Flow through simplified vocal tract geometries

A. Van Hirtum
Gipsa-lab, UMR CNRS 5216, Grenoble Universities, France

Production of speech utterances such as fricatives involves a complex interaction of turbulent airflow with the particular geometry of the vocal tract. The current study introduces simplified mechanical rigid static vocal tract geometries consisting of a rectangular channel to which 1 or 2 constrictions are inserted allowing to study flow-obstacle and jet-obstacle interaction. Different constriction geometries and constriction degrees are considered. The flow through different geometries is predicted by simplified flow models. Quantitative and qualitative comparison of modelled and measured values is assessed.

1 Introduction

Aero-dynamic and aero-acoustic principles are introduced in speech production studies dealing with fricatives since the sixties [2]. The pioneering work is further developed by experimental as well as modeling studies, e.g. [10, 3]. As a result, the underlying mechanism of fricative sound production is well understood as noise produced due to the interaction of a turbulent jet, produced by a constriction somewhere in the vocal tract, with a downstream wall or obstacle. Consequently, the position and shape of articulators like tongue and teeth will determine the generation and development of the jet as well as its downstream interaction with a wall or obstacle as is indeed observed on human speakers [6, 8]. Experimental and simulation studies are performed in order to characterise and quantify the influence of articulators position and shape on the sound produced [10, 7]. Nevertheless, the mentioned studies focus on the acoustics of fricative noise production. Therefore, flow data providing a systematic characterisation issuing from configurations relevant to human fricative production, i.e. moderate Reynolds Re numbers covering the range 2000 < Re < 10^4 and low Mach Ma number, are few. Obviously, model validation would benefit from additional flow data providing quantitative information on the flow field as pointed out by a.o. [3] in the framework of fricative production. The present study aims to contribute to the systematic study of flow data relevant to describe the jet-obstacle interaction occurring during fricative production.

A rectangular rigid mechanical replica, inspired on the work presented in [10], is prososed in order to mimic the jet-obstacle interaction. The replica consists of a constricted portion between the ‘tongue’ and the ‘palatal plane’ upstream of an obstacle representing a ‘tooth’ for which the constriction can be varied. Its dimensions are taken to be relevant to the human physiology of an ‘averaged’ male adult vocal tract [6, 8]: length ≈ 180mm, unconstricted height ≈ 16mm, width ≈ 21mm, constriction height between the tongue and the palatal plane ≈ 3mm, tooth length ≈ 3mm. The gap between the constricted vocal tract portion and the obstacle as well as the constriction degree at the obstacle are systematically varied. In addition, the flow conditions are varied so that the relevant range of Reynolds numbers is experimentally assessed. The flow is characterised by measuring the volume flow rate and performing point pressure measurements at different positions along the replica. The gathered data are compared to the outcome of one-dimensional flow models commonly used to model the flow in physical models of phonation in order to validate to which degree the applied models are suitable to model the flow through the entire upper airway from the larynx up to the lips. Since it is obvious that the studied flow is to complex to be represented by a laminar flow model, assumed in Bernoullis equation, several ‘ad-hoc’ corrections are assessed.

2 One-dimensional flow models

Considering a rectangular channel with two constrictions, see Fig. 1, the total pressure difference $\Delta P_{tot}$ is:

$$\Delta P_{tot} = \Delta P_1 + \Delta P_2 + \Delta P_3 + \Delta P_4 + \Delta P_5,$$

with:

$$\Delta P_1 = P(x = 0) - P(x = i_1),$$
$$\Delta P_2 = P(x = i_1) - P(x = is_1),$$
$$\Delta P_3 = P(x = is_1) - P(x = i_2),$$
$$\Delta P_4 = P(x = i_2) - P(x = is_2),$$
$$\Delta P_5 = P(x = is_2) - P(x = i_3).$$

It is assumed that no pressure loss occurs in the uniform inlet portion so that $P_0 = P(x = 0) = P(x = i_1)$ and $\Delta P_1 = 0$. The pressure losses $\Delta P_i$ in the remaining portions with varying area $A_{is_i}$, with subscript $i$ denoting the upstream position and subscript $si$ the downstream position, can be modeled by application of a combination of the following terms from which the pressure distribution $p(x)$ follows immediately [1]:

$$\Delta P^lcr_i = \phi \frac{\rho}{2} \left( \frac{1}{A_{is_i}} - \frac{1}{A_i^2} \right)$$
$$= \phi \frac{\rho}{2} \frac{1}{A_i^2} \left( \frac{A_{is_i}^2}{A_{is_i}^2} - 1 \right)$$

(1
Figure 1: Two-dimensional geometry $h(x)$ with fixed width $w$. The unconstricted channel height is denoted $h_0$ and two constrictions are inserted, $[i_1\ si_1]$ and $[i_2\ si_2]$. The x-axis indicates the main flow direction.

\[ \Delta P_{i}^{pos} = \phi \frac{12 \mu}{w} \int_{x_i}^{x_{si}} \frac{dx}{h(x)^3} \quad (2) \]

\[ \Delta P_{i}^{exp} = \phi^2 \frac{\rho}{2} \left[ \frac{1}{A_i^2} \left( \frac{A_i^2}{A_{si}} - 1 \right) + \left( 1 - \frac{A_i}{A_{si}} \right)^2 \right] \]
\[ = \phi^2 \frac{\rho}{2} \frac{1}{A_i A_{si}} \left( 1 - \frac{A_i}{A_{si}} \right) \quad (3) \]

\[ \Delta P_{i}^{con} = \phi^2 \frac{\rho}{2} \frac{1}{A_{si}^2} \left[ \left( 1 - \frac{A_{si}^2}{A_i^2} \right) + \frac{1}{2} \left( 1 - \frac{A_i}{A_{si}} \right) \right] \]
\[ = \phi^2 \frac{\rho}{2} \frac{1}{A_{si}^2} \left( 1 - \frac{A_i}{A_{si}} \right) \left( 1 + \frac{1}{2} \left( 1 + \frac{A_i}{A_{si}} \right)^{1/2} \right)^{1} \quad (4) \]

\[ \Delta P_{i}^{ben} = \phi^2 \frac{\rho}{2} \left[ \frac{C_{ben}}{A_{si}^2} \right], \quad Re_D < 2 \cdot 10^5 \rightarrow C_{ben} > 1 \quad (5) \]

\[ \frac{\Delta P_{i}^{ben}}{\Delta P_{i}^{con}} = \left( \frac{C_{ben}}{C_{con}} \right)^{0.2}, \quad (6) \]

\[ C_{con} = \left( 1 + \frac{1}{2} \left[ 1 + \frac{A_{si}}{A_i} \right]^{-1} \right)^{-1/2}, \quad (8) \]

\[ C_{ben} = 1.1 \left( \frac{2 \cdot 10^5}{Re_D} \right)^{0.2}. \quad (9) \]

The position of flow separation is fixed and no correction for its changing position is needed. Viscous losses, on the contrary, are known to be important in case of low Reynolds numbers, i.e. low velocity or small height $h(x)$. Therefore the Bernoulli equation is corrected for viscosity by adding a viscous pressure loss term (2) denoted $\Delta P_{i}^{pos}$. This term is obtained by assuming a fully developed Poiseuille velocity profile. So far, pressure recovery by flow reattachment upstream the flow separation point is neglected. Pressure recovery is estimated by evaluating the quasisteady momentum equation. The resulting expression (3) describes the pressure recovery as a portion of the Bernoulli loss term (1). The magnitude of the recovery depends on the area ratio $A_i/A_{si}$ at the position of flow separation $A_i$ and the expanded area $A_{si}$ downstream the constriction. It is clear that (3) assumes a uniform flow profile over area $A_{si}$ so that the pressure recovery becomes proportional to $1 - (A_i/A_{si})^2$. On the other hand zero pressure recovery is expected in case a narrow jet flow is assumed to be maintained, so that $A_{si} = A_i$ and the loss term becomes zero since $(1 - A_i/A_{si}) = 0$. In addition to the extreme cases of no recovery or uniform flow, an intermediate value for the pressure recovery is expected when assuming an expanding jet geometry to which (1) can be applied. A geometrical correction for jet expansion is easily obtained by applying an expansion angle $\theta_{jet}$ to the uniform narrow jet as:

\[ A_{jet} = [h_i + C_{jet} \cdot tan(\theta_{jet}) \cdot (x_{si} - x_i)] \cdot w, \quad (10) \]

with expansion angle $\theta_{jet} \sim 4.2^\circ$ and model constant $C_{jet}$ set to 1 or 2 accounting for one-side or two-side geometrical expansion of the narrow two-dimensional jet [9].

The constricted portion indicated $[i_2, si_2]$ in Fig. 1 can be seen as a thin square-edged contraction for which separation might occur depending on the Reynolds number at the leading edge, $x = i_2$, instead of the trailing edge, $x = si_2$. In case separation occurs, the flow through the constriction is accelerated and a pressure loss occurs as reported in (5) where $C_{con}$ can be seen as a discharge coefficient whose value can be estimated from geometrical considerations (4), (8) or as an ‘ad-hoc’ orifice coefficient (5) [1].

The expressions (1) up to (5) assume the main flow direction to be along the x-axis. Although, in particular when the distance between the down- and upstream constriction is reduced, the main flow direction in the gap between both obstacles is likely to be perpendicular to the x-direction. In this case, the geometry can be seen as a sharp 90° bend for which the pressure loss can be described with (7) in which the coefficient $C_{ben}$ is either estimated from the volume flow rate and the geometry (9) or chosen as an ‘ad-hoc’ bending discharge coefficient [1]. Alternatively, a change in flow direction in the narrowed portion between both constrictions can be simply accounted for by exchanging height and length in this section of the channel and applying the previous mentioned terms, (1) up to (5), in order to determine the pressure distribution $\rho(x)$ along the main flow direction.
3 Mechanical replica and setup

The rigid ‘in-vitro’ replica consists of two constrictions, $C_1$ and $C_2$, inserted in a uniform rectangular channel as schematically depicted in Fig. 1 and detailed in Fig. 2. The unconstricted channel has length $L_0 = 180$mm, height $h_0 = 16$mm, width $w = 21$mm and aspect ratio $w/h_0 = 1.3$. The shape of both constrictions $C_1$ and $C_2$ is fixed. Their lengths in the $x$-direction yield $l_1 = 30$mm for $C_1$ and $l_2 = 3$mm for $C_2$. The aperture $h_1$ is fixed to 3mm, which corresponds to a constriction degree of 81%. The distance of the trailing edge of $C_2$ to the channel exit, $L_2$, is fixed to 6mm. The distance of the trailing edge of $C_1$ with respect to the channel exit, $L_1$, can be varied as well as aperture height $h_2$ of constriction $C_2$. Therefore, besides the inlet height $h_0$, the pressure distribution is determined by the set of geometrical parameters $\{h_1, L_1, h_2\}$ of which $L_1$ and $h_2$ can be varied. In order to validate the pressure drop three pressure taps are assessed at positions $p_0 = 30$mm, $p_1 = 160$mm and $p_2 = 173$mm from the channel inlet. Experimentally assessed combinations of $\{L_1, h_2\}$ are summarised in Table 1 and schematically illustrated in Fig. 3. Since the position of the pressure tap $p_1$ is fixed, $p_1$ is situated in the gap between $C_1$ and $C_2$ for large $L_1$ or along $C_2$ in case $L_1$ is closer to the channel exit.

4 Flow and pressure data

The geometrical configurations depicted in Fig. 2 and Fig. 3 are assessed for upstream pressures $P_0 < 4000$Pa. The associated bulk Reynolds numbers, defined as $Re = \phi/(\nu w)$, are $Re \in [0 \, 15000]$. Fig. 4 shows the measured values of $Re$ and downstream pressures $(P_1, P_2)$ as function of $P_0$ and the geometrical parameters $(L_1, h_2)$.

Table 1: Assessed geometrical configurations.

<table>
<thead>
<tr>
<th>$L_1$ or $L$ and $h_2$ [mm]</th>
<th>Constriction degrees [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$</td>
<td>$h_2$</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>33</td>
<td>24</td>
</tr>
<tr>
<td>25</td>
<td>16</td>
</tr>
<tr>
<td>19</td>
<td>10</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 4(a) illustrates the measured relationship $Re(P_0)$. For $h_2 = 0.6$mm corresponding to a constriction degree of 96% the relationship $Re(P_0)$ is seen to be nearly independent of $L_1$ since the relative difference is inferior to 5% for all assessed volume airflows. For $h_2 = 1.5$mm the presence of both constrictions becomes notable since increasing $L_1$ from 12 to 33mm slightly decreases $P_0$ with 8% and further down to 12% in absence of $C_1$. The same tendency is observed more clearly as the aperture $h_2$ is increased. The relative pressure decrease with increasing $L_1$ from 12 down to 33mm yields 25%, 44% and 51% for $h_2 = 2.6$, $h_2 = 5.5$ and $h_2 = 6.8$mm and decreases further to 38%, 78% and 87% in absence of $C_1$. Consequently, the pressure drop increases when the gap between both constrictions is narrowed indicating that pressure recovery is favored in case of a wide gap between both constrictions.

In absence of $C_2$, i.e. $h_2 = h_0 = 16$mm, pressure recovery is mainly determined by constriction degree of 81% due to the fixed aperture of $h_1 = 3$mm. Consequently, varying $L_1$ from 33 up to 1mm results in a fairly constant pressure drop $P_0$ regardless the volume airflow rate. The slight pressure increase, inferior to 4%, for increasing $L_1$ is the result of a small pressure recovery in the channel. Fig. 4(b) illustrates the pressures measured at $p_1$ normalised to the upstream pressure, $P_1/P_0$. As illustrated in Fig. 3 the relative position of the pressure tap $p_1$ with respect to the trailing edge of constriction $C_1$ depends on $L_1$. From Fig. 4(b) it is seen that in absence of $C_1$ the pressure ratio $P_1/P_0$ collapses to a single curve, which is independent from $h_2$ and the volume airflow velocity $\phi$. Nevertheless, the pressure loss is increasing with input pressure up to 30% firstly due to friction since the friction factor is Reynolds number dependent and secondly due to the development of entry flow in the uniform inlet portion of the channel with length 13cm [4]. In addition, since the aspect ratio $h_0/w = 1.3 \approx 1$, three-dimensional flow development is likely to occur [9]. Inserting constriction $C_1$ in absence of constriction $C_2$, i.e. $h_2 = h_0 = 16$mm, leads to a pressure drop as expected from the terms discussed in section 2. For $L_1 = 1$ up to $L_1 = 33$mm the pressure tap $p_1$ is situated consecutively along the converging portion of $C_1$, at the minimum constriction and finally in the gap between both constrictions, so that the associated pressure drop is seen to increase from ±40% up to ±100%, i.e. $P_1 = 0$. The pressure drop $P_1/P_0$ measured in presence of both constrictions $C_1$ and $C_2$ is intermediate to the previous configurations: a lower limit is reached in absence
of $C_1$ and an upper limit in absence of $C_2$. As for $Re(P_0)$ shown in Fig. 4(a) the influence of $L_1$ on pressure $P_1$ is most notable for large $h_2 > h_1 = 3\text{mm}$, i.e. 6.8 and 5.5mm, for which the pressure loss is seen to decrease with 12% or more as the gap $L_1$ becomes wider. In addition, the pressure loss $P_1/P_0$ measured for $h_2 > h_1 = 3\text{mm}$ is more pronounced as for smaller $h_2$, i.e. $h_2 \leq h_1 = 3\text{mm}$, for which the pressure loss $P_1/P_0 > 0.5$. Consequently, the relative pressure drop $P_1/P_0$ reduces as $h_2$ decreases since the pressure drop across $C_2$ is increasing.

From the previous discussion of measured $P_1/P_0$ values and from the model terms presented in section 2, accounting for pressure recovery in the gap between both constrictions is expected to be important for $h_2$ in the range $h_0 > h_2 > h_1$ and much less for $h_2 < h_1$. Fig. 4(c) reports measured pressure losses $P_2/P_0$ observed at pressure tap $p_2$. The pressure drop for $C_2$ is most important for small apertures $h_2$ resulting in negative pressures for $h_2 \leq 1.5\text{mm}$ with an order of magnitude about 10% of $P_0$. Nevertheless, the pressure drop is more pronounced for $h_2 = 1.5\text{mm}$ than for $h_2 = 0.6\text{mm}$. This might be due 1) to viscosity as seen from (2), 2) a consequence of the strong asymmetry resulting in a downstream shift of the minimum pressure [5] or 3) related to a small recirculation zone at the position $p_2$. Varying $L_1$ is seen to influence $P_2/P_0$ in particular for small apertures $h_2 \leq 1.5\text{mm}$ for which the presence of $C_1$ is seen to decrease the pressure drop for the assessed flow conditions.

### 5 Validation of 1D flow models

The pressure distribution is estimated from models taking into account different terms, (1) up to (7), as discussed in section 2. Resulting models $q$ and their principle features are summarised in Table 2. The assessed geometry and the total pressure difference corresponding to the measured upstream pressure, i.e. assuming $\Delta P = P_0$, are model input parameters from which the volume airflow velocity and pressure distribution along the ‘in-vitro’ replica geometry, parameterised by $L_1$ and $h_2$, are estimated.

Model estimations of the volume airflow velocity and of the pressures at the positions of the pressure taps, i.e. $\hat{P}_1$, $\hat{P}_2$ and $\hat{\phi}$, can be quantitatively compared to experimentally observed values for each set of input parameters $(P_0, L_1, h_2)$ in order to determine the model accuracy. Consequently, the accuracy of the model estimations for $\hat{P}_1$, $\hat{P}_2$ and $\hat{\phi}$ is sought as function of $(P_0, L_1, h_2)$ for each model $q$. Relative error functions $\zeta^{(q)}_1(\hat{P}_1, P_0, L_1, h_2)$, $\zeta^{(q)}_1(\hat{P}_2, P_0, L_1, h_2)$ and $\zeta^{(q)}_2(\hat{\phi}, P_0, L_1, h_2)$ are obtained for each model, denoted by superscript $q$, as:

\begin{equation}
\zeta^{(q)}_1(\hat{P}_m, P_0, L_1, h_2) = \frac{|\hat{P}_m - P_m|}{P_0}, \quad \text{with } m \in \{1, 2\} \tag{11}
\end{equation}

\begin{equation}
\zeta^{(q)}_2(\hat{\phi}, P_0, L_1, h_2) = \frac{|\hat{\phi} - \phi|}{\phi}, \tag{12}
\end{equation}

where as before $P_0$, $P_m$ and $\phi$ indicate the measured values. An error function $\zeta^{(q)}_k$ for all $N_0(L_1, h_2)$ assessed $P_0$-values is defined as:

\begin{equation}
\zeta^{(q)}_k(\cdot, L_1, h_2) = \frac{1}{N_0} \sum_{r=1}^{N_0} \left( \zeta^{(q)}_k(\cdot, P_0_r, L_1, h_2) \right), \tag{13}
\end{equation}

for which the summation index $r$ in $P_0_r$ sums over all $N_0$ assessed $P_0$ values for each geometrical configuration $(L_1, h_2)$ whereas $\zeta^{(q)}_k$ as well as the variable $\cdot$ are...
defined by (11) for \( k = 1 \) (\( \cdot = P_m \)) and (12) for \( k = 2 \) (\( \cdot = \phi \)). From (13) the overall best mean model error \( \zeta^{(q)}(L_1, h_2) \) with respect to all assessed models \( q \) is thus straightforwardly quantified as the model \( \hat{q} \) minimising the cost function \( J(q) \) as expressed in (14) and (15):

\[
J(q, L_1, h_2) = \frac{1}{3} \left( \zeta_1^{(q)}(\hat{P}_1, L_1, h_2) + \zeta_2^{(q)}(\hat{P}_2, L_1, h_2) + \zeta_2^{(q)}(\hat{\phi}, L_1, h_2) \right),
\]

(14)

\[
\hat{q}(L_1, h_2) = \arg \min_q J(q, L_1, h_2).
\]

(15)

The overall best mean model errors \( J(\hat{q}, L_1, h_2) \) are plotted in Fig. 5. Fig. 6 depicts the corresponding averaged errors \( \zeta_k^{(q)}(\cdot, L_1, h_2) \) (13) for \( \hat{P}_1 \), \( \hat{P}_2 \) and \( \hat{\phi} \). In addition to the error values (13), the errorbars in Fig. 6 illustrate the sensitivity of the model accuracy for variations of the upstream pressure \( P_0 \). In general, the error sensitivity increases as the error values \( \zeta_k^{(q)}(\cdot, L_1, h_2) \) increases. The overall best mean model error yields

\[
J(\hat{q}, L_1, h_2) < 30\% \text{ for all } (L_1, h_2) \text{ except in absence of } C_1, \text{ denoted } L_1 = \text{ none}. \text{ In absence of } C_1, \text{ the errors for } h_2 > 1.5 \text{ are significantly larger than in presence of } C_1, \text{ so that the upper limit for the overall model accuracy increases to } J(\hat{q}, L_1, h_2) < 50\%. \text{ From Fig. 6 is observed that in presence of } C_1 \text{ large overall errors } J(\hat{q}, L_1, h_2), \text{ e.g. } h_2 = 5.5 \text{ compared to } h_2 = 1.5 \text{ in Fig. 5, are due to large errors of } \zeta_1^{(q)}(\hat{P}_2, L_1, h_2) \text{ and/or } \zeta_2^{(q)}(\hat{\phi}, L_1, h_2). \text{ In absence of } C_1, \text{ the error } \zeta_1^{(q)}(\hat{P}_1, L_1, h_2) \text{ is seen to increase as well explaining the increased overall best mean error upper limit of } J(\hat{q}, L_1, h_2) < 50\% \text{ instead of } J(\hat{q}, L_1, h_2) < 30\%. \text{ The models resulting in the overall best mean model error } J(\hat{q}, L_1, h_2) \text{ (14), illustrated in Fig. 5, are summarised in Table 3. From Table 3 is seen that for } h_2 = 16 \text{ as well as } h_2 = 0.6 \text{ accounting for viscous effects, i.e. } \hat{q} = \text{ Poi}, \text{ results in the sought errors } J(\hat{q}, L_1, h_2) \text{ regardless the value of } L_1. \text{ For intermediate values, } 0.6 < h_2 < 16, \text{ the overall best mean model errors } J(\hat{q}, L_1, h_2) \text{ are obtained for models } \hat{q} = \text{ Con} \text{ or } \hat{q} = \text{ Ben} \text{ depending on } (L_1, h_2). \text{ It is observed that inserting } L_1 \text{ upstream from } h_2 \text{ and moving it further downstream, i.e. decreasing } L_1, \text{ causes a model shift from } \hat{q} = \text{ Con} \text{ to } \hat{q} = \text{ Ben}. \text{ So, in case of a large gap } L_1 \text{ between both constrictions } C_1 \text{ and } C_2, \text{ the narrowed passage at } C_2 \text{ can be modelled as a sudden constriction whereas for smaller } L_1 \text{ the narrowed passage } C_2 \text{ can be approximated as a bend in the geometry. The transition, i.e. constriction } \rightarrow \text{ bending, is seen to depend on the value of the aperture } h_2. \text{ The models resulting in the overall best mean model error yields an error of } 0.6 \text{ for } h_2 = 0.6 \text{ and a sudden constricted passage of } 16 \text{ for } h_2 = 16. \text{ Consequently, besides } h_2, \text{ the models do not predict any constricted passage.}

6 Conclusion

A rigid ‘in-vitro’ replica is proposed in order to study airflow through the human vocal tract during fricative production. Two geometrical parameters are experimentally studied: the position of an upstream ‘tongue’ shaped constriction in the main flow direction \((L_1)\) and the constriction degree of a ‘tooth’ shaped downstream obstacle \((h_2)\). The shape of both obstacles is extremely simplified in order to limit the number of geometrical and flow parameters to be taken into account.

Measured pressures and volume airflow rates are compared to the outcome of one-dimensional flow models assuming a laminar incompressible irrotational and one-dimensional flow governed by Bernoulli’s equation to which corrections are applied for viscosity, sudden geometrical expansion, sudden geometrical constriction and a bending geometry. In presence of the ‘tongue’ shaped constriction, the accuracy for each set of geometrical parameters \((L_1, h_2)\) expressed as a mean error for all predicted quantities and all imposed upstream pressures yields < 30%. The model resulting in the minimum errors varies as function of \((L_1, h_2)\). For very small \((\leq 58\%)\) or very large \((\geq 96\%)\) constriction degrees at the ‘tooth’ the most accurate model is obtained by accounting for viscosity regardless the value of \(L_1\). For intermediate constriction degrees, in the interval \((58\%\text{ to }96\%\)), narrowing the gap between both constrictions, i.e. decreasing \(L_1\), causes the most accurate model to shift from constriction to bending. Therefore, the geometrical parameter \(L_1\), although not explicitly appearing as a parameter in the validated one-dimensional models, does determine the appropriate corrective term in terms of the cost function in addition, it is interesting to note that the best model accuracy is poorest for ‘tongue’ constriction degrees \((\approx 60\%)\) for which the influence of \(L_1\) on the measured pressures is most significant. Consequently, one-dimensional flow models can be applied to describe the flow through the vocal tract when accounting for the relevant corrections in order to compensate, on geometrical considerations, for the non realistic assumption of a laminar and irrotational flow. This way the approach of one-dimensional flow modeling, commonly used in physical phonation models, can be extended to the vocal tract. Nevertheless, several topics for further research can be formulated. With respect to modelling, more complex flow modeling is moti-
Figure 6: Model errors $\zeta_k^j(P_1, L_1, h_2)$ (13) for $\tilde{P}_1, \tilde{P}_2$ and $\tilde{\phi}$ for the models $\hat{q}(L_1, h_2)$ corresponding to $J(\hat{q}, L_1, h_2)$ presented in Fig. 5 and summarised in Table 3 versus $h_2$ as function of all assessed $L_1$: a) $\zeta_1^j(\tilde{P}_1, L_1, h_2)$, b) $\zeta_2^j(\tilde{P}_2, L_1, h_2)$ and c) $\zeta_2^j(\tilde{\phi}, L_1, h_2)$. Absence of the downstream obstacle $C_1$ is denoted $L_1 = \text{none}$.

Table 3: Overview of the selected models $\check{q}(L_1, h_2)$ resulting in the overall best mean error $J(\check{q}, L_1, h_2)$ (14) whose value is plotted in Fig. 5. Models are referred to as outlined in Table 2. For completeness also the constriction degree due to $h_2$, i.e. $\vartheta_2(h_2) = 1 - h_2/h_0 [\%]$, and the constriction degree of the gap between both constrictions due to $L_1$, i.e. $\vartheta_1(L_1) = 1 - (L_1 - 9)/h_0 [\%]$, are indicated as well.

<table>
<thead>
<tr>
<th>$\vartheta_1(L_1)$</th>
<th>Decreasing $L_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vartheta_2(h_2)$</td>
<td>$h_2 = \text{none}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>58</td>
<td>6.8</td>
</tr>
<tr>
<td>65</td>
<td>5.5</td>
</tr>
<tr>
<td>84</td>
<td>2.6</td>
</tr>
<tr>
<td>91</td>
<td>1.5</td>
</tr>
<tr>
<td>96</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Acknowledgments

Financial support of the Agence Nationale de la Recherche is gratefully acknowledged.

References


