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Modeling of the nonlinear distortion in electrodynamic loudspeakers caused by the voice-coil inductance

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The electrodynamic loudspeaker is a strongly nonlinear system. The main causes of nonlinearity are the nonlinear stiffness of the suspensions and the nonuniform distribution of the magnetic flux density along the air gap in the magnetic circuit. A third cause, the nonlinear voice-coil inductance is often underestimated. The electrical impedance of the voice-coil in the low frequency range is considered usually as a pure resistance, and the influence of its inductance is neglected : this is a reason of the underestimation of its nonlinearity. The voice-coil inductance depends on both the displacement of the voice-coil and the current intensity flowing through it. Additionally, the reluctance force proportional to the square of the current appears in the mechanical part of the system and depends on the voice-coil displacement too. This paper studies the influence of the voice-coil nonlinearity. The differential equation system has been derived and is solved using numerical methods. The harmonic distortions as well as the intermodulation ones have been computed. The results show that the influence of the voice-coil nonlinearity is significant – particularly for intermodulation distortions. This influence is weaker than the influence of the force factor nonlinearity, but stronger than the influence of the suspensions nonlinearity. The influence of the different terms in the nonlinear differential equation system is also tested.

1 Introduction

The nonlinearities of electrodynamic loudspeakers lead to harmonic and intermodulation distortions in the sound production. Their sources have been studied and described by many authors, [1] - [10]. Among them, the suspensions and the motor are prominent sources. The suspensions create now well-known nonlinearities, and the manufacturers have developed materials and designs to minimize their drawbacks. Some alternative designs have also been proposed which suppress the classical suspensions [11] - [12] . Other important cause of distortion is the nonlinearity of force factor *Bl*. It strongly depends on the displacement of moving system. The nonlinearity of a voice-coil inductance is usually underestimated as the source of nonlinear distortion. In low frequency range the inductance is usually neglected in the equivalent circuit of the loudspeaker as well as in the analysis. However, if in the exciting signal the low and high frequency component appear, the nonlinearity of voice-coil inductance produces very annoying intermodulation distortion. The aim of this paper is to analyse the distortions produced by the voice-coil inductance and compare these distortions with distortion produced by the suspension and *Bl*-product nonlinearity.

2 Modeling of electrodynamic loudspeaker with inductance and eddy currents

The electrical equivalent circuit of an electrodynamic loudpeaker is presented in Fig. 1. It consists of three parts: electrical which is represented by the voice coil resistance R_E and inductance L_E ; transducer represented by a gyrator with gyration constant *Bl* (called also force factor) and mechanical represented by mass of the moving system *mms*, stiffness *k* and mechanical resistance of the suspension *r*.

The voice coil inductance is not a pure inductance. The eddy currents induced in the magnetic circuit of the loudspeaker react with the current in the voice-coil and cause the decrease of the inductance and the appearance of a real part of the impedance. The influence of eddy currents can be represented by the resistor R_μ connected in parallel with voice-coil inductance. However, it is a coarse simplification. The equivalent circuit of the loudspeaker is presented in Fig. 1.

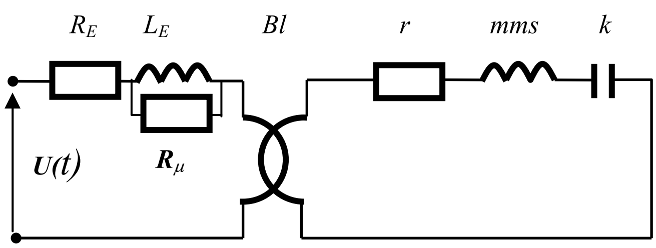

Fig. 1. Electrical equivalent circuit of the loudspeaker with the inductance of the voice-coil and eddy-current resistance

The calculation has been conducted for following constant parameters: R_E = 3.3 ohm, *mms* = 0.009 kg, *r* = 1.1 Ns/m, *k* = 7692 N/m, *Bl* = 5.5 Tm, R_μ = 2.2 ohm. Linear term of the inductance was L_{E0} = 0.0017 H. The excitation was sinusoidal, and the voltage amplitude was 1 V. The sound pressure has been computed on the axis at the distance of *d*=1m from diaphragm center, the diameter of the diaphragm was equal to 0.2 m. The loudspeaker was described with a system of ordinary differential equations. The calculation was realized using Mathcad14[1] worksheet. Figures 2 and 3 present SPL and loudspeaker impedance for linear parameters.

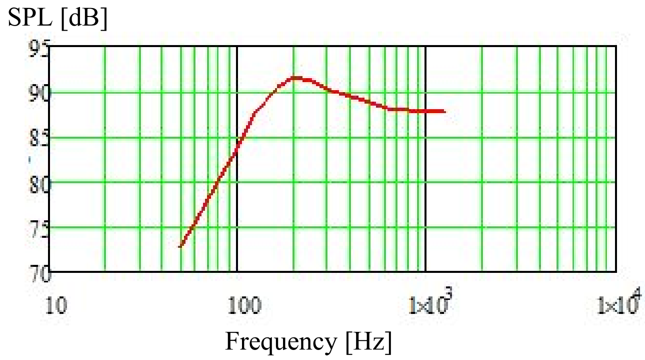

Fig. 2. Frequency response of the loudspeaker with inductance and eddy current resistor but without nonlinearity

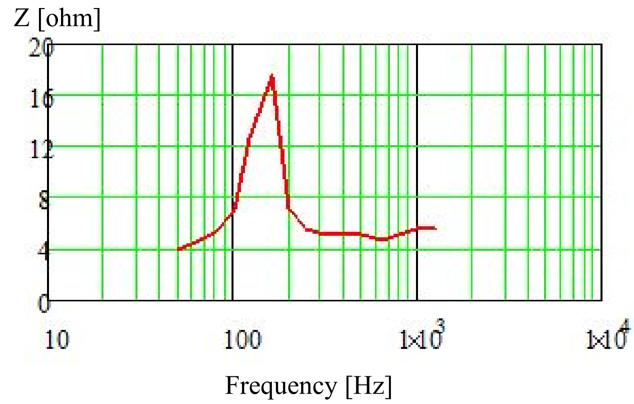

Fig. 3. Electric impedance modulus vs. frequency of the loudspeaker with inductance and eddy current resistor but without nonlinearity

For higher frequencies the resistor R_μ blocks the inductance L_E and the impedance curve becomes flat in this frequency range. However, the total electrical resistance is the sum of R_E and R_μ, and the sensitivity decreases. The results of modeling show that the model of eddy currents based on one resistor blocking the inductance is too simple.

3. Model of loudspeaker with inductance depending nonlinearly on the electric current

The voice coil inductance nonlinearly depends on electric current and on the displacement. In this section the model of the loudspeaker with the inductance depending on the current flowing in the voice-coil is considered. The nonlinear dependence of the inductance on the electric current is modeled with polynomial of the second order:

$$L_E(i) = L_{E0}\left(1 + ai + bi^2\right) \tag{1}$$

[1] Mathcad14 is the Trade Mark of Parametric Technology Corporation

In the electrical side of the system the derivative of magnetic flux appears:

$$\frac{d\Psi}{dt} = \frac{d[L_E(i)\cdot i]}{dt} = L_E\frac{di}{dt} + \frac{dL_E}{di}\cdot\frac{di}{dt}\cdot i =$$
$$= L_{E0}\left(1 + 2ai + 3bi^2\right)\frac{di}{dt} \tag{2}$$

Then, the effective nonlinearity is higher than the nonlinearity of the inductance only. The differential equation system for a loudspeaker with nonlinear inductance depending on the current only has the form:

$$L_{E0}\left(1 + 2ai + 3bi^2\right)\frac{di}{dt} = U(t) - R_E i - Blv$$
$$\frac{dx}{dt} = v \tag{3}$$
$$\frac{dv}{dt} = -\frac{r}{mms}v - \frac{k}{mms}x + \frac{Bl}{mms}i$$

The solution of this equation system are functions of time: current, displacement and velocity. On the basis of the third equation of (3) the acoustic pressure can be computed. The acoustic pressure at the distance r (r was assumed 1 m) is given by the formula [13]:

$$p = \rho\frac{\omega v S}{2\pi r} = \rho\frac{S}{2\pi r}\left(\frac{dv}{dt}\right) \tag{4}$$

where: ρ – air density, S – surface of the diaphragm. After solving equation system (3), the right hand side of third equation of this system is put into equation 4 as the acceleration.

In Figs. 4 and 5 the examples of sound pressure and current for frequency 200 Hz and voltage amplitudes U_0= 1V and 10 V are presented.

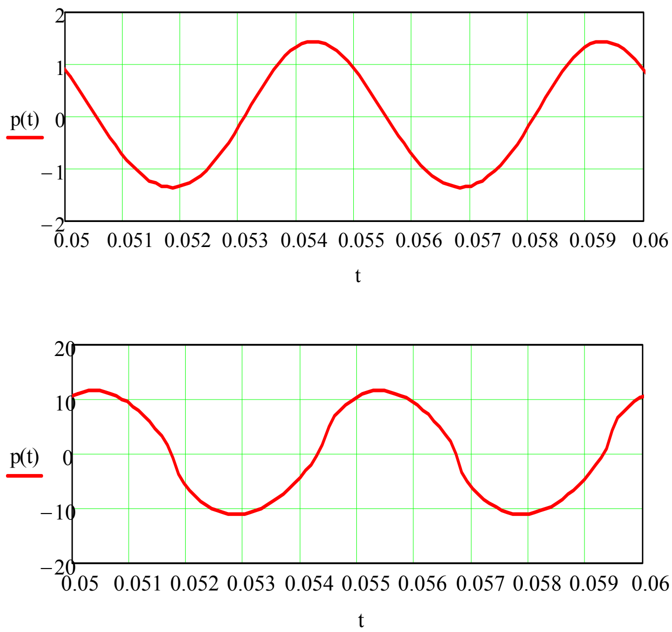

Fig. 4. Acoustic pressure vs. time for frequency f=200 Hz. Upper: U_0=1V, Lower: U_0=10V

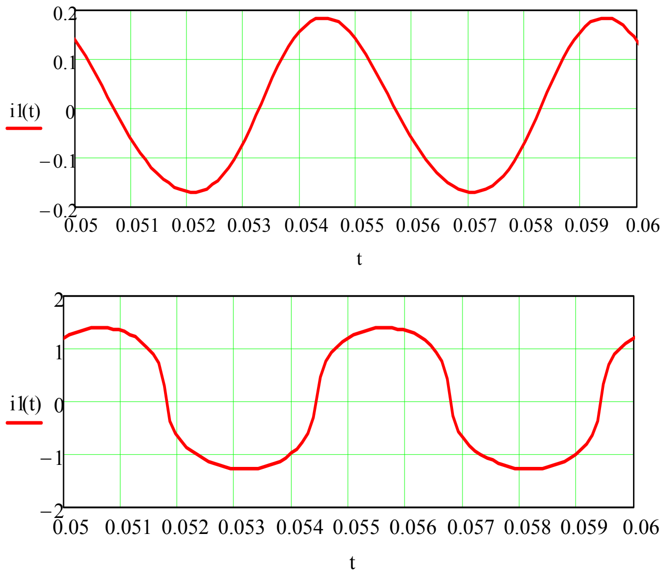

Fig. 5. Current in the voice-coil vs. time for frequency f = 200 Hz. Upper: U_0=1V, Lower: U_0=10V

The calculations have been carried out for following nonlinear coefficients: a=-1.68, b=7.58. The solutions depend strongly on the amplitude of exciting voltage. The spectra of sound pressure levels for both excitation amplitudes are presented in Figs. 6 and 7.

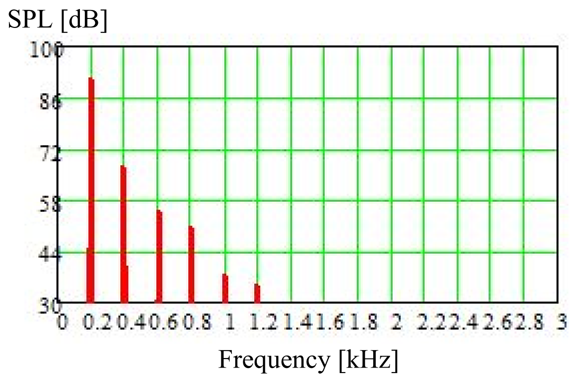

Fig. 6. Harmonic distortion of the acoustic pressure for the system with nonlinear inductance (U_0= 1V)

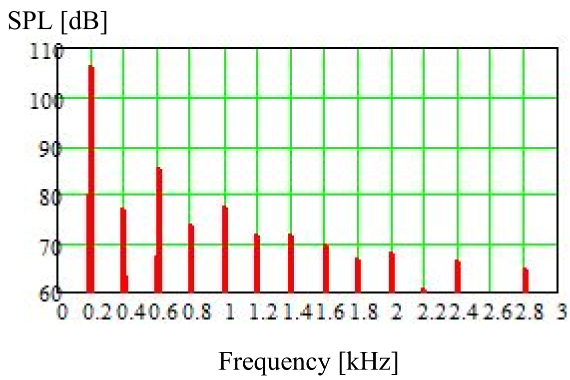

Fig. 7. Harmonic distortion of the acoustic pressure for the system with nonlinear inductance (U_0= 10V)

The output signal for excitation voltage 1 V seems to be undistorted, however in Fig 6 the distortion components are clearly visible. Of course, for high excitation level the distortion products are also high.

4. Model of loudspeaker with inductance depending nonlinearly on the displacement

In this section the dependence of the voice-coil inductance is modeled by the linear function:

$$L_E(x) = L_{E0}(1 + Ax) \tag{5}$$

At the electrical side of the loudspeaker system the derivative of the magnetic flux has the form:

$$\frac{d\Psi}{dt} = \frac{d[L_E(x)i]}{dt} = L_E\frac{di}{dt} + \frac{dL_E}{dx}\cdot\frac{dx}{dt}\cdot i \tag{6}$$

where derivative of displacement is the velocity. Then, the equation system has the form:

$$L_{E0}(1 + Ax)\frac{di}{dt} = U(t) - R_E i - Blv - L_{E0}A\cdot i\cdot v$$

$$\frac{dx}{dt} = v \tag{7}$$

$$\frac{dv}{dt} = -\frac{r}{mms}v - \frac{k}{mms}x + \frac{Bl}{mms}i + \frac{1}{2}\frac{L_{E0}A}{mms}\cdot i^2$$

The last term in third equation of system (7), depending on the square of a current, is called a reluctance force [14]. In fact, the inductance is nonlinear with regard to the current and the displacement. One of the consequence of the dependence with the displacement is the fact that an additional term appears in the force applied to the moving part: the reluctance force is added to the Laplace force.
The value A for calculations is equal to -50 m^{-1}. Fig. 8 presents harmonic distortions created by nonlinear inductance for excitation frequency 200 Hz and voltage 10 V, computed with Mathcad14 worksheet.

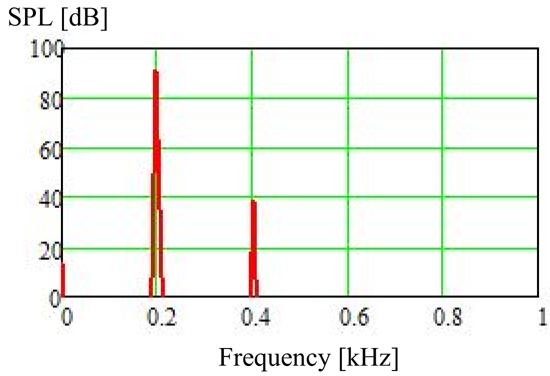

Fig. 8. Harmonic distortion of the acoustic pressure for the system with inductance nonlinearly dependent on displacement

It is clearly visible that the distortion level is very low. Only the second harmonic appears and its level is 50 dB lower than the level of fundamental component. However, the nonlinear inductance produces for an actual signal not only harmonic components but also the intermodulation components. In the simplest case, when the system is excited with 2 components – of low and of high frequency, the intermodulation product appears as the component of sum and difference components. In Fig 9 the intermodulation distortion produced by the system described with equation system (7) is presented. The system was excited with two components: frequency 50 Hz and voltage 10 V, and frequency 1250 Hz and voltage 2.5 V. The result of calculation is presented in Fig. 9.

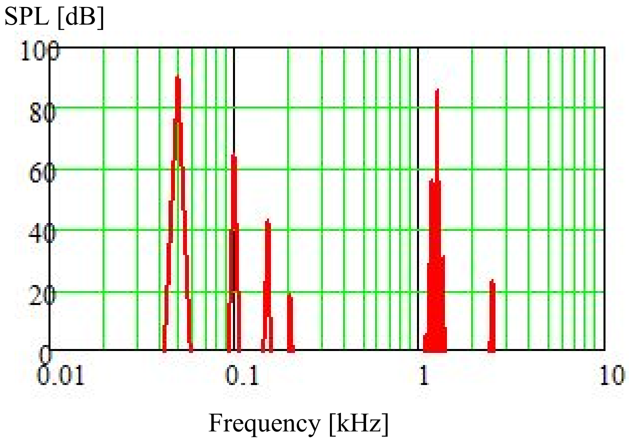

Fig. 9. Intermodulation distortion produced by the system with nonlinear inductance. Excitation: f_1=50 Hz, U_1=10 V, f_2=1250 Hz, U_2=2,5 V.

The intermodulation distortions are very annoying in the human perception. It is interesting to test the contribution of all nonlinear terms in the equation system (7). The results of this testing is presented in Figs. 10-12.

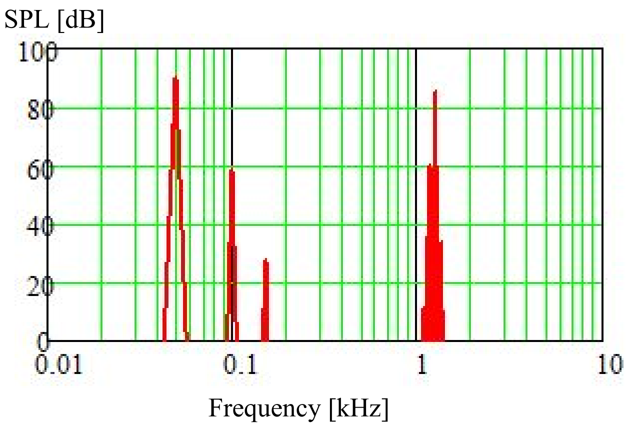

Fig. 10. Contribution of the nonlinear term at the left hand side in the first equation of the system (7) in the intermodulation distortion produced by the system with nonlinear inductance. Excitation the same as in Fig. 9

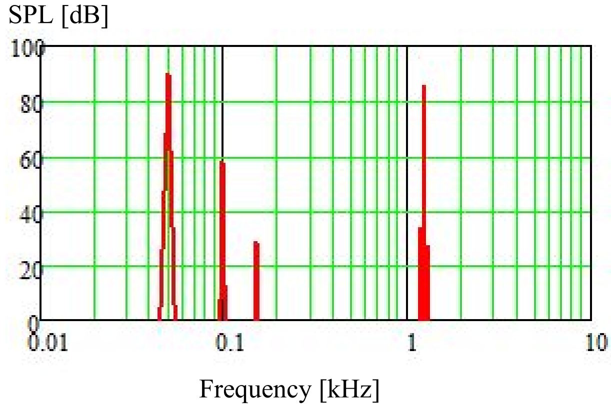

Fig. 11. Contribution of the nonlinear term at the right hand side in the first equation of the system (7) in the intermodulation distortion produced by the system with nonlinear inductance. Excitation the same as in Fig. 9

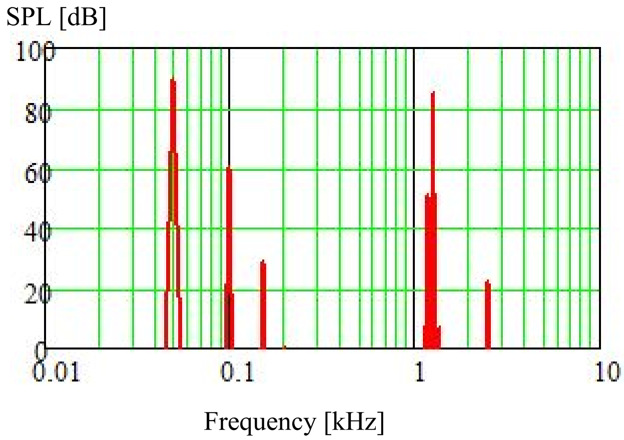

Fig. 12. Contribution of the nonlinear term at the right hand side in the third equation of the system (7) (the reluctance force) in the intermodulation distortion produced by the system with nonlinear inductance. Excitation the same as in Fig. 9

The most important cause is the main nonlinearity of the inductance. It is interesting to mark that the level of intermodulation product of the second order f_2-f_1 and f_2+f_1 is even higher than this level when all nonlinear terms are taken into account (compare Figs. 9 and 10. The reluctance force produces relatively high level of the second harmonic of the higher component of excitation. The mixed nonlinear term current velocity produces the lowest distortions levels.

5. Comparison of intermodulation distortion produced by nonlinearity of *Bl*, stiffness of suspensions and inductance, depending on displacement

In this part ranking of nonlinearities of three physical parameters depending on the displacement has been tested.

The tested parameters are: *Bl*-factor, stiffness of the suspensions and voice-coil inductance. The rest values of these parameters were chosen to be the same as in previous parts, i.e. L_{E0}=0.0017 H, Bl_0= 5.5 T·m, k_0=7692 N/m. All the models of nonlinearities are similar, i.e. quadratic function of displacement, and parameters of these functions were the same, except of sign, because protect character of each nonlinearity. Then, the nonlinearity of three parameters are described by following equations:

$$k(x) = k_0\left(1 + 200x + 20000x^2\right) \qquad (8)$$

$$Bl(x) = Bl_0\left(1 + 200x - 20000x^2\right) \qquad (9)$$

$$L_E(x) = L_{E0}\left(1 - 200x + 20000x^2\right) \qquad (10)$$

The displacement *x* is given in meters. All coefficients in equations (8 – 10) have realistic values. Dependences (8) – (10) are drawn in Figs. (13) – (15).

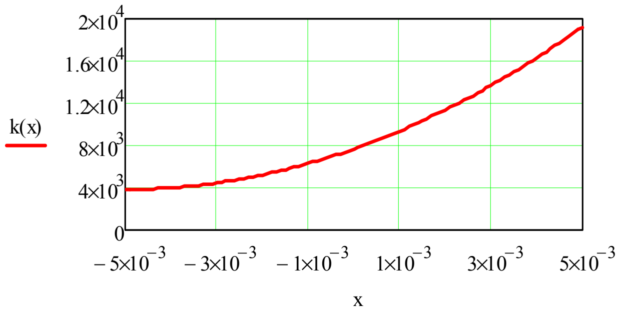

Fig. 13. The dependence of suspension stiffness on displacement

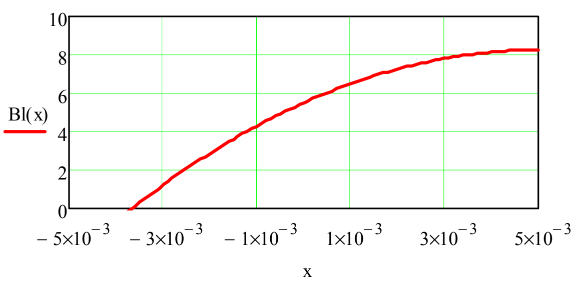

Fig. 14. The dependence of force factor on displacement

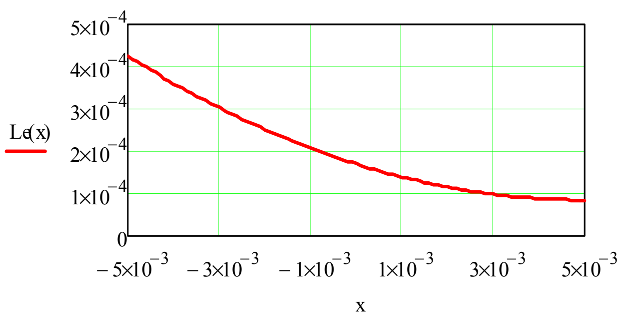

Fig. 15. The dependence of voice-coil inductance on displacement

The results of modeling of intermodulation distortion with the same excitation as described in section 4 are presented in Figs. 16, 17 and 18 to show the influence of the nonlinearity of stiffness, force factor and inductance, respectively.

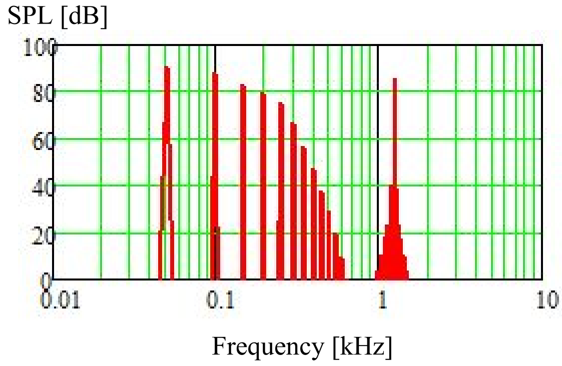

Fig. 16. Intermodulation distortion caused by nonlinear suspension stiffness given by Eq. (8). Excitation: f_1=50 Hz, U_1=10 V, f_2=1250 Hz, U_2=2,5 V.

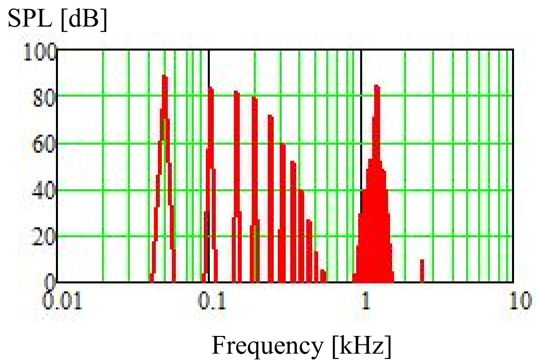

Fig. 17. Intermodulation distortion caused by nonlinear force factor *Bl* given by Eq. (9). Excitation the same as in Fig. 16.

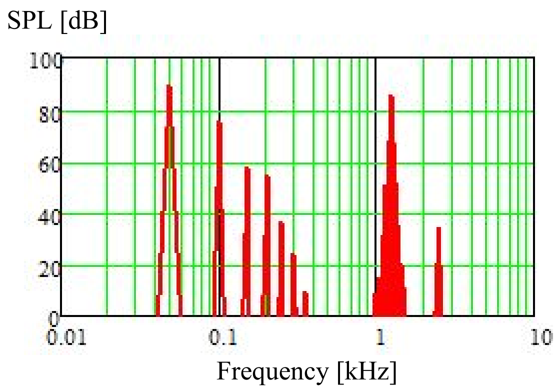

Fig. 18. Intermodulation distortion caused by nonlinear voice-coil inductance given by Eq. (10). Excitation the same as in Fig. 16.

It can be observed that the nonlinear inductance causes the smallest harmonic distortion in low frequency range, but the highest one in high frequency range. The highest intermodulation distortion is produced by nonlinear *Bl*, however, this distortion produced by nonlinear inductance is comparable. The intermodulation distortion produced by stiffness nonlinearity is significantly smaller.

6 Conclusion

The results show a great usefulness of modeling. In an actual loudspeaker the different causes of nonlinear distortion occur commonly and it is impossible to separate them from measurement results. However, the experiment is necessary. The methods of measurements of suspension stiffness and *Bl*-factor has been developed, the method of measurement of inductance nonlinearity is prepared. The nonlinearity of inductance using AC signal as function of both: the current and the displacement has been developed. It has been proved that it is very difficult to conclude about DC inductance nonlinearity on the basis of these results.

It also has been proved that the nonlinearity of voice-coil inductance is a significant source of distortion, particularly in high frequency range and for intermodulation distortions.

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