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Recent progress in vibration reduction using Acoustic Black Hole effect

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The current paper presents theoretical and experimental results of vibration reduction of beams and plates using Acoustic Black Hole (ABH) effect. ABH is a passive technique which uses properties of wave propagation in beam or plates of varying thickness. It is based on related to gradual decrease of the thickness that leads to the decrease of phase and group velocity of flexural waves. Thus, under certain conditions, it can be shown that waves stop propagating and do not reflect back from the edges. However, the practical implementation of the above theory is rather cumbersome task from a technological standpoint because very thin structures should be manufactured. This is why the area of ABH should be treated additionally using a very thin layer of conventional damping materials. One dimensional and two dimensional configurations for the design of ABH are presented, adapted to the cases of beam and plates respectively. In the present study a brief theory of ABH and number of numerical and experimental results are given. A numerical approach able to evaluate the reflection and impedance matrices of a beam and consequently the point mobility is outlined. The experimental results encompass vibration reduction of elliptical plates. Moreover, a thermal ABH has been designed and tested using a beam made of a shape-memory polymer. As a result, all theoretical and experimental outcomes demonstrate excellent vibration reduction properties, thus, ABH could be used as an alternative approach to vibration mitigation. The advantages and limitations of this technique are discussed as well.

1 Introduction

Acoustic Black Hole (ABH) is a relatively new approach to damping structural vibrations of beams and plates. The theory of ABH is based on the fact that flexural waves in beams and plates slow down if their thickness decreases. This was successfully used by Mironov [1] to establish a power-law relationship between local thickness *h* and the distance from the edge *x*:

$$h(x) = \varepsilon x^m \quad (m \geq 2), \tag{1}$$

that provides an infinite travel time for a wave to reach the edge of a beam when the truncated thickness of the profile tends to zero. Under such circumstances the flexural waves stop propagating and the reflection coefficient becomes equal to zero. This is the basic principal of ABH as described by Mironov.

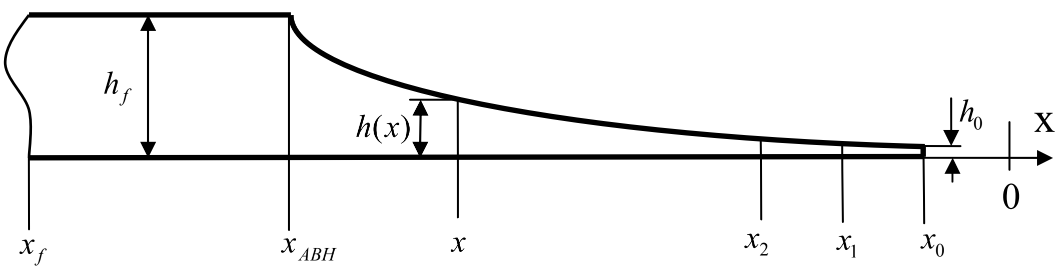

Figure 1: 1-D ABH configuration, corresponding geometrical values are given in Table 1.

Because of technological difficulties manufactured beams and plates with power law profiles always exhibit truncations at certain distance x_0 from the coordinate origin, with a non-zero thickness (see e.g. Fig. 1). This is why a reflected wave by truncated edge always occurs, which partly cancels the effect of ABH and makes its practical application unattractive. However, a recently proposed approach has combined the use of power-law profile wedge (ABH) with a thin absorbing film covering fully or partially the treated area [2-4] leading to the so called ABH effect. Thus, the additional use of a conventional damping technique could overcome to some extent the unpleasant consequences of the truncated thickness profile. In more details, Krylov has used a geometrical acoustic approach to describe the propagation of flexural waves towards a free edge. The effect of a thin absorbing film was taken into account using the model of Ross-Ungar-Kerwin [5] for constrained layer damping in the special case of extensional damping (no constraining layer). As a result, the reflection coefficient was analytically expressed as simplified formulae for different power law profiles of order m=2, 3, 4 and sinusoidal profile.

Furthermore, on the basis of Krylov's approach a number of experimental and numerical results have been reported in [6, 7]. They studied vibration reduction in elliptical plates using ABH pit placed in one of their focii. Instead of analytical description of the problem, a novel numerical approach has been applied [7]. The integration technique is based on state vector formalism which leads to Riccati equation for structural impedance, thus, calculating a system of ODE's one could obtain the impedance matrix and, consequently, the matrix of reflection coefficients. Therefore, more precise evaluation of ABH effect can be achieved taking into account different geometrical discontinuities of structure and absorbing film.

Besides the physical reduction of the thickness, the phase velocity could be decreased by modifying Young's modulus of the material. Thus, if a beam or plate material has the property to reduce its modulus of elasticity as a function of spatial coordinate x then ABH can be designed even without decreasing the thickness. Thus, employing a material whose rigidity depends on spatial coordinates could be used as an alternative of the above mentioned two ABH approaches, designed by using a power-law profile and additional absorbing layer.

The aim of this paper is to report recent numerical and experimental results of designing, modelling and manufacturing ABH as an effective passive vibration damper. For this purpose, in section 2 the basic principles of the numerical model of a 1-D ABH are presented. In section 3 brief notes are given regarding the modelling of thermo-mechanical ABH. The next section is devoted to numerical and experimental results obtained for beams, and elliptical, rectangular, and polygonal plates. In the last section a number of conclusions are withdrawn.

2 Modelling of an ABH

The varying thickness of non-uniform 1-D structure shown in Fig. 1 is given by Eq. (1) for m=2. Its ABH model is based on classical beam theory: Euler-Bernoulli hypothesis are assumed. The vibrational state of the beam can be described by four variables: the displacement w, the local slope θ, the shear force F and the bending moment M. Harmonic motion is supposed (time factor $e^{i\omega t}$ is assumed) and all variables depend only on the spatial coordinate x. In this context, the four variables can be grouped in a state vector:

$$\mathbf{X} = \begin{bmatrix} w & \theta & F & M \end{bmatrix}^T , \tag{2}$$

Eq. (2) is a compact formulation of the Euler-Bernoulli model [8, 9]. The state vector is composed of two kinematic and two force variables and the local impedance matrix $\mathbf{Z}$ can be defined as follows:

$$\begin{bmatrix} F & M \end{bmatrix}^T = j\omega \mathbf{Z} \begin{bmatrix} w & \theta \end{bmatrix}^T , \tag{3}$$

where $\mathbf{Z} = \begin{bmatrix} Z_1 & Z_2 \\ Z_3 & Z_4 \end{bmatrix}$.

It can be shown that the impedance matrix $\mathbf{Z}$ is the solution of the Riccati equation:

$$\frac{\partial \mathbf{Z}}{\partial x} = -\mathbf{Z}\mathbf{H_1} - j\omega \mathbf{Z}\mathbf{H_2}\mathbf{Z} + \frac{\mathbf{H_3}}{j\omega} + \mathbf{H_4}\mathbf{Z} , \tag{4}$$

where

$$\mathbf{H} = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/E_1 I_1 \\ \hline -\rho_1 A_1 \omega^2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right] = \begin{bmatrix} \mathbf{H_1} & \mathbf{H_2} \\ \mathbf{H_3} & \mathbf{H_4} \end{bmatrix}.$$

In the above equation ρ_1 is the material density of the beam, A_1 is the area of the beam's cross-section, E_1 is the Young's modulus of the beam, and I_1 is the moment of inertia of the beam's cross-section. The matrices $\mathbf{H_1}$, $\mathbf{H_2}$, $\mathbf{H_3}$ and $\mathbf{H_4}$ are characteristic matrices of the propagating medium. If impedance $\mathbf{Z}$ can be specified at one point of the medium, solving Eq. (4) gives the way to compute $\mathbf{Z}$ at any coordinate and to derive the response of the medium to any excitation force.

Once the structural impedance $\mathbf{Z}$ is obtained, the reflection matrix $\mathbf{R}$ can be easily defined using a standard wave approach:

$$\mathbf{R} = \left[j\omega \mathbf{Z}\mathbf{E_2} - \mathbf{E_4} \right]^{-1} \left[\mathbf{E_3} - j\omega \mathbf{Z}\mathbf{E_1} \right], \tag{5}$$

where the matrices $\mathbf{E_1}$, $\mathbf{E_2}$, $\mathbf{E_3}$ and $\mathbf{E_4}$ are the components of the eigenvector matrix $\mathbf{E}$ of the matrix $\mathbf{N}$ = -j$\mathbf{H}$ [8]. The scalar components, R_1, R_2, R_3 and R_4, of the reflection matrix represent the reflection of evanescent and propagating flexural waves in the beam and their coupling. R_1 corresponds to propagating waves whereas R_4 corresponds to evanescent waves. The coupling between these two types of waves is characterised by R_2 and R_3.

The complex bending stiffness of the compound beam (beam covered by an absorbing film) can be expressed using the bending stiffness of the beam only [5]:

$$EI(1+i\eta) = E_1 I_1 \left[\begin{array}{c} \left(1+i\eta_1\right) + e_2 h_2^3 \left(1+i\eta_2\right) + \\ \dfrac{3\left(1+h_2\right)^2 e_2 h_2 \left[1-\eta_1\eta_2 + i\left(\eta_1+\eta_2\right)\right]}{1+e_2 h_2\left(1+i\eta_2\right)} \end{array} \right], \tag{6}$$

where EI is the bending stiffness of the compound beam; $E_1 I_1$ is the bending stiffness of the beam only; η is the loss factor of the compound beam; η_1 loss factor of the beam's material; η_2 is the loss factor of the absorbing film's material; E_1 and E_2 are Young's moduli of beam's and absorbing film's materials, respectively; $e_2=E_2/E_1$; δ is the thickness of the absorbing film; h is the local thickness of the beam; $h_2=\delta/h$.

Furthermore, the above modelling of 1-D ABH could be easily extended to an elliptical plate [7]. The shape of elliptical plates induces a focalisation of the waves towards one of its focii if the excitation is applied in the other focus. This geometrical focalisation phenomenon is used for studying ABH in elliptical plates. As was mentioned above, a 1-D ABH is a non-uniform beam having a power-law profile. The corresponding 2-D ABH configuration consists of an axisymmetric pit which thickness gradually decreases to (theoretically) zero towards its centre according to Eq. (1), see Fig. 2. Thus, considering an inner ABH placed in one of the focii, all the generated waves will reach it either directly or after reflections from the free edges. The waves excited in this way could be considered as vibrational rays starting from the driving focus and ending to the damped focus, see Fig. 2. Consequently, the 1-D ABH model can be considered as a phenomenological model for the complex 2-D elliptical configuration.

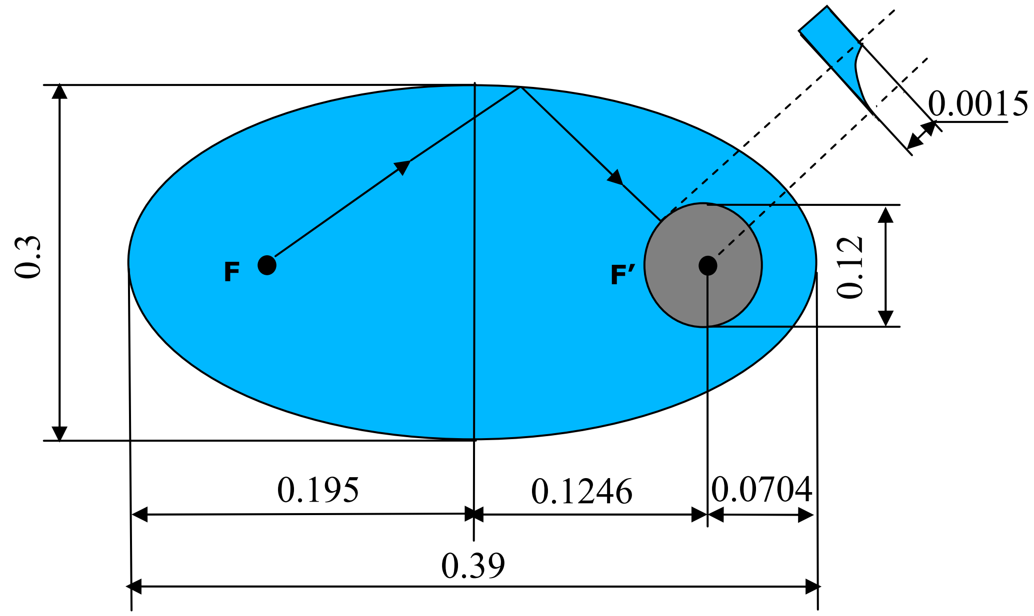

Figure 2: Model of elliptical plate flexural vibrations approximated by 1-D vibrational rays.

3 Thermal modelling of an ABH

ABH phenomenon relies on gradually decreasing phase velocity and its efficiency is improved if the material loss factor increases. These two factors could be modelled not only by decreasing the thickness of the beam and using additional absorbing layer but also by employing a shape memory polymer that is subjected to an appropriate temperature gradient. Using the temperature profile the storage and loss moduli could be modified in order to use the numerical approach described above for studying thermal ABH.

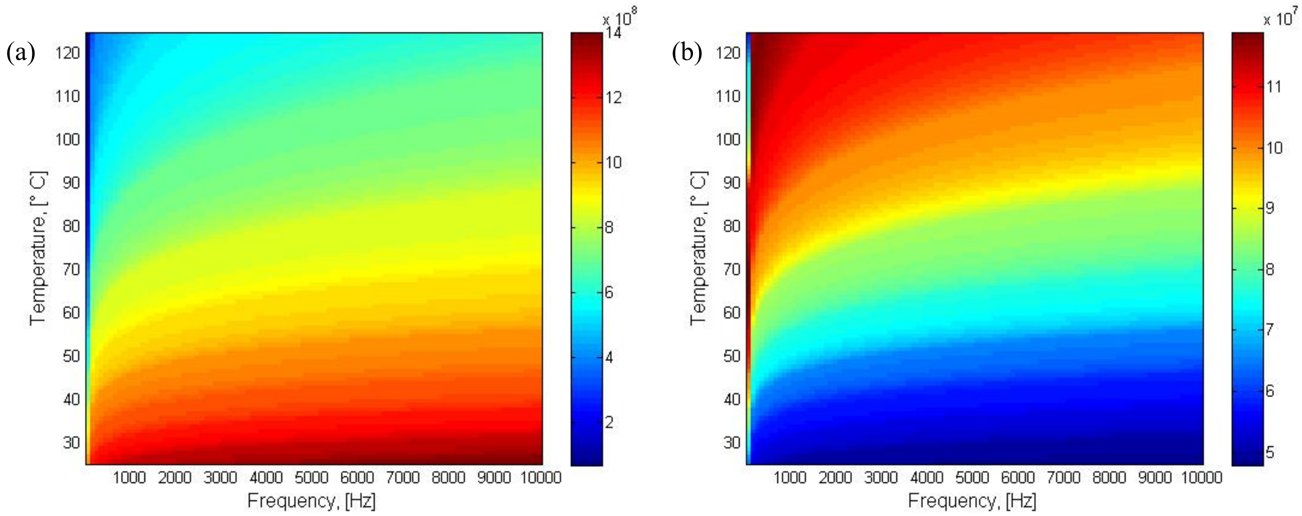

Figure 3: Young’s moduli as a function of temperature and frequency: (a) - storage modulus E’ and (b) – loss modulus E”.

Dynamic mechanical analysis of the polymer material under consideration has been conducted using BOSE Electro Force equipment [10]. As a result, the storage modulus E’ and loss modulus E” were measured as a function of temperature for different frequencies of a cyclic harmonic load [10]. In order to obtain Young’s modulus as a function of temperature and frequency, the time-temperature correspondence that characterises the behaviour of polymer materials was used. This relation allows the measured material properties for a short frequency interval at different temperatures to be replaced by a single frequency curve at a reference temperature. The corresponding shift factor a_T is defined by Williams, Landel and Ferry (WLF) equation [11]:

$$\log a_T = \frac{-C_1(T-T_0)}{C_2+T-T_0}, \tag{7}$$

where T is the temperature at a given moment, $T_0 = 45$ °C is the reference temperature, $C_1 = 18.30$ and $C_2 = 70.54$ are constants characterising the polymer under consideration. Thus, using the master curves and WLF equation the storage E' and loss E" moduli can be specified as a function of temperature T and frequency f (see Fig. 3).

$$E(x,f) = E'(x,f) + jE''(x,f) \tag{8}$$

Once the Young's modulus is expressed as a function of temperature and frequency it can be further transformed to a function of space coordinate and frequency using the temperature profiles measured at each experiment. Thus, for the numerical simulations of a thermal ABH the specified varying Young's modulus $E(x,f)$ from Eq. (8) replaces the constant Young's modulus E used for the mechanical design of an ABH.

4 Results

4.1 Illustration of ABH effect in a beam structure

The efficiency of ABH is estimated by the reflection matrix **R**, which can be computed from impedance matrix **Z**, as shown in Eq. (5). This latter is the solution of Eq. (4), computed with boundary conditions $\mathbf{Z}(x_0)=0$, describing the fact that the end of the beam at $x=x_0$ is free. Numerical simulations are presented for a beam defined by Fig. 1 and Table 1.

Geometrical characteristics, (m)	
$x_0 = -0.01$	$h_f = -0.0015$
$x_{ABH} = -0.06$	$b = 0.0015$
$x_f = -0.08$	$m = 2$
Material characteristics	
Beam	**Absorbing film**
$E_1 = 70$ GPa	$E_2 = 0.5$ GPa
$P_1 = 2700$ kg/m^3	$\rho_2 = 950$ kg/m^3
$\eta_1 = 0.005$	$\eta_2 = 0.05$

Table 1: Geometrical and material characteristics of the beam under consideration

In Fig. 4 the reflection coefficient R_1 is presented as a function of frequency in order to illustrate ABH effect. It is shown that reflection coefficient R_1 for a beam with ABH covered by 700 µm (solid curve) is about 20 % reduced at 10 kHz, whereas for beam with ABH only without any absorbing film (dashed curve) this reduction is only 7 % and for a uniform beam covered by 700 µm absorbing film is even smaller – around 4 %. Therefore, ABH effect leads to much more decrease of reflected waves compared to any of the individual treatment of the beam – ABH only or damping treatment only. Note that the oscillations of reflection coefficient decrease and their periodicity becomes larger when the frequency is increased. Therefore, at theoretical frequencies close to infinity the reflection coefficient does not exhibit any oscillations. The origin of these oscillations might be due to the sharpness and length of the profile.

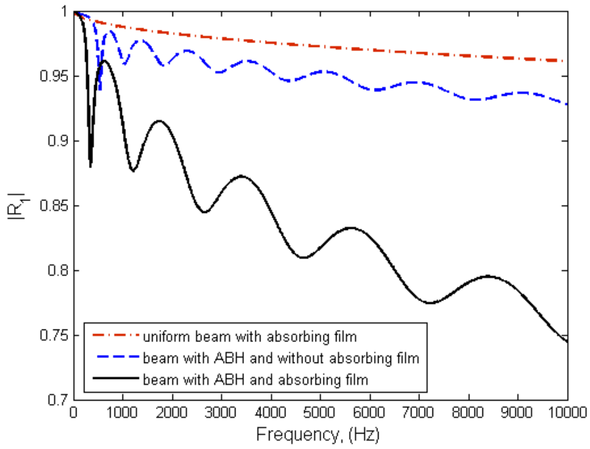

Figure 4: Reflection coefficient for uniform beam (dash-dotted curve), beam with ABH (dashed curve), and beam with ABH covered by absorbing film (solid curve).

4.2 Applying ABH to elliptical plates

The models that were tested are as follows: elliptical plate with ABH and without ABH, elliptical plate with disk of resin placed at the location of the ABH and elliptical plate completely covered by resin. The equipment used was a Polytech Vibrometer Scanning Head – OFV 056, an impedance head Bruel&Kjaer type 8001, a Bruel&Kjaer Conditioning Amplifier, an Amplifier LM 3886, and a Shaker LDS V201. The plates were hanged vertically ('free-free' boundary conditions) and excited by the shaker using a periodic chirp signal. A number of point mobilities and velocity fields were measured.

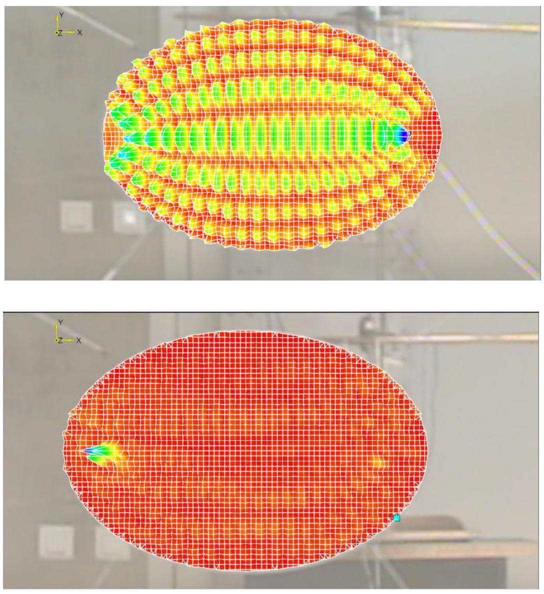

Figure 5: Velocity fields of an elliptical plate without ABH – above, and with ABH – below.

Fig. 5 shows velocity fields of plates without ABH (a) and with ABH (b) at 7613 Hz. The driving force was applied to the left focus whereas the ABH is in the right one. It can be seen that the spatial patterns of the plate without ABH are rather symmetric and equally distributed with some small increases in the area of the right focus, whereas those of the plate with ABH exhibit a concentration of the velocity field in the area of ABH. This concentrated activity is due to both ABH affect and focalisation effect. Thus, the rest of the plate is quite silent

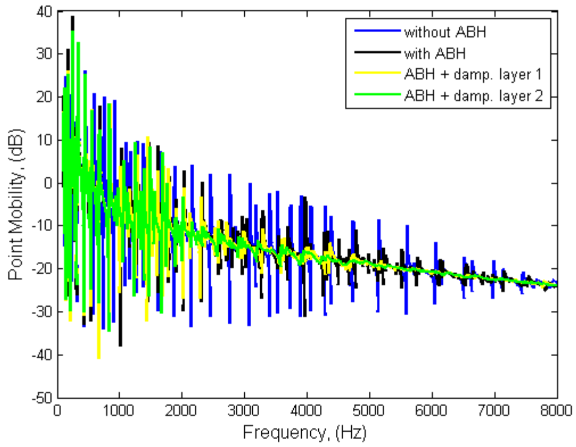

Figure 6: Measured point mobility of the elliptical plate with and without ABH, and with and without damping.

compared to the plate without ABH. Note that due to the focalisation effect treating the right focus of the plate with absorbing film reduces plate vibrations as well. However, this decrease is much smaller compared to the reduction of the plate with ABH and this can be seen in the measured point mobility functions.

The graphs in Fig. 6 represent point mobilities measured at the left focus of the plate. The measured point mobility of the plate with ABH and without an absorbing film is compared to the corresponding point mobility of the plate covered by an absorbing film. Moreover, the point mobility of the plate without ABH is presented as well. Similarly to Fig. 4, the point mobility of the plate with ABH and covered by different absorbing films exhibit the largest reduction. In the range above 2 kHz, the point mobility of the plate with ABH covered by absorbing film 2 was reduced more than 15 dB, compared to the mobility of a plate without ABH. Absorbing film 1 has smaller loss factor compared to absorbing film 2 and respectively its point mobility is larger.

4.3 Vibration damping using a thermal ABH effect

A polymer, vertical, uniform, cantilever beam has been tested in order to demonstrate the thermal ABH effect. The polymer beam was firmly clamped in the vertical plane, see Fig. 7. The impact excitation was initiated by a hammer PCB 086D80, thus, providing a broadband excitation. The response was measured by a laser vibrometer head together with LMS Scadas III acquisition system including Test Lab software. The temperature gradient was introduced by a nickel-chromium resistance wire under high electrical current. The temperature of the beam was measured using an infrared camera. A thermal image and its corresponding temperature profile can be seen in Fig. 7. as well.

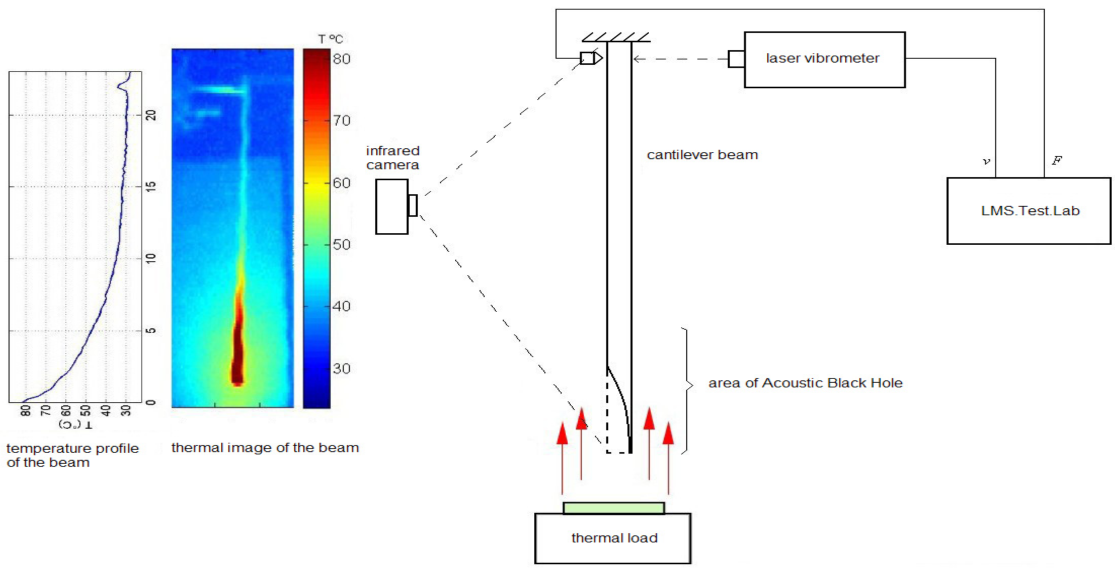

Figure 7: Schematic of the experimental setup.

Fig. 8 shows the measured temperature profiles and corresponding point impedances of the uniform beam under test. The vibration behaviour of the sample without using a thermal load has clear resonance peaks which become drastically reduced once a temperature gradient (i) is applied. Furthermore, it is clearly noticed in Fig. 8 (c) that for the higher temperature profile (ii) the resonance behaviour of the uniform beam is diminished. This confirms that an ABH can be designed controlling the Young's modulus by a temperature gradient even without mechanical decrease of the beam's thickness.

Rising the temperature, the storage modulus *E'* decreases whereas the loss modulus *E"* increases. Furthermore, the analysis shows that phase velocity does not change significantly whereas the loss factor of the beam increases rapidly with the temperature. These characteristics are in contrast to pure mechanically designed ABH where the thickness decreases and reduces the phase velocity and the wavelength. Therefore, it can be concluded that thermal

ABH reduces structural vibrations of the polymer material mainly due to the increase of the loss factor of the structure whereas a mechanical ABH due to the decrease of the phase velocity of propagating waves.

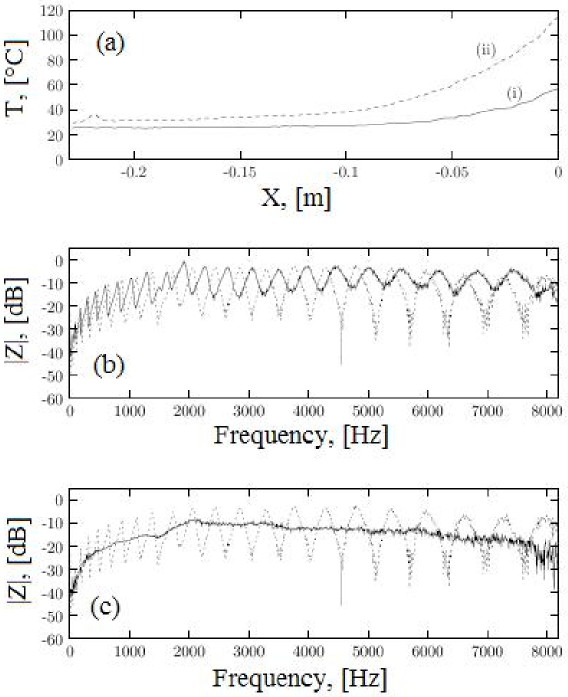

Figure 8: Measured temperature profiles (a) and point impedances (b) corresponding to profile (i) and (c) corresponding to profile (ii).

The different way of damping of vibrational energy between thermally and mechanically designed ABHs defines different frequency regions of their application. For ABH with decreasing thickness vibration reduction is rising at high frequencies [2-4,] whereas for thermal ABH it is relatively independent from the frequency except at very low frequencies. In fact, using certain materials with appropriate characteristics a thermal ABH could be designed such that structural vibration reduction will be due to both the decrease of phase velocity and increase of the material loss factor in equal parts. Thus, the vibration damping using thermal ABH could be further improved extending the frequency range of application and overcoming the required, sophisticated thickness profile.

4 Conclusions

In the present paper, the recent progress in modelling and designing a new alternative approach to damping structural vibrations in beams and plates has been reported. It was outlined the theoretical fundament of ABH and the opportunity to study numerically this phenomenon in 1-D structures. The experimental part contains measurements on elliptical plates. All of them demonstrated significant reduction of resonant peaks of measured point mobilities. This is due to the focalisation of propagation waves towards ABH and to the ABH effect itself. The thermal modelling of ABH showed that the measured vibration reduction is mainly due to the increased loss factor of the material rather then due to the decrease of phase velocity as it is in mechanically designed ABH. Nevertheless, a thermal ABH could suppress vibrations even better than mechanical ABH in event that a material with appropriate characteristics could be found. Finally, in the light of all reported results, it can be concluded that the application of ABH as an alternative method of damping structural vibrations becomes a realistic opportunity.

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References

[1] M.A. Mironov, Propagation of a flexural wave in a plate whose thickness decreases smoothly to zero in a finite interval, *Soviet Physics—Acoustics* **34** (1988), pp. 318–319.

[2] V.V. Krylov and F.J.B.S. Tilman, Acoustic 'black holes' for flexural waves as effective vibration dampers, *Journal of Sound and Vibration,* (2004) **274**, 605–619.

[3] V.V. Krylov, New type of vibration dampers utilising the effect of acoustic 'black holes', *Acta Acustica united with Acustica,* (2004) **90** (5), 830–837.

[4] V.V. Krylov and R.E.T.B. Winward, Experimental Investigation of the Acoustic Black Hole Effect for Flexural Waves in Tapered Plates, *Journal of Sound and Vibration*, (2007) **300**, 43-39.

[5] D. Ross, E.E. Ungar and E.M. Kerwin Jr., Damping of plate flexural vibrations by means of viscoelastic laminae. In: J.E. Ruzicka, Editor, *Structural Damping*, Pergamon Press, Oxford (1960), 49–87.

[6] F. Gautier, J. Cuenca, V.V. Krylov, L. Simon, Experimental investigation of the acoustic black hole effect for vibration damping in elliptical plates, Acoustic'08, Paris, June 29th – July 4th, 2008.

[7] V.B. Georgiev, J. Cuenca, F. Gautier, M.A. Moleron Bermudez, L. Simon, V.V. Krylov, Numerical and experimental investigation of the acoustic black hole effect for vibration damping in beams and elliptical beams. EURONOISE, October 26-28, 2009, Edinburgh, Scotland, (on CD).

[8] Z. Wang, A.N. Norris, Waves in cylindrical shells with circumferential submembers: a matrix approach, *Journal of Sound and Vibration* (1995), **181**(3), 457-484.

[9] F. Gautier, M.H. Moulet, J.C. Pascal, Reflection, transmission and coupling coefficients of longitudinal and flexural wave at beams junctions. Part I: measurement methods, Acta Acustica United with Acustica, **92** (2006), p.982-997.

[10] Jan Klesa, Experimental evaluation of the rheological properties of veriflex shape memory polymer. *Master's project. Department of Applied Mechanics, Franche-Comté University*, 2009.

[11] M.L. Williams, R.F. Landel, and J.D. Ferry, The temperature dependence of relaxation mechanisms in amorphous polymers and other glass-forming liquids. *Journal of the American Chemical Society*, 77:3701-3707, 1955.