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A mixed analytical-numerical model for the S-matrix computation of bidimensional lined ducts with HQ tubes

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The Herschel-Quincke (HQ) tubes, consisting in putting tubes in derivation along a main acoustic wave guide, are used as passive devices to control fan noise. In order to assess the efficiency of this system, a new mixed analytical-numerical model is presented. The technique relies on combining Finite Element techniques to accurately describe the HQ tube with an integral representation for the acoustic pressure in the main duct. The presence of acoustic liners on the walls of the duct is taken into account via an appropriate modal decomposition of the Green's function. We show that our algorithm allows a very fast and accurate computation of the scattering matrix of such a system with a numerical complexity that grows very mildly with the frequency. Results show that 'nearly' optimal configurations can be quickly identified with a very small computational expense.

1 Introduction

One of the most significant sources of noise of an aircraft is due to the propelling system. This noise, which is present during all phases flight around airport, can be decomposed into several types : classical jet noise outside the exhaust nozzle and inner turbo machinery noise (fan, compressor, turbine & combustion). In particular, fan noise is responsible for pure tones at the Blade Passage Frequency (BPF) harmonics, due to the interaction between the rotor wakes and the stator vanes. In order to reduce noise level in modern turbofan engines, sound waves generated by the fan are typically absorbed by acoustic lining covering the duct engine. Though efficient, these passive liners seem to have reach their limit and there is still a need for considering other passive techniques to reduce further the sound radiation from the duct outlet. In this context Herschel-Quincke tubes concept could prove to be a reliable option.

In 1833, Herschel [8] first discussed the idea of using acoustic interferences of tones by simply connecting a tube to the main duct in view of reducing the transmitted acoustic waves. Thirty three years later, Quincke [14] experimentally validated Herschel's theory and many works and experiments have been carried out to explain physical phenomena and explore the potentiality of this system as a noise control device [3, 18].

The assessment of the efficiency of such a system requires a precise knowledge of the acoustic field in the duct. Though standard Finite Element (FE) software could, in principle, be used for this purpose, a full 3D FE model would be extremely demanding as the number of variables is expected to grow like f^3 (f is the frequency). This can have a negative impact when, for instance, some efficient optimizations (geometry of the HQ tubes and their positions) are needed.

Assuming plane wave propagation, the resonance behavior of two duct combination was first established analytically by Selamet *et al.* [16] and then extended to a multiple duct configuration [17]. The proposed approach is simple to implement and allows a very fast computation of the transmitted wave but it is unfortunately limited to low frequency applications. To make some progress, Brady [2] proposed a two-dimensional model including multi-modal analysis in the main duct using a Green's function formalism. The three-dimensional model was then extended by Hallez [6]. Finally, Poirier [12, 13] proposed an improvement by taking into account the exact shape of the interface between the main cylindrical duct and the HQ tube. All the authors just cited simplified their analysis by assuming that the acoustic velocity is *constant* over the duct-tube interface. Furthermore they all modelized the HQ tubes as if they were straight waveguides in which only plane waves are allowed to propagate.

Because these assumptions are known to break down as the frequency increases (see for instance Tang & Lam [19]), Maréchal *et al.* [10] proposed an enhanced model by taking into account (i) the exact shape of the HQ tube(s) and (ii) the non uniformity of the acoustic velocity on the interfaces. Authors show that these improvements can be made with a relatively small additional computational cost while leading to very accurate results even in the mid-frequency regime. In the present paper, we shall show that the technique can be also extended and applied for ducts with acoustic liners on its walls. This is particularly important as early experiments combining typical acoustic liners and HQ tubes already showed promising results for reducing the transmitted acoustic power [4, 1].

2 Problem statement

The problem under consideration is illustrated in Fig. 1. It consists of a two-dimensional lined main duct (domain Ω) of height h which is connected to a single HQ tube. The inlet and outlet pipes (regions I and II) are identical, each having rigid walls at its boundaries. We wish to evaluate the scattering matrix (or S-matrix) of this acoustic system, that is given incident pressure waves P_{I}^{+} and P_{II}^{-}, we compute the scattered waves P_{I}^{-} and P_{II}^{+}. We call Γ_1 (resp. Γ_2) the lined wall of the main duct with impedance Z_1 (resp. Z_2). In the main duct as

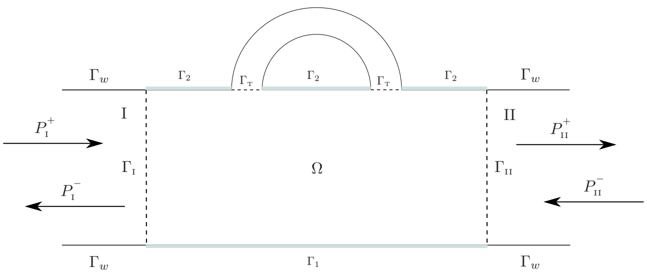

FIGURE 1: Main duct with one Herschel-Quincke tube

well as in the HQ tube, the acoustic pressure p satisfies the Helmholtz equation

$$\Delta p + k^2 p = 0. \tag{1}$$

All boundaries under consideration are rigid (i.e. $\partial_n p = \mathrm{i}\omega\rho\mathbf{v}\cdot\mathbf{n} = 0$ where $\mathbf{n}$ is the outward unit normal) except on the lined walls where the local impedance condition is prescribed:

$$Z_i = \frac{1}{Y_i} = \frac{p}{\mathbf{v}\cdot\mathbf{n}}, \quad i = 1, 2. \tag{2}$$

Here, we adopt the $\mathrm{e}^{-\mathrm{i}\omega t}$ -convention, $k = \omega/c$ is the wave number, c the celerity, ω the angular frequency, ρ the fluid density and $\mathbf{v}$ denotes the acoustic velocity. Finally, we require that p and its normal derivative (i.e. the normal velocity) to be continuous across the interface Γ_{T}. The transmission conditions at the artificial boundaries Γ_{I} and Γ_{II} are given from the pressure wave field in the inlet and outlet pipes. This is expressed as the usual modal series

$$P_j^{\pm} = \sum_{m=0}^{\infty} A_{j,m}^{\pm} \psi_m(x) \mathrm{e}^{\pm \mathrm{i}\beta_m z} \tag{3}$$

where j =I or II. Here the pair (ψ_m, β_m) defines the classical propagative (or evanescent) mode in the rigid pipe. These modes are conveniently normalized so that the orthogonality property holds

$$\int_0^h \psi_m(x)\psi_{m'}(x)\,\mathrm{d}x = \delta_{mm'}. \tag{4}$$

3 Mixed numerical-analytical model

In this section, we shall present the main ingredients of the method, that is (i) the establishment of a numerical impedance matrix describing the dependence of the pressure and its normal derivative on both interfaces of the HQ tube and (ii) the Green's formalism in the main duct.

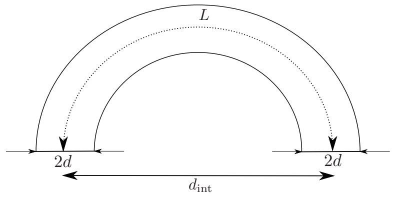

FIGURE 2: Parameters of a HQ tube

Impedance matrices considered in Ref. [2, 6, 16, 17] are built with the restriction that only plane waves are allowed to propagate in the HQ tube. HQ tube parameters are described in Fig. 2 : d_{int} is the distance between interfaces, d the tube half-width and L the average HQ tube length. Note that the tube is not necessarily of circular shape as it is suggested in the figure. Under the plane wave assumption, the 2×2 impedance matrix has the explicit form

$$\mathbf{Z}(\omega) = \frac{1}{k\sin(kL)} \begin{bmatrix} \cos(kL) & 1 \\ 1 & \cos(kL) \end{bmatrix}, \tag{5}$$

where it is understood that the pressure and its normal derivative at the interface are connected via the impedance condition

$$\mathbf{p}_{\mathrm{int}} = \mathbf{Z}(\omega)\,\partial_n \mathbf{p}_{\mathrm{int}}. \tag{6}$$

The vector $\mathbf{p}_{\mathrm{int}}$ (resp. $\partial_n\mathbf{p}_{\mathrm{int}}$) simply contains the value of the constant pressure (resp. normal derivative pressure) on both interfaces. In order to take into account the acoustic particle velocity variation at the interfaces, numerical impedance matrices can be built via finite element discretization of the acoustic pressure field in the HQ tube. The first step is to compute a set of eigenmodes $(\mathbf{\Phi}_n, \omega_n)$ of the HQ tube with rigid wall conditions on the boundary. We define $\mathbf{D}(\omega)$ the diagonal matrix with its diagonal entries: $(\omega_n^2 - \omega^2)^{-1}$. Once the number N_m of modes has been chosen, a numerical impedance matrix can be recovered as follows

$$\mathbf{Z}(\omega) = \left(\tilde{\mathbf{\Phi}}\,\mathbf{D}(\omega)\,\tilde{\mathbf{\Phi}}^{\mathrm{T}} + \mathbf{R}(\omega)\right)\tilde{\mathbf{F}}_{\mathrm{int}}. \tag{7}$$

Here, $\tilde{\mathbf{\Phi}} = (\tilde{\mathbf{\Phi}}_1 \cdots \tilde{\mathbf{\Phi}}_{N_m})$ stands for the matrix containing the eigenmodes in its columns and the tilde symbol means that we only retain the nodal values of these eigenmodes on the interface. $\tilde{\mathbf{F}}_{\mathrm{int}}$ define the classical finite element discretization matrix at the interfaces. Similarly, the vector $\mathbf{p}_{\mathrm{int}}$ (resp. $\partial_n\mathbf{p}_{\mathrm{int}}$) in (6) now contains the value of the pressure (resp. the normal derivative) at each node of the FEM mesh on the interface. The interest of such a decomposition is that when the frequency of interest is taken well below the highest modal resonant frequency (i.e. $\omega \ll \omega_{N_m}$), the correction term $\mathbf{R}(\omega)$ is quasi-constant so we can take $\mathbf{R}(\omega) \approx \mathbf{R}(0)$ and store the so-called static correction matrix once for all. Thus, the computation of (7) is a very fast and simple procedure.

In the main duct, the theory starts by introducing the lined-walled duct Green's function satisfying the usual modal radiation condition on both ends of the main duct, i.e.

$$G(\mathbf{x},\mathbf{x}_0)=\sum_{n=0}^{\infty}\frac{\tilde{\psi}_n(x)\tilde{\psi}_n(x_0)}{-2\mathrm{i}\tilde{\beta}_n}\,\mathrm{e}^{\mathrm{i}\tilde{\beta}_n|z-z_0|} \qquad (8)$$

where $\mathbf{x}=(x,z)$ and $\mathbf{x}_0=(x_0,z_0)$ are two points in the propagative domain Ω. Function $\tilde{\psi}_n$ is the transverse mode satisfying the lined-wall conditions and $\tilde{\beta}_n$ is the associated axial wave number. Low order modes are numerically calculated with a technique described by Kravanja [9] and validated by Nennig *et al.* [11] whereas higher order modes are recovered using asymptotic expansions.

Using the Green's theorem, the pressure anywhere in the lined duct (Ω domain) is given via the integral representation

$$\begin{aligned} p(\mathbf{x}) &= \int_{\Gamma_{\mathrm{I}}\cup\Gamma_{\mathrm{II}}}(G\,\partial_n p - p\,\partial_n G)\ \mathrm{d}\Gamma(\mathbf{x}_0) \\ &+ \int_{\Gamma_{\mathrm{T}}} G\,(\partial_n p - \mathrm{i}\rho\omega Y_2 p)\ \mathrm{d}\Gamma(\mathbf{x}_0). \end{aligned} \qquad (9)$$

The discretization of this equation is carried out in two steps. First, collocating (9) at the FEM nodes of the HQ tube interface leads to

$$\begin{aligned} \mathbf{K}_{\mathrm{T,T}}\,\partial_n\mathbf{p}_{\mathrm{int}} &+ \mathbf{K}_{\mathrm{T,I}}\,\mathbf{A}^{-}_{\mathrm{I}} + \mathbf{K}_{\mathrm{T,II}}\,\mathbf{A}^{+}_{\mathrm{II}} \\ &= \mathbf{F}_{\mathrm{T,I}}\,\mathbf{A}^{+}_{\mathrm{I}} + \mathbf{F}_{\mathrm{T,II}}\,\mathbf{A}^{-}_{\mathrm{II}} \end{aligned} \qquad (10)$$

where vectors $\mathbf{A}^{\pm}_{j}$ contain the modes amplitudes $A^{\pm}_{j,m}$, (j=I,II). The first block matrix

$$\mathbf{K}_{\mathrm{T,T}}=\mathbf{Z}(\omega)-\mathbf{G}+\mathrm{i}\rho\omega Y_2\mathbf{G}\mathbf{Z}(\omega) \qquad (11)$$

stems from the self interaction of the acoustic pressure at the tube interface. Here the Green matrix $\mathbf{G}$ stems from the discretization of the second integral in (9). Other matrices are built by simply substituting,

$$p=P^{+}_{j}+P^{-}_{j} \quad \text{and} \quad \partial_n p=\partial_n(P^{+}_{j}+P^{-}_{j}) \qquad (12)$$

with (j=I,II) in the first integral of (9). This operation requires the computation of the coupling coefficients C_{mn} given by the overlap integrals

$$C_{mn}=\int_0^h \psi_m(x)\tilde{\psi}_n(x)\ \mathrm{d}x. \qquad (13)$$

which also arise in standard mode matching techniques. The system (10) is completed by taking the evaluation point $\mathbf{x}$ in the integral equation on the inlet and outlet boundaries. An additional set of equations is then produced by projecting (9) onto the hard-wall modes basis to give

$$\begin{aligned} \mathbf{K}_{j,\mathrm{T}}\,\partial_n\mathbf{p}_{\mathrm{int}} &+ \mathbf{K}_{j,\mathrm{I}}\,\mathbf{A}^{-}_{\mathrm{I}} + \mathbf{K}_{j,\mathrm{II}}\,\mathbf{A}^{+}_{\mathrm{II}} \\ &= \mathbf{F}_{j,\mathrm{I}}\,\mathbf{A}^{+}_{\mathrm{I}} + \mathbf{F}_{j,\mathrm{II}}\,\mathbf{A}^{-}_{\mathrm{II}} \end{aligned} \qquad (14)$$

for both boundaries j=I and II. Now, by calling

$$\mathbf{K}=\begin{bmatrix} \mathbf{K}_{\mathrm{T,T}} & \mathbf{K}_{\mathrm{T,I}} & \mathbf{K}_{\mathrm{T,II}} \\ \mathbf{K}_{\mathrm{I,T}} & \mathbf{K}_{\mathrm{I,I}} & \mathbf{K}_{\mathrm{I,II}} \\ \mathbf{K}_{\mathrm{II,T}} & \mathbf{K}_{\mathrm{II,I}} & \mathbf{K}_{\mathrm{II,II}} \end{bmatrix},\ \mathbf{F}=\begin{bmatrix} \mathbf{F}_{\mathrm{T,I}} & \mathbf{F}_{\mathrm{T,II}} \\ \mathbf{F}_{\mathrm{I,I}} & \mathbf{F}_{\mathrm{I,II}} \\ \mathbf{F}_{\mathrm{II,I}} & \mathbf{F}_{\mathrm{II,II}} \end{bmatrix},$$

HQ tube	d (m)	d_{int} (m)	L (m)
1	0.02337	0.0985	0.0593
2	0.02337	0.0985	0.0393
3	0.00837	0.0985	0.0793
4	0.00337	0.0485	0.0393

Table 1: Parameters of the HQ tube.

the global matrix system can be solved to give

$$\begin{pmatrix} \partial_n\mathbf{p}_{\mathrm{int}} \\ \mathbf{A}^{-}_{\mathrm{I}} \\ \mathbf{A}^{+}_{\mathrm{II}} \end{pmatrix} = \mathbf{K}^{-1}\mathbf{F}\begin{pmatrix} \mathbf{A}^{+}_{\mathrm{I}} \\ \mathbf{A}^{-}_{\mathrm{II}} \end{pmatrix}. \qquad (15)$$

In practice, the summation in (3) is limited to the number of propagative modes as well as some evanescent modes which are included to ensure a precise approximation of the pressure field in the inlet and outlet boundaries. Thus, the square matrix $\mathbf{K}$ is of a relatively small size in the present study.

4 Results

This section presents some numerical results computed with our numerical model. The width of the main duct is $h=2a=0.04859$ m and the length of the liner is 0.6 m. The results are presented with respect to the dimensionless variable ka. The study is carried out from very low frequency up to $ka=22.3$ (which corresponds to 5000 Hz) with a stepsize of 10 Hz. In the overall frequency range, the incident pressure is a plane wave. The first cut-off frequency occurs at $ka=\pi/2$ which corresponds to $f\approx 3523$ Hz. For the lined wall, the impedance value are chosen as to be in line with perforate plates encountered in the aeronautic industry. Thus, we take $Z_1=2+2\mathrm{i}$ and $Z_2=1+1\mathrm{i}$. The expected frequency dependence of the impedance is not taken into account here but this can be easily included in the analysis. The Transmission Loss (TL) is defined as the ratio of transmitted acoustic power with respect to the incident one, that is

$$\mathrm{TL}=-10\log_{10}\left(\frac{1}{\beta_0\left|A^{+}_{\mathrm{I},0}\right|^2}\sum_{m\geq 0}\beta_m\left|A^{+}_{\mathrm{II},m}\right|^2\right), \qquad (16)$$

where the summation is limited to propagative modes only.

In the present study, four different HQ tubes which parameters are given in Table 1 are investigated. In order to validate the method, a full finite element model is used. Radiation conditions at both ends of the duct have been implemented using the DtN map [5, 7, 15]. In Fig. 3 are plotted the TL calculated from our mixed model and the FE model (the HQ tube 1 is considered here). The very good agreement validates the present method and the small discrepancies noticeable at high frequency are thought to be due to the FE model which starts loosing accuracy. Note that it takes about 40 minutes to produce these results when using the FE model whereas only 9 minutes are needed with the mixed model. More importantly, the number of FE nodes (around 25000) is very large compared to number of variables used in our model (i.e. the number of nodes at the interface + the number of modes in the summation in (3)) which does not exceed 30. At higher frequency, this number is expected to grow very mildly with the frequency whereas the FE model would quickly become intractable because of the computational overhead.

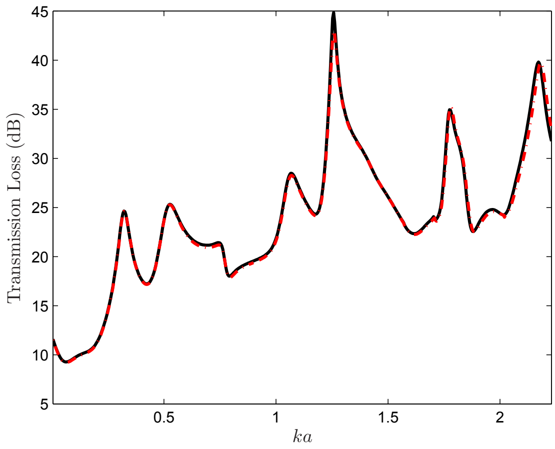

FIGURE 3: HQ tube in a lined duct : —— mixed model, -·- full finite element model.

Finally, we shall demonstrate that our mixed model can be used efficiently in order to find a 'nearly' optimal configuration with a very small computational expense. Given a set of HQ parameters (Table 1), the TL curves have been computed up to 5000 Hz for 4 scenarios: (i) no HQ tube, (ii) tube 2, (iii) tube 3 and (iv) tube 4. Here the position of the tube is fixed to a constant value for all configurations. The four curves are plotted in Fig. 4. It is clear from these results that a 'nearly' optimal configuration is given by the tube 4. In the present example, by 'nearly' optimal we mean that the inclusion of the HQ tube in the liner should provide better Transmission Losses than if only the liner was present in the frequency range of interest. Here again, we have to point out that the overall cost for computing these results is negligible. It is clear that the use of the FE method would be more computer time consuming especially if time allocated for the mesh preparation is taken into account. Depending on the problem, our algorithm could be enhanced even further. For instance, the four block matrices $\mathbf{K}_{j,j'}$ $(j, j'$=I,II) do not dependent on the position and the geometry of the HQ tube, thus these can be computed only once for all. The inclusion of another HQ tube next to the first one can be done at very small expense since the additional block matrices involving the tube (indice T) are, up to some phase shifts, similar to the original one. It goes without saying that all these improvements are being considered in the near future.

5 Conclusions

To conclude, it has been shown, from these early results, that the mixed model allows to identify the combined effects of the acoustic liners as well the presence of HQ tubes giving access to a precise knowledge of the transmitted sound field. Compared to standard FEM, the proposed model allows very fast computation of the Transmission Loss with a computational complexity that increases very mildly with the frequency. The algorithm has been developed in the bidimensional case in order to assess its potentiality but work is ongoing to extend the model to 3D configurations so that real geometries could be tackled.

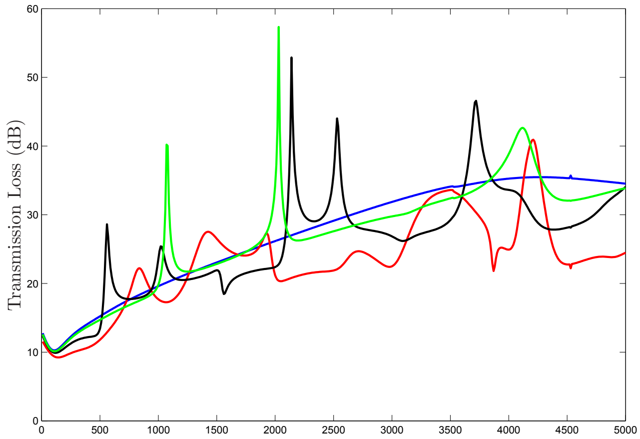

FIGURE 4: TL curves for an incident plane wave: —— liners without HQ tube, —— HQ tube 2, —— HQ tube 3 and —— HQ tube 4.

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