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Direct quasistatic measurement of acoustical porous material Poisson ratio

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This paper proposes a quasistatic method for Poisson's ratio estimation of isotropic acoustical porous materials. The method is based on longitudinal and transverse displacements measurements of a cylindrical sample of the material under study, this sample being submitted to an harmonic and uni-axial compression. At the same time, damping and stiffness of the material can be measured, as the sample reaction force and vertical displacement are also measured. For a given shape factor defined as the radius over twice the thickness of the sample, a polynomial relation between displacements ratio and Poisson's ratio are used. The Poisson ratio is the solution for which the difference between the polynomial and the measured value is null. The method is validated numerically and applied to the Poisson's ratio estimation of a melamine foam which is close to be isotropic.

1 Introduction

Most of poro-viscoelastic materials can be well modeled with Biot-Allard equations. This model requires the assessment of geometrical parameters such as porosity, flow resistivity, tortuosity and characteristic lengths together with viscoelastic parameters associated to the material skeleton, in order to describe the macroscopic geometry of materials. Several characterization methods of porous materials viscoelastic parameters exist (a review of those methods is done on [2]) but Poisson's ratio is the most tricky parameter to get ([7], [6]). Sim et al ([10]) show how to get complex Poisson ratio in correlating resonant measurements to finite element simulations. The methodology, described in [3], is about the same as the Sim's one, but uses quasistatic tests. In both cases, Poisson ratio is found by doing measurement on two samples having different shape factors. Here, the purpose is to show how to get the Poisson ratio directly, in measuring at the same time longitudinal and lateral displacement of a cylindrical sample. In fact, in the articles [4] and [9], it is proved that the ratio of those displacements only depends on Poisson's ratio, for a given shape factor. Considering a cylindrical sample under quasistatic test, a new methodology to assess this parameter is described in this article. In the second section, the different steps of the methodology are detailed. The measurement set-up is described and the abacus construction from polynomial relations is discussed. It is finally explained how to derive Poisson 's ratio from measurements and the abacus. In the third part, the methodology is applied to a melamine foam. The conclusion summarize the main points of this work.

2 Methodology

2.1 Displacements and mechanical impedance measurements

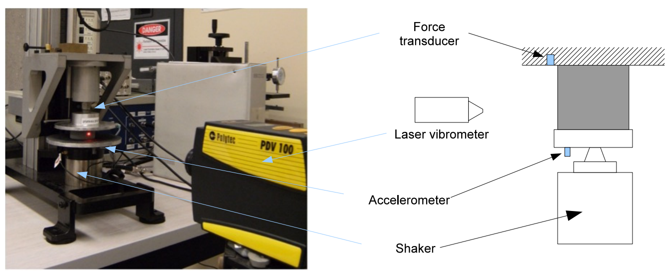

Figure 1: Picture (left) and scheme (right) of the experimental set-up

The experimental set-up is the one described in [9]. A cylindrical sample is slightly prestressed between two rigid plates (as shown in picture 1). The sample is excited via the lower plate. The goal is to measure the reaction force of the upper plate and the induced lateral displacement of the sample. The frequency range under study should be below the material first compression resonance frequency, say from 20 Hz to 90 Hz. In fact, for upper frequencies the fluid effect on the skeleton is not negligible anymore and the the coherence is bad because of the dynamic behavior of the system. For lower frequencies, the coherence is also bad because of non linearity of the system.

A force transducer is used to measure the reaction force of the upper plate. An accelerometer measures the acceleration of the driven lower plate (the related displacement is written $\tilde{W}_L$), and finally, a laser vibrometer measures the lateral velocity of the sample at mid thickness (the related displacement is written $\tilde{W}_V$).

The lower plate is vertically excited by means of an electrodynamic shaker. Sand paper is put on the lower plate surface to prevent the sample from sliding laterally. The upper rigid plate is supposed to fixed (its displacement is null).

Some reflected product is put in the material surface (on a small area) to allow the laser beam to be reflected.

The frequency response function (FRF), also named displacement ratio, is defined as:

$$\tilde{T}(\omega) = \frac{\tilde{W}_V(\omega)}{\tilde{W}_A(\omega)} \tag{1}$$

$\tilde{T}(\omega)$ is the ratio of the integrated vibrometer signal and the twice integrated accelerometer signal.

Moreover, the dynamic complex stiffness writes:

$$\tilde{K}_m(\omega) = \frac{\tilde{F}(\omega)}{\tilde{W}_A(\omega)} = K_m(\omega)(1 + j\eta) \tag{2}$$

Where $K_m(\omega)$ is the dynamic stiffness and η is the loss factor of the material. It is well known that elastic properties may depend on the prestrain rate (see [1]). As it can be seen from table 1 showing the mean measured stiffness on the studied frequency range, a 5% strain induces a stiffness variation of 12%, and the variation is of 41% for a 10% strain. In order to be in the linear range of the material behavior, the imposed prestress should not induce more than 5% strain, however, the prestress should still be enough for the signal coherence to be close to unity.

Strain %	Mean measured stiffness (N/m)
1	5265
5	5885
10	7433

Table 1: Stiffness as a function of prestrain rate, for material M1

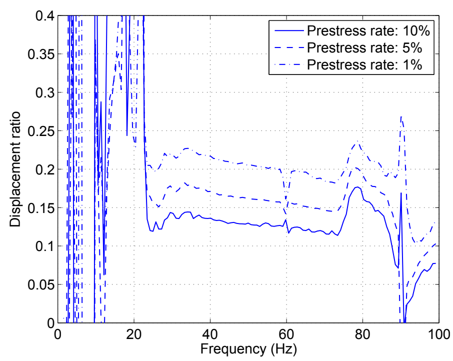

Figure 2: Amplitude of the FRF $\tilde{T}$ as a function of frequency and for several prestrain rates, tests done on M1 sample

The plot on figure 2 shows an example of measured FRF for several compression rate of a melamine foam sample (which will be called M1). The strain rate is measured with a calliper rule. The strain rate actually influences the ratio $\tilde{T}$ in the studied frequency range since the curves are not superposed. One also observes the non-linearities of the system for frequencies lower than about 20 Hz. In the table 2 reporting the mean value of the ratio (calculated in the studied frequency range), one sees that the latter is increasing of about 20% when the strain is increasing of 4%. The sample should then not be prestressed with a prestrain rate higher than 5%, otherwise, Poisson's ratio could be underestimated. In fact, the imposed prestrain implies an initial lateral displacement. The measured lateral displacement would then be diminished, which leads to a decrease of $\tilde{T}$ and a lower Poisson's ratio.

Poisson's ratio isotropy of each material is finally verified in XY plane (see scheme 3 in next section for axis definition). For this purpose, the displacement is measured in different points of the cylinder radius, Z coordinate being fixed at mid-thickness. The observed variation along the radius is not relevant.

Strain %	Mean measured T
1	0.195
5	0.154
10	0.122

Table 2: Mean value of the ratio $\tilde{T}$ amplitude as a function of prestrain rate, for material M1

One should finally keep in mind to apply to the sample a prestress rate inducing a strain lower than 5% of the initial thickness. If the later condition is fulfilled, the thickness variation is negligible: it is not necessary to take this variation into account.

2.2 Polynomial relations and abacus construction

This section presents how the abacus (that will be used for Poisson ratio calculation) is constructed from polynomials. The frequency response function $\tilde{T}$, defined in equation 1, can also be written as:

$$\tilde{T} = Q_s(\nu) \tag{3}$$

where Q_s is a polynomial depending only on Poisson's ratio for a given shape factor s. s is defined as the diameter over twice the thickness.

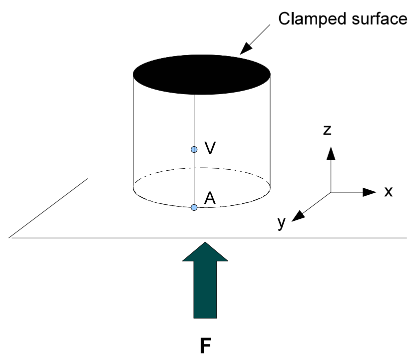

Figure 3: Geometry and boundary conditions of the simulated cylinder

Cylinders of different shape factors are simulated with FEMAP/NX-Nastran[1]. The chosen shape factors are: 0.1, 0.2, 0.4, 0.6, 0.75, 1 . The scheme in figure 3 illustrates the used cylinder geometry. The used mesh type is hexagonal. There are 10 nodes per centimeters in order to be sure that the mesh is converging.

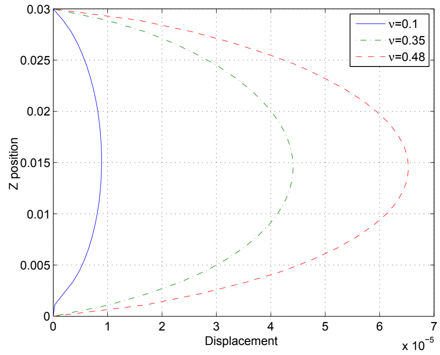

Figure 4: Radial displacement as a function of the point position along a vertical edge

For a shape factor of 1 and three Poisson ratios (0.10, 0.35, 0.48), the curves giving the evolution of the radial displacement as a function of the node position along a vertical edge, is drown on figure 4. The displacement are calculated for an imposed force F equal to 10 N. Each degree of freedom is fixed where the clamped condition has to be fulfilled (all displacements are null). On the surface where the force is imposed, X and Y displacements is set to 0 in order to prevent sliding in those directions. The calculus is done for a static load. The ratio of lateral displacement at mid-thickness of the cylinder (point V on the scheme) and imposed vertical displacement (point A on the scheme) is computed for the different cylinders and the following Poisson's ratios: 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.48 .

The obtained values are then interpolated with the Polyfit function of matlab. Polynomials are of degree 3 in terms of Poisson's ratio variable. One obtains 6 polynomials in terms of Poisson ratio for each shape factor. Figure 5 shows the evolution of the displacement ratio T as a function of Poisson's ratio, for different shape factors. The curves tend towards 0 for null Poisson ratio, which is logical since in this case the transverse displacement is null.

2.3 Poisson's ratio estimation

Knowing sample shape factor s_m as well as the measured value T_m of the displacements ratio, Poisson's ratio corresponding to the couple (s_m,T_m) has to be found. The polynomial Q_{s_m} is calculated in interpolating the polynomial Q for the shape factor s_m and each Poisson ratio. The equation to be solved for each measured frequency is then:

$$\tilde{T}_m(\omega) - Q_{s_m}(\nu) = 0 \qquad (4)$$

The real solution being in the range 0-0.49 corresponds to Poisson's ratio. The intermediate value theorem ensures the solution uniqueness for a given shape factor. According to Bolzano theorem, the following

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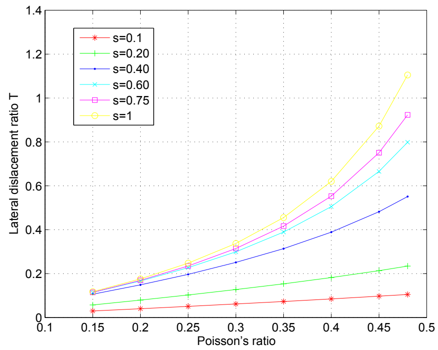

Figure 5: Abacus giving the simulated FRF evolution as a function of Poisson ratio, for different shape factors

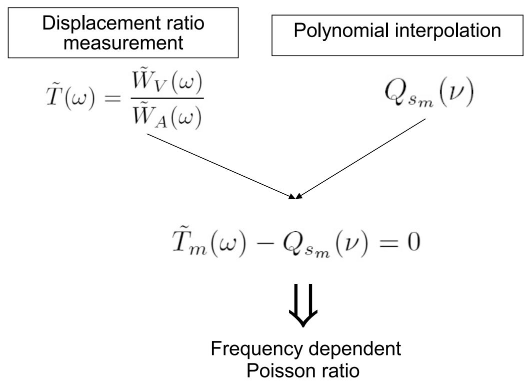

Figure 6: Poisson ratio obtention methodology

condition should hold for the solution to exist:

$$(\tilde{T}_m - Q_{s_m}(0))(\tilde{T}_m - Q_{s_m}(0.49)) < 0 \qquad (5)$$

If equation 5 is not verified, there is no physical value for the Poisson ratio and the measurement has to be repeated. The curves for lower shape factors are flatter which means that this condition is unlikely to be satisfied. One recommendation is that the samples shape factor should be higher than 0.4.

3 Application

In this part, the principle is applied to a polymeric cellular foam. The displacements are measured following the set-up described in section 2.1. The frequency range of interest is [20-80] Hz. The density of the tested melamine foam is 9 kg/m^3.

Figure 7 shows the frequency evolution of Poisson ratio as obtained from the proposed methodology. One observes that the Poisson ratio is almost constant with frequency. Note that measurement on several sample have to be done for each material in order to get rid of the non-homogeneity of the materials.

Table 3 presents Young's modulus, Poisson's ratio and loss factor mean values averaged over the frequency range of interest for each sample. The averages over the 3 samples are also done in order to get material properties. Young's modulus and displacement ratio are directly computed with the signal analyzer.

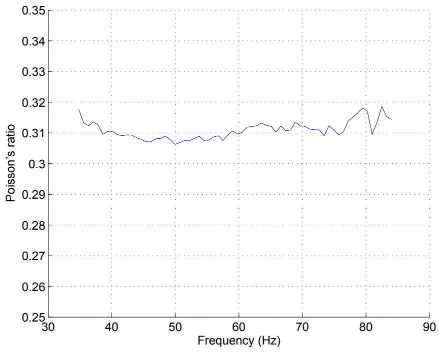

Figure 7: Poisson's ratio as a function of frequency for a sample of melamine foam

Sample	E (N.m^{-2})	η	ν
I	148380	0.08	0.36
II	128800	0.08	0.30
III	120000	0.07	0.32
Mean value	132400	0.08	0.33
Std	14500	0.01	0.03

Table 3: Estimated parameters for the tested melamine foam. E: Young's modulus, η: structural damping, ν: Poisson's ratio

Material M1	E	η	ν
Direct quasistatic method	132 400	0.08	0.33
Resonant method	169 400	0.08	0.47
Relative difference (%)	28	0	42

Table 4: Values of the elastic parameters for material M1 obtained from quasistatic and resonant methods. Relative difference between parameters for these two methods are also reported.

The results obtained from the proposed method are compared in table 4 with those obtained from the resonant method originally described by Pritz ([5]). The Poisson's ratio, calculated as described in [3], appears to be much lower with the proposed method than with the resonant method. The difference observed between the two methods for the estimated values of Young's modulus can partly explain the difference between the estimated values of Poisson's ratio. Indeed, the uncertainty in the estimation of the Young's modulus affects the estimated value of the Poisson's ratio.

To sum up, here are the different steps to be followed in order to get acoustical material Poisson's ratio:

- Simultaneous measurement of vertical and lateral displacements for different samples of material (and reaction force for Young's modulus and structural damping estimation)
- Poisson's ratio calculation for each sample, for each frequency of the studied frequency range using the proposed abacus
- Calculation of Poisson's ratio mean value, for each sample
- Calculation of the mean value of each parameter for the material

A material with low Poisson's ratio has been tested without any success. In this case, the lateral displacement is weak and it is difficult to measure it properly. The values of the ratio T could become very high and there would not be physical solution to the equation 4. The direct measurement of Poisson ratio is easy but requires precautions, particularly with the sample cutting should be done very carefully. Otherwise, erroneous values could be found.

4 Conclusion

A fast methodology as been described in order to get Poisson's ratio of porous materials, on a frequency range between 20 to 80 Hz. The theoretical and experimental part of the methodology have been described. The feasibility of the methodology has been proved and applied to an acoustical porous materials. The results seems coherent with those found in using other approaches. The difficulty in Poisson's ratio measurement has been pointed out. The found quasistatic parameters can be used as initial values for frequency dependent Young's modulus and structural damping determination via optimization (cf [8]). The advantage of the method is to be quick. The drawbacks is that it is sensitive to sample cutting and that this measurement is then tricky. The boundaries conditions have also to be well controlled. Further investigations have to be done in order that the numerical boundary conditions are as close as possible to the experimental one.

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