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A reduced global semi-active vibroacoustic control strategy, statement and preliminary validations:

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In this talk, a control strategy is presented and numerically tested. This strategy aims to achieve the potential performances of fully active systems with a reduced energy supply. These energy needs are expected to be comparable to the power demand of semi-active systems, while system performance is intended to be comparable to that of fully active configuration. The underlying strategy is called " global semi-active control". This control approach results from an energy investigation based on the management of the optimal control process. Energy management encompasses storage and convenient restitution. The proposed strategy monitors a given active law without any external energy supply by considering purely dissipative and energy-demanding phases. Such a control law is offered here along with an analysis of its properties. Moreover, a numerical experiment of a cantilever beam subjected to external perturbations is proposed to validate findings.

1 Introduction

Traditionally, there were two categories of vibration control based on the power flow in dynamic subsystems namely passive and active. Active strategies have good performances but necessitate an external power supply to apply control in opposite of the passive ones. A third intermediate strategy called regenerative control, has been introduced since the early nineties [1]. A regenerative system is a system that is not passive, yet, on average, more energy flows into it than out of it. It is a promising system since it offers the possibility of being self-sustainable which would reduce the dependence on an external energy source as for active systems.

Researches in regenerative systems is still an intensively addressed subject. Gupta and al. [2] presented two different regenerative electromagnetic shock absorbers (linear and rotatory) that use the dissipated energy resulting from the roughness of the roads to enhance the damping. Recent works introduced a regenerative control approach called "global semi-active control" [3]. Based on the reuse of the energy coming from the system vibrations, the energy management is realized through a switching between two control schemes that are the optimal active and the semi-active ones. It seems to effectively enhance the vibrations control performances. They offered the required algorithms to calculate the energy management term which will decide the phase switching. Numerical results for discrete systems with a very limited number of degrees of freedom like a quarter vehicle suspension or a deck system were given. They demonstrated that vibration attenuation capacities of the proposed strategy approach those of the pure active one and exceed those of the semi-active one. The stored energy seems to increase which lets hope to significant reduction in the energy consumption for real systems.

In this paper, the vibration control is specifically applied to continuous flexible structures. Indeed, emerging from their potential modes interferences, the control solution might be unstable. It is here necessary to design a multi-mode controller that can effectively suppress vibrations at and near specific natural frequencies of interest, but does not introduce unwanted vibrations at other natural frequencies (i.e., spillover) [4]. An other specific issue to be addressed is a consequence of these system's high order. The higher the order is, the larger is the amount of real time calculations is. This drawback can limit the controller performances or even prevent it from working properly.

2 Modal space representation

Flexible structures are inherently distributed parameter structures with infinite degrees of freedom which request a high computational effort if using the full system model. That is why, modal reduction methods are preferred. Yet, a feedback controller based on a finite reduced modal model may destabilize the residual modes. This part of unmodeled dynamics may lead to spillover problems on real applications [5] [6]. Since an excited structure has preferable modes of vibration which depend on the spectral content of the excitation, the lower order modes are assumed to be the most significant to the system global response. This way, the full order model can be reduced to those modes with a faithful restitution of the dynamic behavior.

In the following example, a cantilever beam subjected to external forces is considered for vibration control study. The full model equations of motion are derived according to the following Equation with $x(t)$ being the generalized nodal displacement vector.

$$M.\ddot{x}(t) + C.\dot{x}(t) + K.x(t) = Lu(t) \tag{1}$$

where: M, C and K are the structural mass, damping and stiffness matrices of the beam respectively, $\dot{x}(t)$ and $\ddot{x}(t)$ are the velocity and acceleration vectors respectively, L is the location matrix of the control force $u(t)$. The eigenvalue problem is then solved by using the modal transformation matrix Ψ and the modal reduced displacement vector $q = [q_1\, q_2\, \ldots q_i]^t, (i = 1 \ldots n)$ such that $x = \Psi.q$. The equations of motion relatives to the the reduced n eigenmodes, are now uncoupled and can be written as:

$$\ddot{q} + diag(2\xi_i w_i)\dot{q} + diag(w_i^2)q = f(t) \tag{2}$$

where w_i and ξ_i are the natural eigenfrequency and the damping ratio of the i^{th} mode respectively, and $diag(2\xi_i w_i) = (\Psi^T M\Psi)^{-1}.\Psi^T C\Psi$, $diag(w_i^2) = (\Psi^T M\Psi)^{-1}.\Psi^T K\Psi$, and $f(t) = (\Psi^T M\Psi)^{-1}.\Psi^T Lu(t) = L^+u(t)$. The modal control force $f(t) = [f_1\, f_2 \ldots f_i]^t$ is related to the physical control vector $u(t)$. Consequently, each control force f_i corresponding to mode i depends on all the modal coordinates which leads to the problem of recoupling our decoupled equations. Methods avoiding this recoupling issue are presented in section 3 . The state space approach is the basis of the current control theories and is strongly recommended in the design and analysis of control systems with a great amount of inputs and outputs [7]. Let $X(t)$ be the state vector such that $X(t) = [q(t)\, \dot{q}(t)]^t$, Equation 1 can be written in the form of a linear, first-order state space differential equation:

$$\dot{X}(t) = AX(t) + Bf(t) \tag{3}$$

with:

$$A = \begin{bmatrix} 0 & I \\ -diag(w_i^2) & -diag(2\xi_i w_i), \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix},$$

where A is the modal state matrix, B is the location matrix of the modal control forces, and I is the n-rank identity matrix.

3 Independent Modal Space Control (IMSC)

Independent modal space control method is used to derive the global control force $f(t)$, since it has the advantage of restating the problem as a set of independent modal equations which will permit decoupling equations and thus simplifying the controller design. For that, the global control force $f(t)$ will be composed of N_c chosen modal feedback forces such that ($f(t) = [f_1(t) f_2(t) \dots f_i(t)]^t$, $i = 1 \dots N_c$) and the modal feedback control force $f_i(t)$ depends on the corresponding modal coordinates $q_i(t)$ and $\dot{q}_i(t)$ through the following equation:

$$f_i(t) = -g_{1i} q_i - g_{2i} \dot{q}_i \tag{4}$$

Thus, the global modal control force is written as:

$$\begin{aligned} f(t) &= [diag(g_1) \quad diag(g_2)] X(t) \\ &= g\, X(t) \end{aligned} \tag{5}$$

The global gain matrix g is calculated through an optimal scheme [8] consisting on the minimization of a quadratic performance index J:

$$J = \int_0^\infty (X^T Q X + f^T R f)\, dt \tag{6}$$

where Q is the positive definite or semi-positive definite weightening matrix and R is the positive factor that weights the importance of minimizing the vibration with respect to the control forces. In stead of minimizing the global performance index J, we chose to limit the study to N_c modal cost functions J_i such that $J = [J_1\, J_2 \dots J_i]^t, i = 1 \dots N_c$ [9]. These latter modal cost functions depend on the modal control force f_i (to minimize the control input effort), the mode states q_i (through minimizing the potential energy of the structure $w_i^2 q_i^2$) and $\dot{q}_i$ (through minimizing the kinetic energy $\dot{q}_i{}^2$) and can finally be written as:

$$J_i = \int_0^\infty [(w_i^2 q_i^2 + \dot{q}_i{}^2) + r_i f_i^2]\, dt, \quad i = 1 \dots N_c \tag{7}$$

The closed-form solution g_1 and g_2 of the gain matrix which permit minimizing the performance index J_i can be obtained by the formulation given by [**?**] such that:

$$\begin{aligned} g_{1i} &= -w_i^2 + w_i \sqrt{w_i^2 + \frac{1}{R}}, \\ g_{2i} &= \sqrt{-2w_i^2 + \frac{1}{R} + 2w_i \sqrt{w_i^2 + \frac{1}{R}}} \end{aligned} \tag{8}$$

By substituting the expression of J_i in equation 6, the expression of matrix Q and R can be deduced to $Q = [diag(w_i^2)\, eye(N_c)]$ and $R = [diag(r_i)]$ respectively.

4 Modal global semi-active vibration control law

The control strategy globally manages the system energy which is composed of two types: the energy extracted from vibrations and stored in the accumulators and the dissipated energy used to power the actuators such these later switch between two control types which are the optimal scheme and the semi-active one. Actuators will operate under the purely-active law when the available energy allows them to or under the semi-active one if not which is possible through extracting energy from vibrations and storing it in the accumulators. It is clear then that storage devices are needed in this control scheme (called accumulators) as well as an energy management device responsible of the switching operation between the two different control types. A major advantage can be instantly deduced from the proposed law which is the reduction of the energy consumption required for the control law.

4.1 Constraints on accumulators and actuators

In order to be able to apply the global semi-active control, storage (accumulators) and actuation (actuators) devices are needed. Consequently, two constraints that may affect the control performances have to be introduced. The first limitation is related to the stored energy amount which itself depends on the accumulators capacities meaning that, at each time t, the stored energy amount has to be bounded by two extreme values E_{min} and E_{max} such that:

$$E_{min} \leqslant E(t) \leqslant E_{max}, \quad t \in [t_0, t_f] \tag{9}$$

The second limitation deals with the control force that the actuators are supposed to deliver. In fact, the control is displayed by piezoelectric actuators (collocated piezoelectric patches bounded on the beam) and the control force $f(t)$ is proportional to the feedback control voltage $V(t)$. In this case, $V(t)$ is the physical control force and matrix L includes the electro-mechanical constants of the piezoelectric patches. The voltage limitation of the of the piezoelectric actuators results in the limitation of the produced control force such that:

$$\|f\| \leqslant f_{max}, \quad f_{max} > 0 \tag{10}$$

Moreover, for stability reasons, actuators saturation must be avoided i.e. they must be able to function regardless of the available stored energy level and also extract energy from the system even if one accumulator is already full. A Boolean function $b(t)$ is therefore introduced to define the sequence disconnection between actuators and accumulators, and we have $b(t) = 1$ when actuators and accumulators are connected and optimal control force is delivered and $b(t) = 0$ otherwise i.e. actuators extract energy and store it in the accumulators which make them operating like conventional semi-active actuators. The switching between these two states is decided in function of an energy management term Γ which will be introduced in the following section.

4.2 Algorithm of the control strategy

The minimization of the cost criterion J resulted in the determination of the optimal control force further denoted $f_{gsa}(t)$ (section 3) , the knowledge of the initial conditions as well as the definition of the constraints on the accumulators and actuators (section 4.1) permits setting the global minimization problem denoted below as (P):

$$\begin{array}{lll} \dot{X} = AX + Bf & & \\ E_{min} \leqslant E(t) \leqslant E_{max}, & \forall t \in [t_0, t_f] & \\ \|f\| \leqslant f_{max}, & f_{max} > 0, & \text{when} \quad b(t) = 0 \\ X(t_0) = X_0, & E(t_0) = E_0 & \end{array}$$

In order to be able to apply the global semi-active control to a multi-modal structure, the optimization problem will be addressed for each mode apart (i.e. each mode will be considered as a 1 d.o.f system). The needed power amount for the piezoelectric actuators to deliver the optimal control force denoted $f_{gsa_i}(t)$ corresponding to each mode i (which is supposed to be supplied from the accumulators) is electrical and can be calculated by [10] through the following expression:

$$\begin{aligned} \dot{E}_i &= b\, w\, C_{PZT} \frac{\left(V_{gsa_i}\right)^2}{2} \\ &= b\, \frac{w\, C_{PZT}}{2\alpha^2} (f_{gsa_i})^2 \end{aligned} \tag{11}$$

where w, α and C_{PZT} are the radial frequency of voltage, the piezoelectric constant and capacitance respectively. Consequently, we will have N_c modal global minimization problems to solve through minimizing their corresponding Hamiltonian functions H_i relative to each mode separately, and we get:

$$\begin{aligned} H_i &= (X_i^t Q_i X_i + f_i^t r_i f_i) + \lambda^t (A_i X_i + B_i f_i - \dot{X}_i) \\ &+ \Gamma_i (\dot{E}_i - b.\, \frac{w\, C_{PZT}}{2\alpha^2} f_i^t\, f_i) - \gamma_{1_i}(E_i - E_{min}) + \gamma_{2_i}(E_i - E_{max}) \\ &+ (-\beta_{1_i}(f_i + f_{max}) + \beta_{2_i}(f_i - f_{max})) \end{aligned} \tag{12}$$

with λ_i, $\gamma_{1_i}, \gamma_{2_i}$, β_{1_i}, β_{2_i} and Γ_i , being the set of required Lagrange multipliers. A_i, B_i, Q_i are the reduced state matrix, actuator location matrix and weightening matrix corresponding to mode i respectively. The minimization of H_i with respect to the state coordinates gives the following expression of λ_i:

$$\begin{aligned} \frac{\partial H_i}{\partial X_i} &= (X_i^t Q_i + \lambda_i^t A_i - \dot{\lambda}_i) \\ &= 0 \\ \Longrightarrow \dot{\lambda}_i &= -Q_i X_i - A_i^t \lambda_i \end{aligned} \tag{13}$$

and the modal control force is obtained by:

$$\begin{aligned} \frac{\partial H_i}{\partial f_i} &= (f_i^t r_i + \lambda_i^t B_i - \Gamma_i\, b.\, \frac{w\, C_{PZT}}{2\alpha^2} f_i^t) \\ &= (r_i f_i + B_i^t \lambda_i - \Gamma_i\, b\, \frac{w\, C_{PZT}}{2\alpha^2} f_i) \\ &= 0 \\ \Longrightarrow f_i &= \frac{-B_i^t \lambda_i}{r_i - \Gamma_i b\, \frac{w\, C_{PZT}}{2\alpha^2}} \end{aligned} \tag{14}$$

The minimization of H_i with respect to the power amount provides the expression of $\dot{\Gamma}_i$ at instant t:

$$\begin{aligned} \frac{\partial H_i}{\partial E_i} &= (-\gamma_{1_i} + \gamma_{2_i} + \dot{\Gamma}_i) \\ &= 0 \\ \Longrightarrow \dot{\Gamma}_i &= \gamma_{1_i} - \gamma_{2_i} \end{aligned} \tag{15}$$

The minimization problem relative to each mode i can then be written as:

$$\begin{aligned} \dot{X}_i &= A_i X_i + B_i f_i \\ \dot{\lambda}_i &= -Q_i X_i - A_i^t \lambda_i \\ f_i &= \frac{-B_i^t \lambda_i}{r_i - \Gamma_i b\, \frac{w\, C_{PZT}}{2\alpha^2}} \\ \dot{\Gamma}_i &= \gamma_{1_i} - \gamma_{2_i} \end{aligned}$$

4.2.1 Available energy

In order to calculate the value of the available energy, we need to have a direct relationship between Γ_i and $\dot{E}_i$. So, we substitute the expression of the control force 14 in 11 and get:

$$\dot{E}_i = b\, \frac{w\, C_{PZT}}{2\alpha^2} . \left(\frac{B_i^t \lambda_i}{r_i - \Gamma_i b\, \frac{w\, C_{PZT}}{2\alpha^2}} \right)^2 \tag{16}$$

An additional constraint relative to the energy management term Γ_i arises from the previous Equation 16. In fact, because Γ_i is related to the amount of energy stored in the accumulators $\dot{E}_i$, it must rely in an eligibility interval $[\Gamma_{min}\, \Gamma_{max}]$ to respect the physical limitations of both actuators and accumulators. When these last conditions are satisfied, we can calculate the value of the energy management term and we get:

$$\Gamma_i = \frac{2\alpha^2}{w\, C_{PZT}} \left(r_i - \frac{B_i^t \lambda_i}{\sqrt{\frac{2\alpha^2 \dot{E}_i}{b\, w\, C_{PZT}}}} \right) \tag{17}$$

However, it is necessary to study the state of the system at instant t_n where a control switching is required i.e. the corresponding value of Γ_i which is equal to its previous value at instant t_{n-1} causes a saturation of the accumulator and thus has to be readjusted to a new one denoted $\hat{\Gamma}_i$ satisfying Equation 16. Let denote $\tilde{\Gamma}_i$ the displayed value of Γ_i at the switching instant from which we can deduce the value of the displayed control force $\tilde{f}_i$ which itself is not satisfying the accumulator limitations, such that:

$$\tilde{f}_i(t_n) = \frac{-B_i^t \lambda_i(t_n)}{r_i - \tilde{\Gamma}_i(t_n) b\, \frac{w\, C_{PZT}}{2\alpha^2}} \tag{18}$$

Now, the value of the readjusted energy management term can be calculated from Equation 17 and we have:

$$\tilde{\Gamma}_i(t_n) = \frac{2\alpha^2}{w\, C_{PZT}} \left(\frac{B_i^t \lambda_i(t_n)}{r_i - \sqrt{\frac{2\alpha^2\, \dot{E}_i(t_n)}{b\, w\, C_{PZT}}}} \right) \tag{19}$$

By replacing the expression of the displayed control force 18 in 19, we get:

$$\hat{\Gamma}_i(t_n) = \frac{1}{\sqrt{\frac{2\alpha^2 \dot{E}_i(t_n)}{b\, w\, C_{PZT}}}} . \left(b\, r_i \tilde{f}_i(t_n)\, \frac{2\alpha^2}{w\, C_{PZT}} + \tilde{\Gamma}_i(t_n) \right) + r_i\, \frac{2\alpha^2}{w\, C_{PZT}} . \tag{20}$$

The value of $\dot{E}_i(t_n)$ in Equation 20 enables us to calculate the value of the maximum available instantaneous force further denoted $\hat{f}_i$ through the relationship 11 and the expression of the readjusted value of Γ_i can be finally obtained by:

$$\hat{\Gamma}_i(t_n) = \left(\frac{b\, r_i \tilde{f}_i(t_n)\, \frac{2\alpha^2}{w\, C_{PZT}} + \tilde{\Gamma}_i(t_n)}{\hat{f}_i(t_n)} \right) + r_i\, \frac{2\alpha^2}{w\, C_{PZT}} . \tag{21}$$

The global management term Γ at instant t_n, when switching is required, is chosen to equal the modal energy management term $\hat{\Gamma}_i$ corresponding to the highest modal control force $\hat{f}_i$ to be applied i.e. the mode with the highest vibrations level. With this procedure, we are sure that at each time path, we are addressing the mode that influences the most the overall response of the beam and thus obtaining the best vibration attenuation performances with the lowest control effort.
From an industrial implementation consideration, readjusting the value of Γ at each time path is not practical and certainly not attractive. So, a suboptimal algorithm is developed in order to avoid this heavy calculation effort and instead, the switching will occur as soon as a saturation in the accumulator is depicted (i.e. limits of $E_i(t)$ are not respected). The algorithm is detailed in Figure 1.

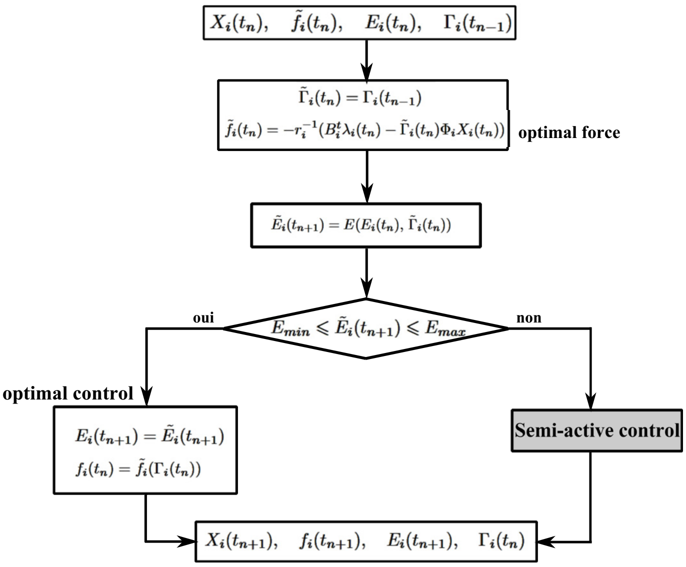


Figure 1: Algorithm of modal global semi-active control

5 Simulations and discussions

In this section, the capacity of the proposed modal global semi-active control (MGSA) to efficiently attenuate a cantilever beam vibrations is investigated and the performances of this method are compared to the purely active ones obtained with IMSC scheme as well as to the semi-active (SA) scheme. The modal response of the beam is initially reduced to the five first modes which appeared to be the most energetic and here the most interesting to the overall response. One of the main aspects of this control is to use some energy harvested from the structure itself. From this observation, one can expect this technique to reduce the structure's vibrations by taking some of its energy at least while stored energy is increasing (Figure 2). Figure 3(a) indicates the frequency response comparisons when using the different control strategies. It is noted that MGSA scheme is able to attenuate the vibrations of the structure with performances approaching those of the IMSC scheme. The precise analysis of the FRF around the first eingenfrequency (Figure 3(b)) actually confirms this remark where MGSA control (green curve) is located between the uncontrolled (solid curve) and the IMSC (dashed curve) responses. This observation is even confirmed by the time response results of the beam for a harmonic excitation

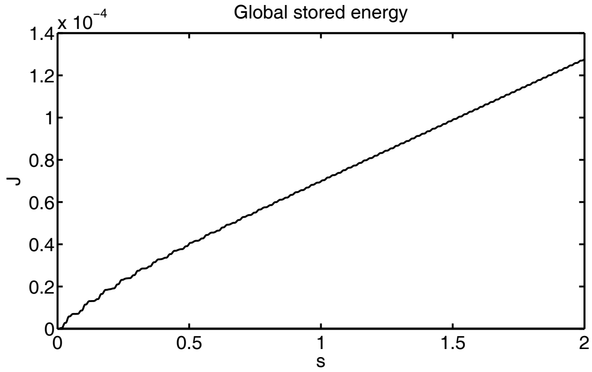
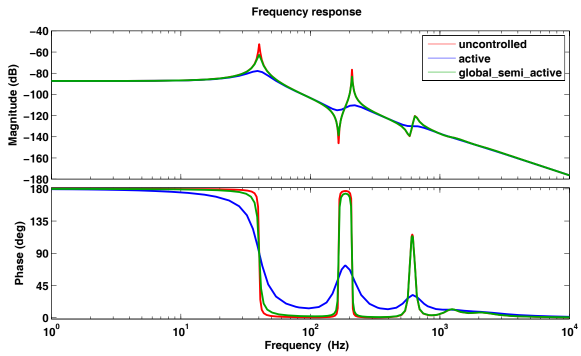
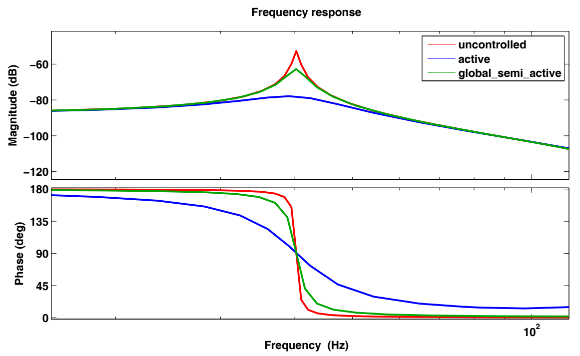


Figure 2: Evolution of the stocked energy



(a) Frequency response of the beam: IMSC(dashed) /MGSA (dotted) comparisons



(b) Zoom of the first mode frequency response

Figure 3: Frequency response of the beam

(Figure 4. The RMS values of the beam displacement

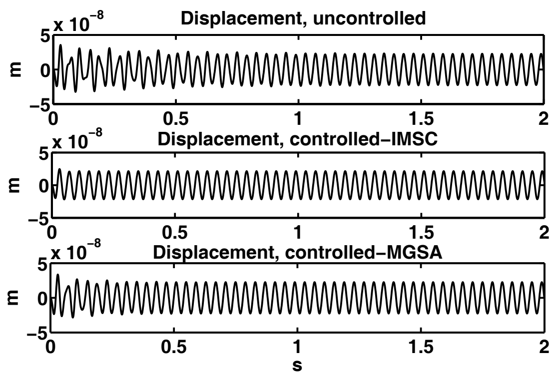


Figure 4: IMSC/ MGSA time response comparisons for harmonic excitations

for the three types of control (IMSC, MGSA and SA) in comparison with the uncontrolled response are respectively: $0.152010^{-7}, 0.161210^{-7}, 0.163210^{-7}$ and 0.163810^{-7} under a harmonic excitation. The performances of the modal global semi-active strategy rank it between the active and the semi-active schemes.

6 Conclusion

The main advantage of the proposed control scheme is its low power requirement since it re-uses the energy of vibrations to supply the actuators. If the stored energy is sufficient to follow the optimal scheme, this last is applied. Otherwise, the controller switches to the semi-active one (dissipative one). The control law was addressed to a cantilever beam and the results showed its good performances approaching those of the optimal law and exceeding those of the semi-active one. Moreover, a reduction in the energy consumption is noticed. It presents an attractive achievement in comparison with the pure active strategy.

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