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Trajectory Design Method for Less Noise and Fuel Consumption from Aircraft Operations

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Aircraft noise and air pollution are considered to be one of the most significant environmental concerns on the local community of modern cities, affecting people living in particular near airports during landings and takeoffs. This study aims to analyze benefits due to optimization of flight parameters in order to reduce noise impact and fuel consumption during takeoffs and landings. The three-dimensional motion of a commercial aircraft over a few minutes time span is described by a dynamic system. A thermodynamic analysis for a turbofan engine is carried out to obtain the state parameters which are useful to express the jet noise and the fuel consumption functions. The obtained model is an optimal control problem (OCP) including a consumption function, an overall sound pressure level, a dynamic system and flight related constraints. A pseudospectral method is suggested and implemented to solve this OCP problem. Results are presented and discussed.

1 Introduction

Advances in engine technology such as high bypass ratio and acoustic liners have helped to reduce the noise emitted by commercial aircraft. Nevertheless, the combination of continuing air transport growth, intolerance of communities towards disturbances, and growing airport neighborhoods present aircraft noise as an increasing problem. In addition, civil air-transport authorities start to care about green house gases and pollutant emissions from aircraft operations.

In response to concerns of exposed populations, modern aircraft are required to respect noise regulations specified in FAR 36 [1] and ICAO annex 16 [2]. Additionally, it is becoming more common for airports to have their own increasingly stringent noise rules and operational restrictions. Low noise operational procedures provide operators a way to respond quickly to noise concerns [3].

In this paper, we develop a flight paths optimization methodology in order to reduce noise footprints and fuel consumption during approaches and takeoffs. We describe the mathematical model which is considered as an optimal control problem. Technical feasibility and flight safety constraints have been taken into account. We present the Stone jet noise model [4] and the method used to calculate accurately its parameters [5] based on the engine inlet and outlet conditions. A Gauss pseudospectral method [6] is suggested to solve the obtained optimal control problem. Simulations are given and discussed.

2 Problem modeling

2.1 Flight dynamics

To design flight paths, we have considered the translational motion of the aircraft. The aircraft angle of attack α and bank angle ϕ are considered as pseudo-controls inputs, together with the throttle δ_x.

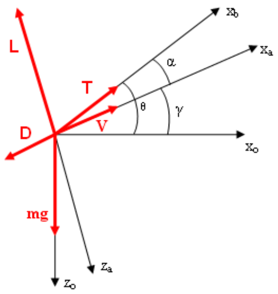
The following assumptions are considered :

1. The motion of the aircraft is described in an inertial frame attached to the Earth. Earth is considered as static and flat.
2. The wind effects are not taken into account.
3. Aircraft is considered as a perfect symmetrical rigid body and uniformly mass distributed. Turns are absent in approach and takeoff phases. Momentum forces are neglected since all external forces cross the gravity center of the aircraft.

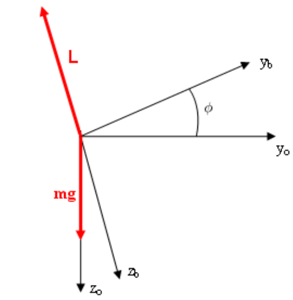
The system describing the point mass aircraft motion in a 3-dimensional frame is (more details can be found in [7])

$$\begin{cases} \dot{x} = v\cos\gamma\cos\chi \\ \dot{y} = v\cos\gamma\sin\chi \\ \dot{h} = v\sin\gamma \\ \dot{v} = \dfrac{T\cos\alpha - D}{m} - g\sin\gamma \\ \dot{\gamma} = \dfrac{(L + T\sin\alpha)\cos\phi}{mv} - \dfrac{g}{v}\cos\gamma \\ \dot{\chi} = \dfrac{(L + T\sin\alpha)\sin\phi}{mv\cos\gamma} \\ \dot{m} = -TSFC \times T \end{cases} \tag{1}$$

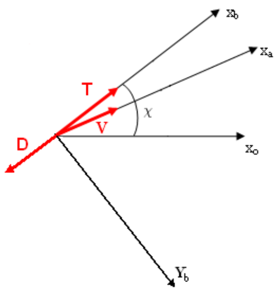
where T is the thrust force, m is the aircraft mass, $D = qSC_D$ is the drag, g is the gravity acceleration, $L = qSC_L$ is the lift, $q = \frac{1}{2}\rho v^2$ is the dynamic pressure, ρ is the air density, and S is the aircraft reference wing



(a) Side view



(b) Face view



(c) Top view

FIGURE 1 – Body frame $R_b(G, X_b, Y_b, Z_b)$, Aerodynamic frame $R_a(G, X_a, Y_a, Z_a)$ and Earth frame $R_0(G_0, X_0, Y_0, Z_0)$

area. v is the aircraft velocity, γ is the flight path angle, χ is the heading angle. C_L and C_D are called lift and drag coefficients and $TSFC$ is the thrust specific fuel consumption factor.

For convenience, we define three vectors $X = (x, y, h, v, \gamma, \chi, m)^T$, $U = (\delta_x, \alpha, \phi)^T$ and p. The first two vectors implicitly depend on time $t \in [t_0, t_f]$ and may be noted $X(t)$ and $U(t)$. p contains time-independent parameters (gravitational acceleration, wing reference area...). Notation $\dot{x} = \frac{dx}{dt}$ is the first derivative of a component with respect to time. The start time t_0 and end time t_f are assumed constant. Problems with free start or end time can easily be transformed to problems with fixed start and end times. The vector valued function $X : [t_0, t_f] \rightarrow \mathbb{R}^7$ is restricted to be a solution of the nonlinear ordinary differential equation (ODE) in Eq.(1). We call X the state function or state variable because X describes the state of a dynamical system. Generally, the measurable, vector valued function $U : [t_0, t_f] \rightarrow \mathbb{R}^3$, influences the solution of ODE Eq.(1), or -in other words- controls the dynamical system. Therefore we call U the control function.

Let us denote the dynamic system Eq.(1) by

$$\dot{X} = f\left(X(t), U(t), t; p\right) \tag{2}$$

2.2 Constraints

In addition to the dynamic constraint Eq.(2), we consider two types of constraints called boundary conditions and path constraints, defined in the following subsection :

Boundary conditions : The flight simulation must start from a feasible fixed initial state $X(t_0)$ and finish at another feasible endpoint $X(t_f)$. Assume that Φ is the function translating these conditions and it takes its values in the interval $[\Phi_{min}, \Phi_{max}]$. Then, we express the boundary conditions by the following inequalities

$$\Phi_{min} \leq \Phi(X(t_0), t_0, X(t_f), t_f; p) \leq \Phi_{max} \tag{3}$$

Path constraints : During the flight, state and control variables have to be inside an admissible range of values. We call this kind of restriction : path constraints. A simple and general form could be given by :

$$C(X(t), U(t), t; p) \leq 0, \quad \forall t \in [t_0, t_f] \tag{4}$$

2.3 Objectives

Our interest is composed of two sub objectives. First, to minimize noise under the flight path. Second, to minimize the consumed fuel during operations. Several parameters are needed to calculate these two objective functions.

2.3.1 Noise function

We consider the noise radiated from the jet exhaust of the engines. We use the well known Stone model [4] :

$$\begin{aligned} OASPL = 141 + 10\log\left[\left(\frac{\rho}{\rho_{ISA}}\right)^2\left(\frac{c}{c_{ISA}}\right)^4\right] \\ +10\log\left(\frac{A_j}{R^2}\right) + 10\log\left(\frac{\rho_j}{\rho}\right)^{\omega} \\ +10\log\left(\frac{V_j}{c}\right)^{7.5} - 15\log\left[(1 + M_c\cos\theta)^2 + \beta^2 M_c^2\right] \end{aligned} \tag{5}$$

This is an OASPL (overall sound pressure level) assessed at a position (R, θ), where R is the distance to the noise source and θ is the angle measured downstream from the jet exhaust axe. ρ and c are respectively density of air and speed of sound in the free stream conditions. ρ_{ISA} and c_{ISA} are the air density and the sound velocity in ISA (International Standard Atmosphere) conditions. A_j, ρ_j and V_j are fully expanded jet area, jet exhaust density and jet velocity respectively. The following analytic expression is used for ω (in reference [8]) :

$$\omega = \frac{3\left(\frac{V_j}{c}\right)^{3.5}}{0.6+\left(\frac{V_j}{c}\right)^{3.5}} - 1 \tag{6}$$

This expression gives similar values to one suggested in reference [9]. The source convection is introduced to translate the effect of directivity in sound radiation : following Williams [10], the acoustic intensity is multiplied by $\left[(1+M_c\cos\theta)^2+\beta^2M_c^2\right]^{-n/2}$, where $M_c = kV_j/c$ is the convective Mach number. In this formulation we took $k = 0.62$ and $n = 3$, as suggested by Goldstein and Howes [11] and $\beta = 0.2$, essentially given by Larson et al [12]. Refraction corrections are not considered since no spectral composition is used in this study. So, the noise minimization criterion can be expressed by :

$$\begin{aligned} &\int_{t_0}^{t_f} J_1\left(X(t),U(t),t;p\right)\ dt \\ &= \int_{t_0}^{t_f} OASPL\left(X(t),U(t),t;p\right)\ dt \end{aligned} \tag{7}$$

2.3.2 Fuel consumption function

The calculation of the fuel burn in this study is based on the assumptions and equations outlined by Benson [5]. Basically, the instantaneous fuel flow $FF(t)$ can be estimated by :

$$FF(t) = TSFC \times T(t) \tag{8}$$

The thrust specific fuel consumption ($TSFC$) is specified as a function of airspeed. So, we simply state the second objective function as :

$$\begin{aligned} J_2\left(X(t_f),t_f;p\right) &= \int_{t_0}^{t_f} -\dot{m}(t)\ d(t) \\ &= \left[m\left(t\right)\right]_{t_f}^{t_0} = m(t_0) - m(t_f) \end{aligned} \tag{9}$$

where $m(t_0)$ and $m(t_f)$ are the initial and final aircraft mass. Since $m(t_0)$ is a constant, in optimization we have the following equivalence :

$$\min J_2\left(X(t_f),t_f;p\right) \equiv \min -m(t_f)$$

An additive aggregation of J_1 and J_2 is enough for our minimization purpose. We write :

$$\begin{aligned} J(X(t),U(t),t;p) = \\ \int_{t_0}^{t_f} J_1\left(X(t),U(t),t;p\right)\ dt + J_2\left(X(t_f),t_f;p\right) \end{aligned} \tag{10}$$

which is exactly the Bolza form [13] of the cost function in optimal control theory.

2.4 Final form and resolution method

The final formulation of our problem is a one-phase optimal control problem which consists of a Bolza objective function minimization subject to dynamics, boundary and path constraints.

$$\begin{cases} \min\limits_{U\in\mathcal{U}} \int_{t_0}^{t_f} J_1\left(X(t),U(t),t;p\right)\ dt + J_2\left(X(t_f),t_f;p\right) \\ \dot{X} = f\left(X(t),U(t),t;p\right) \\ \Phi_{min} \le \Phi(X(t_0),t_0,X(t_f),t_f;p) \le \Phi_{max} \\ C(X(t),U(t),t;p) \le 0 \end{cases} \tag{11}$$

Many methods for solving optimal control problems are described in the literature (shooting methods [14], collocation methods [15, 16] etc.). We chose a collocation algorithm called Gauss pseudospectral method (GPM) because of its efficiency in the approximations of three types of mathematical objects : the integration in the cost function, the differential equation of the control system, and the state-control constraints. It is noted that unlike previously developed pseudospectral methods such as Elnagar and Kazemi [16], in the one developed by Rao [6], the differential equations are collocated only at the Legendre Gauss points [6] and not at the boundary points. The resulting model after discretization define an NLP. The solution of this NLP is an approximate solution to the continuous-time optimal control problem Eq.(11). The resulting NLP can be solved by an appropriate method taken from the well known nonlinear programming theory.

3 Numerical results and discussions

In this section we present and discuss some numerical results. The problem is implemented using GPOPS-MATLAB® [17] software and run on an Intel Core2 Quad processor (2.66 GHz, 4 GB memory). Derivatives are approximated by the numerical INTLAB derivation method. The NLP resulting after discretization is solved by SNOPT optimization algorithm [18]. Local optimal solutions are obtained with an average order of feasibility error of 10^{-10}. Units are in the International System.

TABLE 1 – Aircraft specifications

Type	Airbus A300-600
Powerplants	Two 262.4 kN General Electric CF6-80C2A1s
Weights	Max takeoff 165900 kg. Operating empty 90965 kg
Dimensions	Wing span 44.84 m, length 54.08 m, height 16.62 m. Wing area 260 m^2.

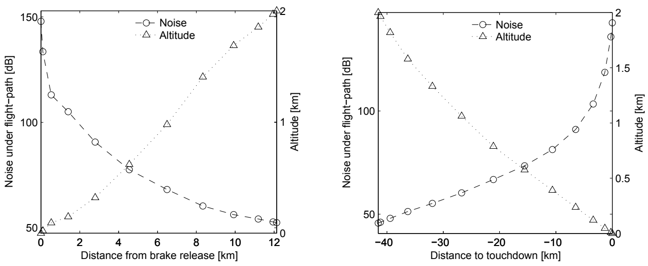
Practically, we introduce boundary conditions and path constraints for takeoff as follows : the start point is $X(t_0) = (0,0,0,75,13,0,140000)^T$ which corresponds to 3D-position $(x(t_0),y(t_0),h(t_0)) = (0,0,0)$, takeoff velocity $v(t_0) = 75$ m/s, flight path angle $\gamma(t_0) = +13°$, heading $\chi(t_0) = 0°$ and an initial mass $m(t_0) = 140000$ kg. The terminal state as $X(t_f) = (\text{free},\text{free},2000,160,3,\text{free},\text{free})^T$. Time range in seconds as $t_0 = 0$ and $t_f = \text{free} \in [t_{f_{min}}, t_{f_{max}}]$.

For landing, the start point is $X(t_0) = (\text{free},0,2000,110,-5,0,125000)^T$

which corresponds to 3D-position $(x(t_0), y(t_0), h(t_0)) = (\text{free}, 0, 2000)$, initial velocity $v(t_0) = 110$ m/s, flight path angle $\gamma(t_0) = -5°$, heading $\chi(t_0) = 0°$ and initial mass $m(t_0) = 125000$ kg. The terminal state as $X(t_f) = (0, 0, 0, 65, 0, 0)^T$. Time range in seconds is $t_0 = 0$ and $t_f = \text{free} \in [t_{f_{min}}, t_{f_{max}}]$.

For takeoff, we may expect that the average climbing slope is about 15% (i.e. around $+8.6°$). The throttle is set to its maximum $\delta_x \approx 1$ for the whole flight and the finesse is quite good (around 17). No engines overheating restriction is included in our current model and we assume that the engines can be run in a full power setting over the considered time span. Thrust cutbacks would show up if this restriction is taken into account or if more weight is applied to the fuel cost function.

During approach, the average slope rate is near -4% (i.e. $-2.77°$) which is very close to one recommended by ICAO [2] corresponding to the continuous descent approach. The throttle is kept at low setting to reduce the jet exhaust speed and therefore emitted noise. The finesse is in average neighboring 14 which is quite good because of modern aircraft have a finesse between 8 and 20.



(a) Noise levels for takeoff (b) Noise levels for approach

FIGURE 2 – Noise levels under flight-path (minimization of fuel & noise)

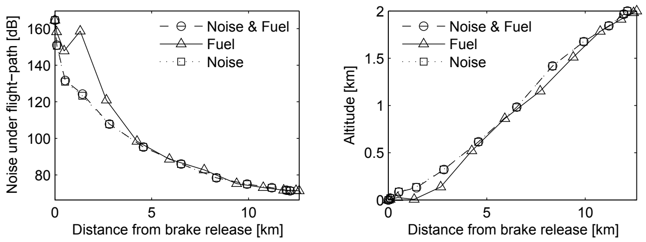
Comparative study :

In this section, we look for the influence of different optimization criteria. Three cases have been tested : minimizing simultaneously noise and fuel consumption, and noise and fuel separately. We observe that the difference is considerable between the case of optimizing the noise (with or without fuel) and that of minimizing only consumed fuel.

During takeoff (Fig. 3), from 1 to 5 km from brake release, the average noise decrease is around -7dB when we include the noise function in the optimization criterion. This decrease is greater in the first kilometers which are related to low altitudes. This observation is quite interesting since noise concern is more important at low altitudes (high disturbance).

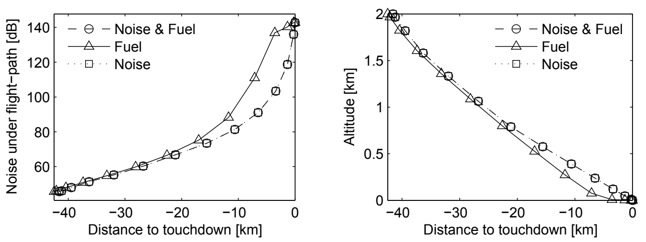
In the approach (Fig. 4), a quite significant noise reduction when we include noise criterion in the optimization model. From -41.5 km to -20 km to touchdown point, the process leads to les than -1dB compared to the case of minimizing only fuel consumption. Fortunately, in the last 20 kilometers we get more significant decrease up to -6.3dB in average.

Concerning the fuel consumption (Fig. 5), in takeoff, a decrease of -1.9% when we minimize only fuel instead of minimizing noise alone or noise and fuel. If we optimize noise and fuel instead of noise alone we obtain -0.23% which is equivalent to -1.27 kg per takeoff (Lyon Saint Exupéry airport : 128397 movements in 2006 $\Rightarrow$ 170 tons of fuel reduction). In the approach operations, -2.5% of fuel burn when we minimize only fuel (Lyon Saint Exupéry airport : 128397 movements in 2006 $\Rightarrow$ 362 tons of fuel benefit). There is no significant fuel consumption decrease between the case of minimizing noise alone and noise and fuel together for approach. Figure 5(b) seems to imply a trade-off between noise and fuel burn on approach.



(a) Noise under flight-path (b) Altitude

FIGURE 3 – Comparison between minimizing : noise and fuel, only noise, only fuel in takeoff



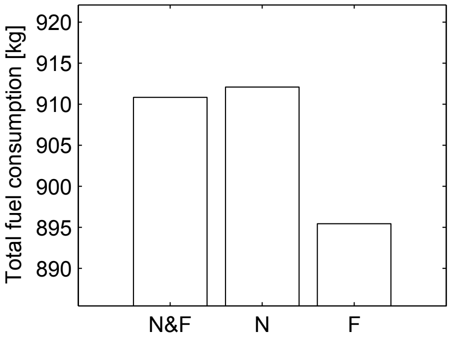
(a) Noise under flight-path (b) Altitude

FIGURE 4 – Comparison between minimizing : noise and fuel, only noise, only fuel in approach

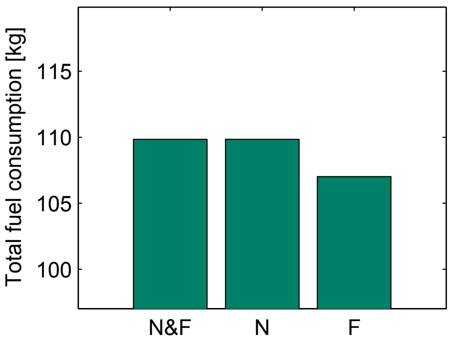
4 Conclusion

The benefits of the flight paths design reducing noise and fuel consumption have been analyzed. The problem is solved by a Gauss pseudospectral method. Simulation results are given and discussed. Different conclusions are carried out and statistical extrapolations are given for Lyon Saint Exupéry International airport to show how fuel consumption, and consequently pollutant emissions, can be decreased. This study shows up a trade-off between noise and fuel burn. The objective of this research and the expected results are considered to be complementary with technological development in engine design aiming to increase fuel efficiency and noise control systems.

Further research are needed to add the other noise sources of the aircraft. It will also be interesting to develop an indirect solving method to compare with current results.



(a) Takeoff



(b) Approach

FIGURE 5 – Total fuel after optimization (N&F : noise and fuel, N : only noise, F : only fuel)

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