorrelated.

If the nonlinear device is measured for several input levels with linear techniques (impedance analyzer), only first harmonic is considered. This kind of measurement can be sufficient if there exists a nonlinear differential equation describing the nonlinear device and solved for the first harmonic [8], but the first harmonic dependency cannot be understood as a nonlinear impedance of the device under test.

5 Measurement of nonlinear impedance / admittance using Synchronized Swept-Sine method

In previous section, it has been shown that all the higher harmonics should be taken into account when dealing with nonlinear systems. The nonlinear system can be described either by Volterra series [4], or by another simpler model such as generalized Hammerstein or Wiener model [5]. However, these nonlinear models are independent of the amplitudes of the input signal, that is, in the case of loudspeaker, of the actual voice-coil position [9]. In case of a loudspeaker and for small voice-coil displacements, the models fit well [10], but the higher the displacement the lower the accuracy of the model.

Recently developed Synchronized Swept-Sine method for identification of nonlinear systems [2], based on an input exponential swept-sine signal [1] allows a robust and fast one-path analysis and identification of the unknown nonlinear system under test. It allows to estimate the nonlinear impedance coefficients Z_n in amplitude and phase (Fig. 2).

First, a swept-sine signal exhibiting an exponential instantaneous frequency is generated and used as the input signal of the nonlinear system under test. If the measured value is impedance, the swept-sine signal is equal to the current passing through the loudspeaker and the voltage is distorted and vice versa for admittance. The distorted signal is convolved with the time-reversal replica of the input signal what separates the higher order nonlinear impulse responses $z_n(t)$ that can be expressed in the more usual form of frequency dependent impedance components $Z_n(\omega)$. Mathematical derivation of the method is detailed in [2].

6 Results : first harmonic vs linear part

In this section, we compare the first harmonic and the linear part measurements as defined in section 4. An electromagnetic loudspeaker with diaphragm diameter 14 cm, impedance 4 Ω and resonance frequency 77 Hz has been measured using nonlinear swept-sine technique described in [2]. This technique permits to measure the frequency dependency of higher harmonics in both amplitude and phase, within one measurement, and to estimate the nonlinear model of the measured nonlinear device in the form of generalized Hammerstein model.

To avoid any nonlinearities from other devices than the loudspeaker, an amplifier with negligible distortion has been used. The used amplifier acts as a source of voltage. The voltage across the loudspeaker is then kept linear and constant in amplitude. The current changes in nonlinear way due to the nonlinear impedance. That is why the nonlinear admittance measurement is used.

The results are presented in Fig. 4-6. In Fig. 5, the admittance frequency dependence first harmonic is depicted for several values of input voltage between 0.5 and 5 Volts. As explained in section 4, the admittance value consequently includes the linear and the odd order contributions. In figure 6, the linear contribution is plotted thanks to the generalized Hammerstein model. This first order admittance represents the purely linear contribution of the nonlinear system under test.

It is noticeable that higher orders influence the first harmonic. In figure 4, the resonance frequency is estimated based on both measured techniques. The incorrect measurement based only on the first harmonics results in a completely different resonance frequency shift than when taking into account the higher orders of the nonlinear system (the first order estimation).

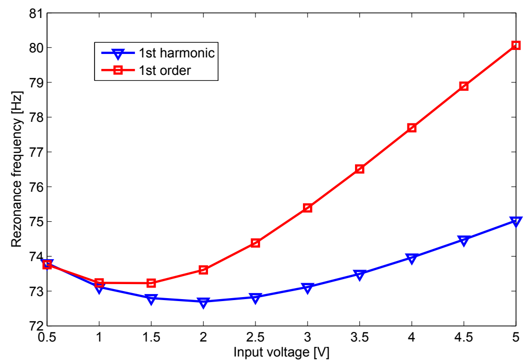

Figure 4: Estimation of resonance frequency of loudspeaker in nonlinear mode using measured data from first harmonic (blue-triangled) and from first order of Hammerstein model (red-squared) for several values of input voltage between 0.5 and 5 Volts.

7 Conclusion

In this paper, two ways of measuring a nonlinear admittance are compared, both theoretically and experimentally. We prove that electrodynamic loudspeaker nonlinear contributions have to be carefully taken into account for example through a generalized Hammerstein modeling [2]. The works in progress aim at physically interpreting both the linear part and the first harmonic measurements regarding a nonlinear Thiele/Small model of electrodynamic loudspeaker.

Acknowledgments

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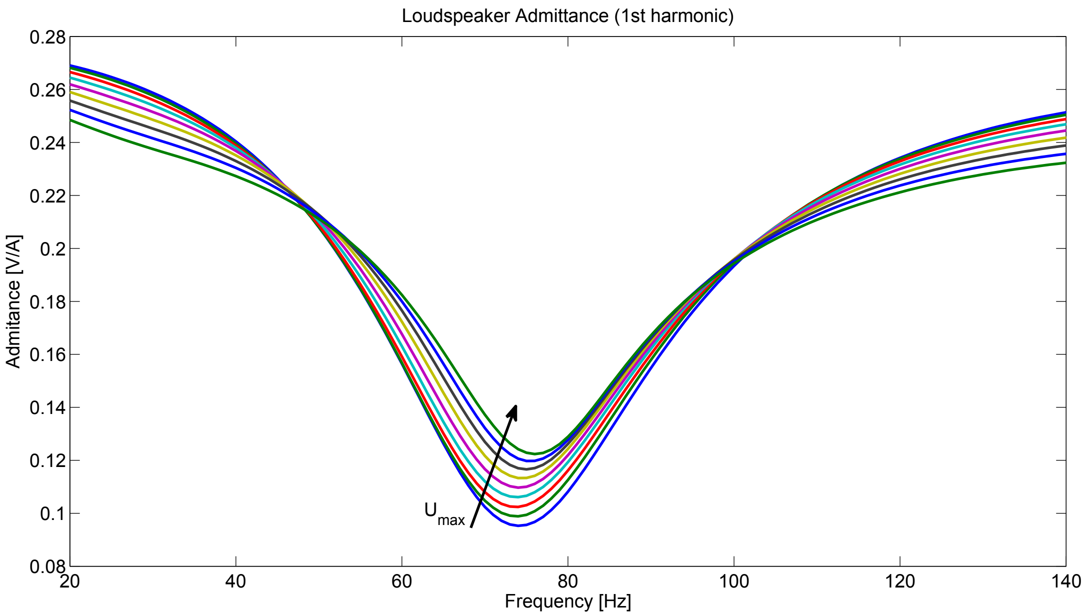

Figure 5: Admittance of the loudspeaker under test measured as a first harmonic of distorted current and linear voltage ratio (influence of higher orders is not taken into account). Measurements done for several values of input voltage between 0.5 and 5 Volts.

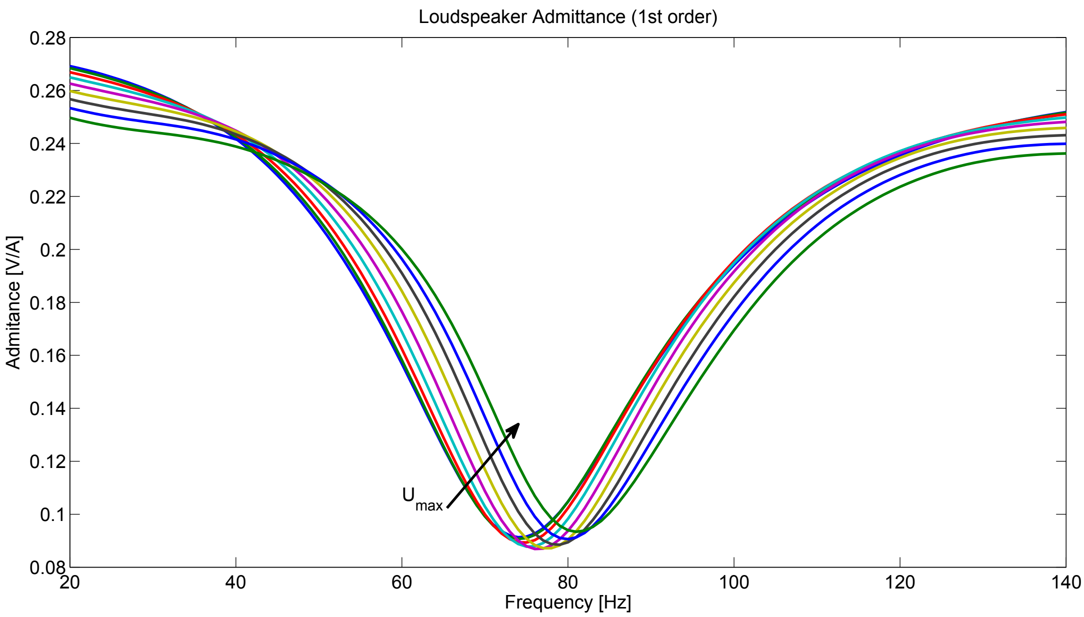

Figure 6: The linear part of the admittance of the loudspeaker under test derived from the Hammerstein model (influence of higher orders taken into account). Measurements done for several values of input voltage between 0.5 and 5 Volts.