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Surface acoustic waves and their associated quasi-particles

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In this contribution a way to associate quasi-particles in inertial or non-inertial motion with surface-wave solutions is shown, especially for Rayleigh and Bleustein-Gulyaev surface waves on elastic or piezoelectric substrates. Perturbations of various kinds are also envisaged. The technique employed for this is based on the exploitation of the wave-momentum equation deduced via Noether's theorem or by direct computation, and associated with the basic field equations.

1 Introduction

After a long period of competition, the wavelike and particle-like visions of some dynamical theories seem to have reached an agreement in their useful complementarity [1]. Both serve to describe a propagating information via their well founded duality. More specifically, wave theory is a non-discretized model as opposed to the particle theory that is a discretized (in some sense) condensed, or even "materialized" model. The wave modelling favours a description of the propagation of information in terms of wave number and frequency. As to the particle model, it pertains to a diffusion of information through certain interactions in terms of momentum and energy. Classically, the duality between the two modellings is concretised by the introduction of the Planck constant as a true translator. For instance, if, in a picturesque vision, we can say that photons are grains of light (luminous vibrations) and acoustic phonons are grains of acoustic vibrations, both in an unbounded space, the particle-like objects studied in the present contribution account in their very definition for the fact that the continuous waves are confined along a guiding surface and also for the type of considered boundary conditions. We can speak of *guided* quasi-particles *or grains of surface acoustic waves* (SAWs). This explains the relative complexity for the proof of their existence and the sometimes farfetched definition of their effective mass.

In the present approach we are very much influenced by the study of solitonic waves whose nickname "solitons" suggests a particle-like behaviour [2]. But contrary to these that are extremely spatially localized nonlinear waves, here we are concerned with harmonic waves of small amplitude and infinite support in propagation space. Here, exploiting the analogy with solitons, we study the quasi-particles that are dual – in the sense of the above emphasized duality – of surface waves or guided waves that are solutions of the elastic wave problems associated with specific boundary conditions such as Rayleigh waves, Lamb waves, etc. Remember that such waves have an energy confined to the vicinity of the appropriate guiding interface. We are thus led to a clear introduction of the notion of *surface acoustic phonon*. Usually such a notion is elusive and strictly based on lattice dynamics [3]. The interest in this study stems from the potentially associated simple interpretation of the interaction between fellow waves or of the interaction of such a wave with material objects (defects, inclusion,…). In order to define the looked for dual quasi-particles, it proves efficient and well adapted to exploit the conservation equations of canonical momentum (wave momentum) and energy, as recently revisited in continuum mechanics [4].

More precisely, we consider here the most popular case of SAWs, the Rayleigh wave propagating along the free surface of an isotropic elastic substrate and the case of so-called Bleustein-Gulyaev SAWs propagating on top of a 6mm-symmetric piezoelectric crystal. The reason for considering these two cases is that the first provides an example of *nondispersive but multicomponent* SAWs (polarized in the sagittal plane), while the second exhibits an example of a nondispersive SAW with only one mechanical component (an SH wave from the mechanical view point) but coupled to electrostatics in the appropriate crystal symmetry. Minimal perturbations to these are also considered such as coupling with an external mechanical medium (superimposed film with its own elastic energy; coupling with an external fluid) in the first case, and coupling with an external dielectric medium (vacuum) in the second case. The case of dispersive waves such as Love and Lamb waves, and that of generalized Rayleigh waves, as well as more complicated surface conditions (Stoneley waves, elastic slip, delamination) and propagation in a dissipative substrate, are left out for the moment.

2 The case of standard Rayleigh waves

2.1 Reminder

The standard equations for dynamical small-strains in a homogenous isotropic body in the absence of body force and in Cartesian tensor notation are

$$\frac{\partial}{\partial t}p_i - \frac{\partial}{\partial x_j}\sigma_{ji} = 0 \ in \ B, \tag{1}$$

with (λ and μ are Lamé coefficients)

$$p_i = \rho_0 v_i \ , \ \sigma_{ji} = \lambda e_{kk}\delta_{ji} + 2\mu e_{ji} \ , \tag{2}$$

$$v_i = \partial u_i / \partial t, \ e_{ij} = (u_{i,j} + u_{j,i})/2 \ . \tag{3}$$

The boundary conditions that accompany (1) read

$$n_j \sigma_{ji} = T_i^d \ or \ u_i = u_{0i} \tag{4}$$

on complementary parts of the supposedly regular boundary ∂B of unit outward **n** of *B*.

The Rayleigh surface acoustic wave (SAW) involves the longitudinal and vertical displacement components (polarization parallel to the sagittal plane), propagating in the $x_1 = x$ direction of wave number $k_1 = k_R$ and with amplitude decreasing exponentially to zero with depth $x_2 = z > 0$. With standard notation

$$k_R = \omega / c_R \ , \ k_L = \omega / c_L \ , k_T = \omega / c_T \ ; \quad c_L = \sqrt{(\lambda + 2\mu)/\rho_0} \ , \quad c_T = \sqrt{\mu / \rho_0} \ , \tag{5}$$

$$\alpha_L = \sqrt{k_R^2 - k_L^2} \ , \ \alpha_T = \sqrt{k_R^2 - k_T^2} \ , \tag{6}$$

the celebrated Rayleigh equation for the Rayleigh speed c_R reads [5]

$$D(c_R; c_L, c_T) := 4\sqrt{1 - (c_R / c_T)^2} \ \sqrt{1 - (c_R / c_L)^2} - \left(2 - (c_R / c_T)^2\right)^2 = 0 \ . \tag{7}$$

2.2 Conservation of wave momentum

Either through the application of Noether's theorem in a variational formulation or by direct computation (by applying $u_{i,k}$ to (1) and rearranging terms) it is shown that in addition to the energy equation, (1) is accompanied, at each regular material point, by a co-vectorial conservation equation, the equation of so-called *wave momentum* [4]:

$$\frac{\partial}{\partial t}p_i^w - \frac{\partial}{\partial x_j}b_{ji} = 0 \ , \tag{8}$$

where ($L = K - W$, *K* kinetic energy, *W* potential energy; b_{ji} = Eshelby stress tensor; cf. [4])

$$p_i^w := -p_k u_{k,i} = -\rho_0 v_k u_{k,i} \ , \ b_{ji} := -\left(L\delta_{ji} + \sigma_{jk} u_{k,i}\right). \tag{9}$$

Once a solution to a standard boundary-value problem for (1) and (4) is know by any means, (8) can be exploited for another purpose. This is the case here.

2.3 Quasi-particle associated with the standard Rayleigh SAW

We integrate (8) in a vertical band (see Figure 1) of the sagittal plane $D = (x_0, x_0 + \lambda_R) \times (0, +\infty)$, and *thickness unity* in the x_3 direction (the solution does not depend on that coordinate) and apply the divergence theorem. The resulting various surface terms are treated in different manners. The term at $x_2 \to \infty$ yield zero as all quantities go to zero by the nature of a surface wave. The terms on the left and right sides of the band compensate one another by virtue of the periodicity of the solution along x_1 and the opposite sign of their unit normals. It remains the term at the surface $x_2 = 0$ with x_1 confined to a wavelength λ_R of the Raleigh wave. The remarkable results here are that the x_1 component of this surface term is nil by computation (average over a wavelength of a $sin \times cos$ term) while the x_2 component vanishes because it is proportional to the "Rayleigh" equation (7) that is satisfied by the solution (this indeed says that the average over one wavelength of the Lagrangian density *L* evaluated at the limiting surface is proportional to the linear "dispersion relation"). A direct evaluation of the integral over the band *D* of the left hand side of (8) yields identically zero, so that there only remains a nonzero $x_1 = x$ component that represents the *inertial motion of a quasi particle* of "mass" M_R (per unit length in the $x_3 = y$ direction) such that

$$\frac{d}{dt}\left(M_R\, c_R\right) = 0 \;, \tag{10}$$

with

$$M_R = \rho_0\, \pi\, f\left(k_R, \alpha_L, \alpha_T\right) A^2 \tag{11}$$

where A is the amplitude of the longitudinal displacement component, and the nondimensional factor f is evaluated as

$$f\left(k_R;\, \alpha_L\,, \alpha_T\right) = \frac{k_R^2 + \alpha_L^2}{4 k_R \alpha_L} + \frac{2\alpha_L^2\, \alpha_T\, k_R^2}{k_R (k_R^2 + \alpha_T^2)^2} + \frac{2\alpha_L\, \alpha_T k_R^2 + \alpha_L^2 (k_R^2 + \alpha_T^2)}{k_R (k_R^2 + \alpha_T^2)(\alpha_L + \alpha_T)} \;. \tag{12}$$

Accordingly, the "mass" contains the whole of information related to the surface wave solution represented by a quasi-particle in inertial motion guided by the limiting surface.

3 The case of Bleustein-Gulayev waves

3.1 Reminder

In this case allowed by the crystal symmetry (6mm with axis perpendicular to the sagittal plane), the SAW consists in only one elastic component of the shear horizontal (SH) type (along x_3) coupled to a quasi-electrostatic potential ϕ via piezoelectricity. (cf.[6], pp.250-254). The surviving field equations from the electromechanical case (compare (1); now **D** is the electric displacement)

$$\frac{\partial}{\partial t} p_i - \frac{\partial}{\partial x_j} \sigma_{ji} = 0 \,,\;\; D_{j,j} = 0 \; in\; B \,, \tag{13}$$

read (∇^2 is the 2D Laplacian operator in the sagittal plane)

$$c_{44}\, \nabla^2 u_3 + e_{15}\, \nabla^2 \phi = \rho_0 \frac{\partial^2 u_3}{\partial t^2}, \qquad e_{15}\, \nabla^2 u_3 - \varepsilon_{11} \nabla^2 \phi = 0 \;, \tag{14}$$

where use has been made of the constitutive equations

$$\sigma_{23} = c_{44} u_{3,2} + e_{15} \phi_{,2} \;,\; \sigma_{13} = c_{44} u_{3,1} + e_{15} \phi_{,1} \;, \tag{15}$$

and

$$D_1 = e_{15} u_{3,1} - \varepsilon_{11} \phi_{,1} \,,\; D_2 = e_{15} u_{3,2} - \varepsilon_{11} \phi_{,2} \,. \tag{16}$$

Here c_{44} is an elastic coefficient, ε_{11} is the dielectric coefficient of the substrate, and e_{15} is a piezoelectric constant. The boundary conditions at $x_2 = 0$ may be of the following types:

$$n_j \sigma_{ji} = \mathbf{0} \;\; \text{(mechanically free surface);} \tag{17}$$

$$n_j \left[D_j\right] = 0 \,; \tag{18}$$

$$\phi = \phi_0 \;\; \text{(electrically grounded surface);} \tag{19}$$

$[\phi] = 0$ (matching to an external electric field, e.g., vacuum of electric permeability ε_0); (20)

In the last case one has to account for the electrostatic equation

$$\nabla^2 \phi = 0 \; for \; x_2 < 0 \;\; . \tag{21}$$

Equations (14) can be re-written as the system

$$\bar{c}_T^2\, \nabla^2 u_3 = \frac{\partial^2 u_3}{\partial t^2} \,,\; \nabla^2 \psi = 0 \tag{22}$$

with

$$\psi = \phi - \left(e_{15} / \varepsilon_{11}\right) u_3 \;,\; \bar{c}_T^{\,2} = \bar{c}_{44} / \rho_0 \,, \quad \bar{c}_{44} = c_{44} \left(1 + K^2\right),\; K^2 = e_{15}^2 / \varepsilon_{11}\, c_{44} \;. \tag{23}$$

For the surface wave problem for $x_2 > 0$ with boundary conditions (17)-(19), one obtains the *Bleustein-Gulyaev SAW solution* with "dispersion" relation [6]

$$D_{GB}\left(\omega, k_1\right) = \omega^2 - c_{BG}^2 k_1^2 = 0 \;;\; c_{BG}^2 = \bar{c}_T^2 \left(1 - \bar{K}^4\right), \tag{24}$$

where $\bar{K}^2 = K^2 / \left(1 + K^2\right)$.

For the surface wave problem for $-\infty < x_2 < +\infty$ with matching conditions (17), (18), and (20) , one obtains the *Bleustein-Gulyaev SAW solution* with "dispersion" relation [6]

$$D_{GB}(\omega, k_1) = \widetilde{D}_{BG}(\omega, k_1)$$
$$:= \omega^2 - \widetilde{c}_{BG}^{\,2}\ k_1^{\,2} = 0\ ; \quad (25)$$
$$\widetilde{c}_{BG}^{\,2} = \ \bar{c}_T^{\,2}\left(1 - \widetilde{K}^{\,4}\right)$$

where

$$\widetilde{K}^{\,2} = \overline{K}^{\,2} / \left(\varepsilon_0 + \varepsilon_{11}\right). \qquad (26)$$

Since $\widetilde{K}^2 < K^2$, $\widetilde{c}_{BG} > c_{BG}$, and the depth penetration length in the substrate is somewhat larger in the present case than in the electrically grounded solution.

3.2 Quasi-particle associated with the Bleustein-Gulyaev SAW

In this case the Eshelby stress in the wave-momentum equation contains an electric contribution and is given by [4]

$$b_{ji} = -\left(L\delta_{ji} + \sigma_{jk}u_{k,i} + D_j\phi_{,i}\right). \qquad (27)$$

Proceeding just like in Paragraph 2.3, for the electrically grounded top surface, we are led to the following Newtonian equation of motion for the associated quasi-particle:

$$\frac{d}{dt}\left(M_{BG}\left(A;\overline{K}\right)\right)c_{BG}\Big) = 0 \qquad (28)$$

with “mass” per unit length in the x_3 direction:

$$M_{BG} = \rho_0 \pi g(K) A^2\ , \ g(K) = \overline{K}^{-2}\ . \qquad (29)$$

The last nondimensional quantity blows up with piezoelectricity $e_{15} \to 0$, but the amplitude *A* also goes to zero in these conditions. Independently of the proof of (28), working directly on the energy equation, it is possible to prove exactly that the total energy of the obtained quasi-particle is Newtonian and reads:

$$E_{qp}(BG) = \frac{1}{2} M_{BG}\ c_{BG}^2 \quad , \qquad (30)$$

so that (28) ad (30) together provide a true Newtonian point-mechanics for this quasi-particle.

The situation is a priori more complicated when the matching condition (20) prevails. The integral of the wave-momentum equation must be evaluated over a whole vertical band extending from $x_2 = -\infty$ to $x_2 = +\infty$ with width equal to one wavelength and a unit thickness in the x_3 direction. But the final result is practically the same as (28)-(29), with c_{BG} and $\overline{K}$ replaced by $\widetilde{c}_{BG}$ and $\widetilde{K}$, respectively. In both cases we have benefited from the remarkable result that the average over one wavelength of the relevant component b_{22} or its jump (second case) is proportional to the identically satisfied dispersion relation, e.g., in the second case

$$-\left\langle \left[b_{22} \right] \right\rangle \left(x_2 = 0 \right) \propto \frac{1}{2} \rho_0 A^2 \widetilde{D}_{GB}(\omega, k_1) \equiv 0\ . \quad (31)$$

4 Perturbation of the motion equation of quasi-particles

Perturbations of the above set SAW problems can be envisaged along three directions. Perturbations in the mechanical boundary conditions can be of two types. For instance the limiting plane may be equipped with its own free energy [7] or the mechanical solution in the substrate must match an external wave solution in a liquid. It is shown that the first case results for the Rayleigh SAW in a perturbation of the “dispersion” relation (7) such as

$$D(c_{RF}; c_T, c_L) + \varepsilon_F\ (c_{RF}/c_T)\sqrt{1 - (c_{RF}/c_L)^2} = 0\ , \ (32)$$

where c_{RF} is the new SAW speed solution and ε_F is a supposedly small parameter defined by

$$\varepsilon_F = F k_T / \mu \ \text{ or } \varepsilon_F(\omega) = (F/\mu c_T)\omega\ . \qquad (33)$$

Here *F* is the free surface energy (or surface elasticity) of the surface at $x_2 = 0$. Note that the Rayleigh SAW becomes *dispersive* because a characteristic length $(l = F/\mu)$ is involved. This results in a perturbation of both the velocity of the associated quasi-particle and its mass (that change in opposite directions, keeping the motion inertial). That is, we have

$$\frac{d}{dt}\left(M_{RF}\ c_{RF}\right) = 0\ , \qquad (34)$$

With

$$c_{RF}(\omega) = c_R(1 - \varepsilon_F(\omega)\alpha)\ , \quad M_{RF} = M_R\left(1 + \varepsilon_F \alpha\right) \ , \qquad (35)$$

and

$$\alpha = \frac{-k_R k_T \sqrt{k_R^2 - k_L^2}}{\left(k_T^2 - 2k_R^2\right)^2 \left(\frac{2}{k_R} + \frac{k_R}{(k_R^2 - k_L^2)} + \frac{k_R}{(k_R^2 - k_T^2)} + \frac{8k_R}{(k_T^2 - 2k_R^2)}\right)} \quad . \quad (36)$$

This parameter depends only on the speeds of the Rayleigh wave, of the transverse wave and of the longitudinal wave. Therefore, it is a known negative constant. Then we observe that c_{RF} increases from c_R to c_T when the angular frequency ω varies between 0 and ω_{CF}. The latter is the characteristic value of the angular frequency that corresponds to the limit for the Rayleigh wave to exist, as the transverse wave still is evanescent.

The second type of perturbation (coupling with an outside fluid; Scholte-Stoneley SAW) is not examined here but will yield complications such as a complex mass of which the imaginary part that is negative reflects the leak of energy to the fluid.

Finally, the last type of perturbation is related, not to an alteration in boundary conditions, but to a change in the constitution of the substrate (e.g., viscoelasticity). In that case we should expect a non-inertial motion of the Rayleigh quasi-particle with a perturbing force in the right-hand side of the motion equation and resulting in a slow down of the quasi-particle. .

5 Perspectives

The studied cases of the standard Rayleigh SAW and the Bleustein-Gulyaev SAW nicely complement each other while being the simplest ones to deal with. Further developments include considering classical Stoneley waves (at the interface of two elastic crystals of appropriate symmetries) and also dispersive Love and Lamb waves, all cases that require, like the second case of Bleustein-Gulyaev waves above, the consideration of a vertical band of integration of the wave-momentum equation including one interface.

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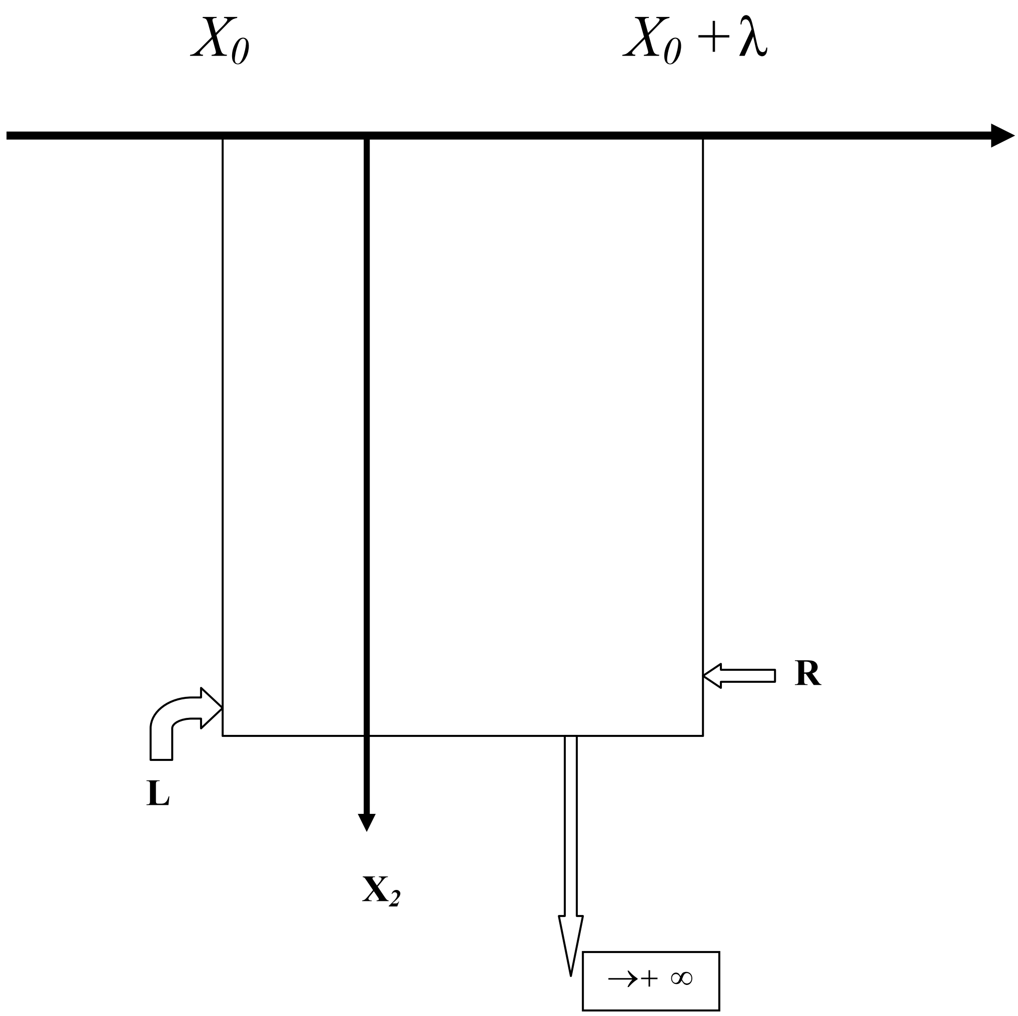

Figure 1: Integration of the equation of wave momentum in the sagittal plane for Rayleigh SAWs.