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Extended angular spectrum method for calculation of higher harmonics

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Ultrasound imaging in the superharmonics band is getting important in the ultrasound community, since all the advantages of the second harmonic imaging modality are further increased at higher harmonics (3^{rd}, 4^{th}, ...). Then, a fast, reliable and relatively accurate modelling and estimation tool of ultrasound field at higher harmonics is of clear interest. Different numerical solutions (in time or frequency domain) of KZK equation are mostly used. Although the KZK simulators are largely used to understand the nonlinear propagation, they are not appropriated to simulate steered beams, propagating through inhomogeneous media. Moreover, simulations based on KZK equation are time consuming. We propose an alternative method by solving the Westervelt equation based on the Angular Spectrum Method (ASM). The idea consists in separating the equation in n equations (n is number of considered harmonics). For each harmonic component, a nonlinear wave equation is deduced, where the n-th harmonic depends on the previously calculated harmonics. Since the solution of the nonlinear wave equation is a known mathematical problem, the calculation is fairly simple. Coordinate transformation can be used to simulate steered propagation. Furthermore, modelling the solution of the nonlinear equation in the frequency domain (hence the name angular spectrum method) facilitates the introduction of the attenuation in the model. The solution of the nonlinear wave equation was implemented in Matlab software. The calculation of ultrasound field up to third harmonic showed good agreement with experimental measurements performed in a water tank. The main beam width and side lobe levels differ between simulation and measurements by lessthan 5%. The calculation of the pressure field was about 15 times faster than the KZK solution.

1 Introduction

Modeling of the nonlinear propagation of the ultrasound wave is of great interest in research community, since it gives valuable insight in the prediction of the behavior of the ultrasound pressure field at higher harmonics. Simulators of nonlinear propagation are usually based on KZK equation [1, 2]. The equation accounts for the thermo viscous absorption and nonlinearity of the medium and diffraction due to the finite size of the transducer. Using a paraaxial approximation, Aanonsen et al. [3] presented a spectral domain calculation method for solution of KZK equation. Lee and Hamilton [4] presented a time domain method for circular symmetric transducers. Christopher [5] presented a method where the diffraction and attenuation substep of the propagation is achieved in time domain, while nonlinear substep is modeled as spectral solution of lossless Burgers equation. This approach is not limited by parabolic approximation and allows propagation through multiple layers. Similar method has been presented also in Ref. [6].

All the aforementioned method use stepping methods. The propagation from source to region of interest is broken in to smaller steps. The effects of nonlinear propagation (absorption, nonlinearity of medium and diffraction) are considered to be independent on small distance. As such they can calculated independently. This approach is computationally very long. Even with the most modern computers the simulation can take up to an hour. Recently [7, 8] have presented a fully angular spectrum method for estimation of the second harmonic. The method is based on the modeling of the diffraction in optics [9]. By doing so there is no need to calculate all the intermediate steps from source to region of interest. This approach is indeed needed, since second harmonic imaging provides several advantages over fundamental [10–12].

Lately also super harmonics (third, fourth, fifth and so forth) are being investigated for possibility of imaging and ultrasound diagnosis. This is possible with the new transducer designs [13]. It has been already shown to improve ultrasound contrast harmonics imaging [14–16] and further enhance tissue harmonic imaging [17]. These findings drive the need for relatively accurate, fast and reliable simulation of the ultrasound field at super harmonics.

In this paper we present an extension of angular spectrum method for calculation up to $3-$th harmonic. Reader will find it very easy to extend the method even in the higher harmonics. Fig.1 is presenting the general

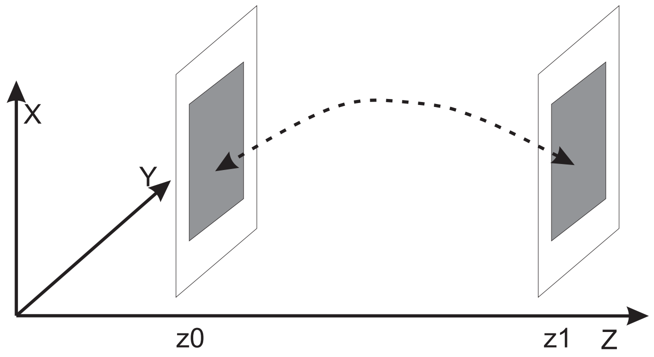

Fig. 1: Propagation of a plane from z_0 to z_1

idea behind modeling nonlinear propagation with angular spectrum method. Consider that X and Y are elevation and transversal coordinates respectively and that wave is propagating in positive Z direction. If wave is know at some distance z_0, then calculation of z_1 is simple matter of (1) calculating angular spectrum using Fourier transform (2) multiplying the obtained spectrum with phase factor and (3) performing an inverse Fourier transform to return back to time domain. With this we have achieved that the calculation of all the intermediate steps are avoided and as such the computation is largely reduced.

We begin first with writing the three most important equations of wave propagation. The reader might recognize them as equation of continuity, motion and state. Next a homogeneous wave equation is developed and later extend it into a set of inhomogeneous wave equation. In Sec. the implementation of the solutions of the harmonics is presented. Later we will show the results and compare them to the measurements. At the end we finish with discussion and conclusion.

2 Theory

Consider that the wave is propagating in the medium which is unbounded, homogeneous and inviscid. Than an infinitesimal element $dV = dx\,dy\,dz$ is regarded as a continuous medium and small enough that all acoustic variables (density ρ, pressure p and temperature T) are uniformly distributed. Further more medium is considered as lossless. The compression or expansion of the medium caused by disturbance produces increase or decrease in density ρ compared to equilibrium ρ_0. The changes of density in dV are related to the traveling wave with by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \tag{1}$$

where ∇ is a Laplace operator and $\vec{u}$ particle velocity. Equation (1) shows that density change occurs with the passage of the wave disturbance and after the wave has passed the density returns to its steady state (e.g., ρ_0).

If the element dV has a mass $dm = \rho\, dV$, the applied disturbance or pressure will move the element according to

$$\rho \frac{\partial \vec{u}}{\partial t} = -\nabla\, p \tag{2}$$

From (2) it is clear that all the disturbances are due to the external forcing pressure p and that there are no vortices (irrational wave flow). Equation (1)-(2) are know as the Euler equations[18].

To connect all the acoustic quantities one more equation is needed. In the case of perfect gas density, pressure P and temperature T are defined by $P = \rho\, r\, T$, where r is the gas constant. A much simpler relation is obtained if restriction is made to thermodynamical process. For ultrasound propagation this is a valid assumption. When the medium, during passage of the ultrasound wave exhibits no temperature change the process is called isothermal. The relation between pressure and density is then $P/P_0 = \rho/\rho_0$. For most cases, propagation of the ultrasound wave is adiabatic and reversible. This means that there is some conversion of energy in to heat, but the entropy remains constant. Then, adiabatic law for perfect gas is

$$\frac{P}{P_0} = \Big(\frac{\rho}{\rho_0}\Big)^{\gamma} \tag{3}$$

where γ is empirically determined constant (ratio of specific heats). Now the speed with which the wave is spreading in the medium is defined as

$$c^2 = \Big(\frac{\partial P}{\partial \rho}\Big)_0 \tag{4}$$

where subscript 0 defines constant entropy. Using (3) and taking equilibrium values, speed of the wave is

$$c_0^2 = \gamma \frac{P_0}{\rho_0} \tag{5}$$

Equations (1)-(3) and the speed of ultrasound wave, give a valid description of propagation for small acoustic pressure.

2.1 Wave Equation

For practical reasons (1)-(3) can be combined in to one single partial differential equation. This is done by taking the derivative of (1)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \Big/ \cdot \frac{\partial}{\partial t} \tag{6}$$

to obtain

$$\frac{\partial^2 \rho}{\partial t^2} + \nabla \cdot (\rho \frac{\partial \vec{u}}{\partial t}) = 0 \tag{7}$$

Next, consider the divergence of (2)

$$\rho \frac{\partial \vec{u}}{\partial t} = -\nabla\, p \Big/ \cdot \nabla \tag{8}$$

by writing $\nabla \cdot \nabla = \nabla^2$, (8) gives

$$\nabla \Big(\rho \frac{\partial \vec{u}}{\partial t}\Big) = -\nabla^2\, p \tag{9}$$

by noting that the left side of (9) in equal to second term on the left side of (7) this two terms can be eliminated. Using (4) to express density in terms of pressure and speed of sound one obtains

$$\nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = 0 \tag{10}$$

where speed of sound was evaluated at equilibrium values of density and pressure. This is the linear lossless wave equation for the propagation of ultrasound wave with thermodynamical speed c_0.

2.2 Nonlinear wave equation

In section 2.1 we considered propagation where nonlinear effects can be disregarded. This is usually true for perfect gases. How ever for propagation of the ultrasound in fluids and tissue nonlinear effects must be considered since pressure density relation of the medium is not a straight forward equation (Equation (3)). Then consideration of the equation of state demands the expansion in Talyor series [19–21]

$$p - p_0 = \left(\frac{\partial p}{\partial \rho}\right)(\rho - \rho_0) + \left(\frac{\partial^2 p}{\partial \rho^2}\right)\frac{(\rho - \rho_0)^2}{2} + \cdots \quad (11)$$

Since Talyor expansion series is truncated after the second term this has second order accuracy. By using (4) and defining

$$A = c_0^2 \frac{\rho_0}{p_0}, \qquad B = \frac{\rho_0^2}{p_0}\left(\frac{\partial^2 p}{\partial \rho^2}\right) \quad (12)$$

Equation (11) can be written as

$$p = p_0 + A\frac{\rho - \rho_0}{\rho} + \frac{B}{2}\left(\frac{\rho - \rho_0}{\rho}\right)^2 \quad (13)$$

the quantities A and B are called nonlinearity parameters and define a nonlinearity coefficient β as

$$\beta = 1 + \frac{B}{2A} \quad (14)$$

Equation of motion for fluids is written as

$$\rho\frac{\partial \vec{u}}{\partial t} + \nabla(p + \mathcal{L}) = 0 \quad (15)$$

where

$$\mathcal{L} = \frac{\rho_0}{2}\vec{u}^2 - \frac{1}{2\rho_0 c_0^2}(p)^2 \quad (16)$$

is called the Langrangian density[22, 23]. Equation of continuity when combined with (11) gives

$$\frac{\partial(p + \mathcal{L})}{\partial t} + \rho_0 c_0^2 \nabla \cdot \vec{u} = \frac{\beta}{\rho_0 c_0^4}\frac{\partial (p)^2}{\partial t} + 2\frac{\partial \mathcal{L}}{\partial t} \quad (17)$$

Noting that propagation of ultrasound wave produces no vortices (irrotational flow)

$$\nabla \times \vec{u} = 0 \quad (18)$$

combining (15) and (17) and linearizing the higher order terms (cubic and higher) we obtain a Kuznetsov equation[1]

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\beta}{\rho_0 c_0^4}\frac{\partial^2 p^2}{\partial t^2} - \left(\nabla^2 + \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\mathcal{L} \quad (19)$$

which is exact to the second order equation and is the same as Eq.9 in [22] for non viscous case. In [23] and [22] it was shown that $\mathcal{L}$ accounts for local nonlinear effects and no cumulative effects. Because of that it is a valid assumption to neglect Langrangian density $\mathcal{L} = 0$, hence (19) reduces to the Westervelt equation [24]

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\beta}{\rho_0 c_0^4}\frac{\partial^2 p^2}{\partial t^2} \quad (20)$$

Comparing (20) to (10) it can be noticed that the only difference is the right side. Since on the right side of (20) we have a forcing term, this is known as nonlinear wave equation and it is going to be in further sections as a driving force behind determining the higher harmonics of the ultrasound wave.

3 Method

Let the pressure with fundamental transmission frequency f_0 at the surface of the transducer $z = z_0$ be

$$p(t, x, y, |z_0) = \Re\{P_0 \exp^{\imath \cdot 2\pi f_0 \cdot t}\} \quad (21)$$

where x and y are the lateral and elevation coordinates respectively. Then the two Fourier transformation pairs can be defined as [25]

$$P(f_x, f_y, f_0|z) = \frac{1}{2\pi}\iiint p(x, y, t|z) \exp^{-\imath\frac{2\pi}{c_0}(f_x \cdot x + f_y \cdot y + f_0 \cdot c_0 t)}\, dxdydt \quad (22)$$

$$p(x, y, t|z) = \iiint P(f_x, f_y, f|z) \exp^{\imath\frac{2\pi}{c_0}(f_x \cdot x + f_y \cdot y + f_0 \cdot c_0 t)}\, df_x df_y df \quad (23)$$

where f_x and f_y are the spatial frequencies. A shorter notation will be used further on, $\vec{f} = (f_x, f_y, f)$.

3.1 Solution of linear wave equation

When $\beta = 0$, nonlinear effects during propagation of the ultrasound wave are disregarded. The ultrasound wave propagation is then described by (10). Inserting (21) in (10) and taking the Fourier Transform yields

$$(\nabla_\perp^2 + k(\vec{f})^2)\underline{P}_1 = 0 \quad (24)$$

where $k(\vec{f}) = \frac{2\pi}{c_0}\sqrt{f^2 - f_x^2 - f_y^2}$ is the wave vector. The fundamental pressure field $\underline{P}_1$ at an arbitrary distance $z = z_1$ is then obtained by multiplying transmitted field $\underline{P}_0$ with a kernel function Hp and taking the inverse Fourier transform

$$p(x, y, t, z_1) = \mathcal{F}^{-1}\{\underline{P}_0 \cdot \exp^{(-\imath k(\vec{f})(z_1 - z_0))}\} \quad (25)$$

for the linear field kernel function is a simple phase factor $\exp^{(-\imath k(\vec{f})(z_1 - z_0))}$. This is a linear solution of the propagating ultrasound wave, since no higher harmonics are predicted by (25).

3.2 Perturbation method for the solution of the nonlinear wave equation

To show how higher harmonics (second, third and so forth) can be deduced from (20) we will closely follow the solution proposed in section 16.4 [26] (note that the author uses a scaled acoustic pressure $q = p/(\rho_0 c_0^2)$). We assume that the generated harmonics are sequential, meaning that if n-th harmonic is generated than also $n-1$,$n-2$. . .,1 are generated. Also we assume that the relation between successive harmonics is

$$p_1 > p_2 > p_3 > \cdots p_{n-1} > p_n \quad (26)$$

Next, let the total pressure be sum of fundamental p_1, second harmonic p_2 and third harmonic p_3

$$p = p_1 + p_2 + p_3 \quad (27)$$

By inserting (27) on the right side of (20) and preforming the square operation gives $p_1^2 + 2(p_1 p_2 + p_2 p_3 +$

$p_1p_3)+p_2^2+p_3^2$. By using the condition from (26) ($p_3 << p_2 << p_1$), higher order terms can be dropped. This leaves ($p_1^2 + 2p_1p_2$). The obtained NPDE is

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\beta}{\rho_0 c_0^4}\frac{\partial(p_1^2 + 2p_1p_2)}{\partial t^2} \tag{28}$$

The solution of (28) is found by first decomposing it into one homogeneous part and two non homogeneous parts

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = 0 \tag{29}$$

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\beta}{\rho_0 c_0^4}\frac{\partial^2 p_1^2}{\partial t^2} \tag{30}$$

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\beta}{\rho_0 c_0^4}\frac{\partial^2(2p_1p_2)}{\partial t^2} \tag{31}$$

It can be noticed that (29) is same as (10), so this is fundamental pressure field of ultrasound wave. The equation (30) is then the second harmonic field, which has been previously shown in [7, 8, 27]. It was reported by [28–30] that (31) presents solution for the third harmonic of the ultrasound field p_3. Further more it is clear that third harmonic can be computed when fundamental p_1 and second harmonic p_2 are known.

3.3 Solution of the nonlinear wave equation

To find the solution of the nonlinear wave equation for the second harmonic begin with taking the Fourier transform of the (30) to obtain

$$(\nabla^2 + 4k(\vec{f})^2)\underline{P}_2 = \frac{-\imath 2\beta k^2(\vec{f})}{\rho_0 c_0^2}\underline{P}_1^2 \tag{32}$$

Previous works like [7, 8, 27] haven shown that the amplitude spectrum of the generated second harmonic $\underline{P}_2$ can be expressed as

$$\underline{P}_2(\vec{f}|z_1) = \frac{-i\beta k(\vec{f})}{2\rho_0 c_0^2}\int_{z_0}^{z_1}\underline{P}_1(\vec{f}|z_1) \circledast \underline{P}_1(\vec{f}|z_1)\cdot \exp^{-\imath k(\vec{f})(z_1-z_0)}\,dz \tag{33}$$

Equation (33) shows that the second harmonic is obtained by multiplying the phase factor with the auto convolution of the amplitude spectrum of the fundamental component and integrating from z_0 until z_1.

To obtain the third harmonic, the same procedure can be applied on (31) only taking care that pressure fields $\underline{P}_1$ and $\underline{P}_2$ are used. In this manner the formulation of the third harmonic is

$$(\nabla_\perp^2 + 9k(\vec{f})^2)\underline{P}_3 = \frac{3\beta k(\vec{f})}{16\rho_0 c_0^2}\underline{P}_2 \cdot \underline{P}_1 \tag{34}$$

The third harmonic at $z = z_1$, calculated by (34) has a similar solution as (33)

$$\underline{P}_3(\vec{f}|z_1) = \frac{-i3\beta k}{16\rho_0 c_0^2}\int_{z_0}^{z_1}\underline{P}_2(\vec{f}|z_1) \circledast \underline{P}_1(\vec{f}|z_1)\cdot \exp^{-ik(\vec{f})(z_1-z_0)}\,dz \tag{35}$$

The generated third harmonic is viewed as a convolution of the amplitude spectrum of the fundamental and the second harmonic. The amplitude spectrum of the fundamental is given by the Fourier transform of (25), while the second harmonic is calculated using (33).

4 Measurements

To test the method a focalized phase array transducer (FPA), with 64 elements and $2.5\,MHz$ center frequency was mounted on a side of a water-filled tank. The transducer geometry was as follows : each element was $12.5\,mm$ in height and $230\,\mu m$ width. Spacing between elements was $k_{erf} = 80\,\mu m$. The focal spot of FPA transdeucer was $z_{FPA} = 70\,mm$. The transducers was attached to a fully programmable ultrasound scanner (Fig. 2).

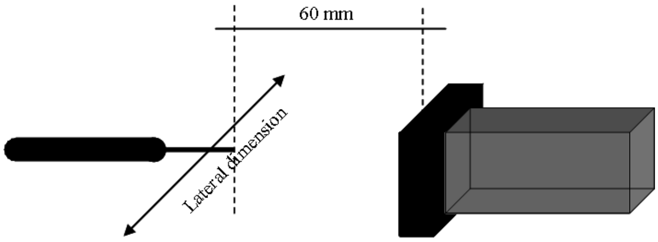

FIG. 2: Measurement setup. The hydrophone is placed in focal spot of the each transducer surface. Lateral profile was measured with a $3D$ xyz system

5 Results

The solutions of Eq.(25), (33) and (35) were implemented in Matlab and compared to the experimental measurements. The parameters of the medium were those corresponding to the water : nonlinearity parameter $\beta = 3.5$, density $\rho_0 = 998.2\,kg/m^3$, speed of sound $c_0 = 1487.3\,m/s$.

The FPA transducer was excited by a 3 cycles long signal with Gaussian envelope and frequency $f_0 = 2.5\,MHz$. The measured and simulated signals are compared on Fig. 3. The acquired pulse was decomposed in to first three harmonics (Fig.3(b), Fig.3(c) and Fig.3(d)). ASM presents some over estimation of the amplitude of the harmonics while phase is in good agreement with measurements. Also notices some round off errors before and after actual pulse, predicted by ASM. These errors come from the aliasing property of inverse Fourier Transform, since angular spectrum method, performs all the calculations in the frequency domain. This error could be overcome by applying an envelope of the signal.

6 Discussion

We have presented a model for estimating the first three harmonics, using the extended nonlinear wave equation. By means of the perturbation method, a simple wave equation was further developed to predict, in addition to fundamental harmonic, also the second and the third harmonic. The measured and simulated time traces from Fig. 3 show a very good similarity. The phase is in excellent agreement. The amplitude of the fundamental pulse is good evaluated. Second harmonic

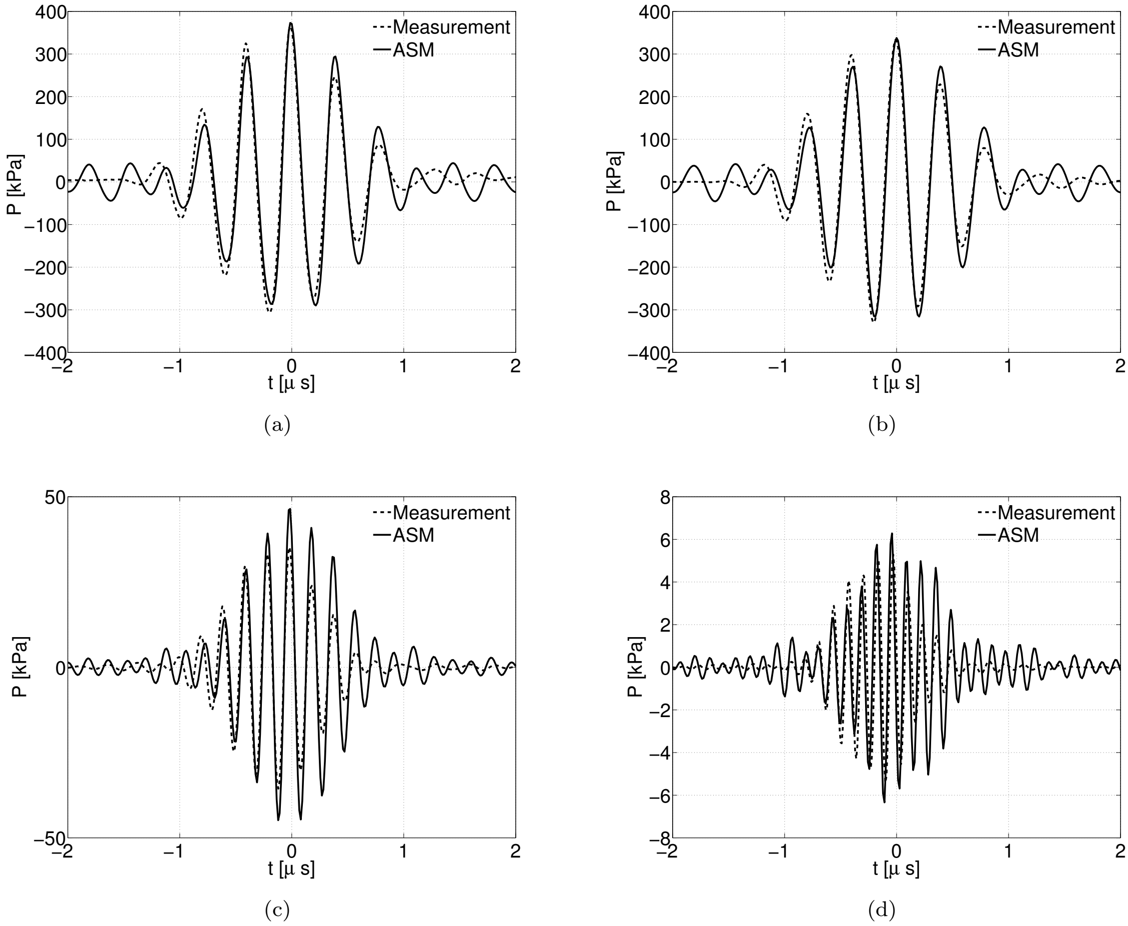

FIG. 3: Comparison between measurements and simulation. Full line presents ASM method results, while dashed line is the measurements. On Fig.3(a) is the acquired pulse, while on the Fig.3(b), Fig.3(c) and Fig.3(d) are the fundamental, second and third harmonic respectively. Note that there is some over estimation of the harmonics, while phase of each harmonic is good predicted. Also note that amplitude scale is not the same for all harmonics, meaning that assumption Eq. (26) is fully acceptable

amplitude is overestimated by $5\,kPa$ while third harmonic amplitude by $3\,kPa$. It should be noted that this error is inherited from truncation order of the Taylor series expansion of the equation of state. On a desktop PC, with dual core Intel(R) processor with CPU clock $3\,GHz$ and $2\,GB$ of RAM memory, the whole fundamental, second and third harmonic field is were evaluated in less than 3 minutes. The accuracy and the speed, with which this was done, suggest that this approach could be a powerful tool for predicting ultrasound field at higher harmonics.

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