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Transparent boundary condition for acoustic propagation in duct with bulk-reacting liner

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The proposed study focuses on the modeling of acoustic wave propagation in infinite guides lined with bulk-reacting sound absorbing material which allows acoustic propagation through the lining. The finite element method is used, it is then necessary to limit the numerical domain by an artificial boundary on which a transparent boundary condition is introduced. The method described in this work consist to write the transparent condition as a Dirichlet to Neumann operator (DTN) based on a modal decomposition of the pressure in the guide. This study is a generalization of recent works with absorbent materials modeled with impedances boundary condition. The presented method can be useful for the acoustic design of silencers, air-conditioning ducts, industrial fans, and other similar applications.

1 Introduction

The study of acoustic wave propagation in ducts remains an important issue for aerospace (turbine aircraft) and automobiles (mufflers) engineering. For these problems, the complexity of geometries often justify the use of the finite element method, but there is also areas equivalent to straight waveguides allowing the use of modal methods when they are assumed infinite. Applications with mean flow and acoustic treatment have been studied in a precedent paper with a locally reacting liner modeled by an impedance condition [1]. Our work concerns here the study of an infinite guide lined with a non locally reacting porous material without mean mean flow. The system is modeled by the Helmholtz equation in the part of the guide containing air while the porous material is modeled with a fluid equivalent model. A new transparent boundary condition described by a DtN (Dirichlet to Neumann) operator is expressed to truncate the infinite waveguide.

2 Description of the problem

We consider a two-dimensional infinite duct of height h_1 containing air at rest. The duct is lined with a bulk-reacting sound-absorbing porous material of height $h_2 - h_1$ as shown in Figure 1. The interface of these two medium is denoted by Γ_I. The walls Γ_1 and Γ_2 in contact with the fluid and the bulk-reacting liner respectively, are assumed perfectly rigid.

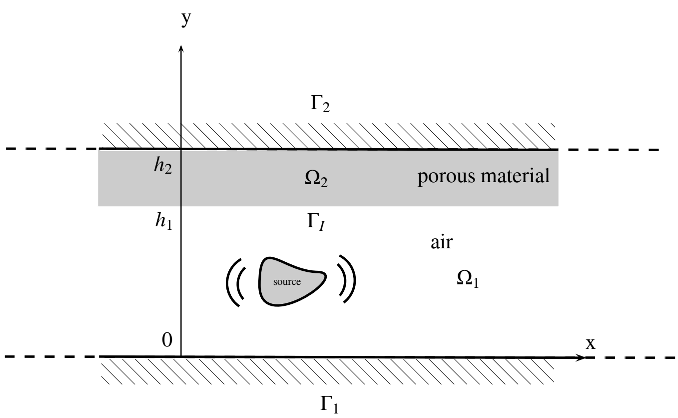

Figure 1: Geometry of the physical domain

We are interested in the radiation of a fixed source located in the guide, the problem is posed in the Oxy plane where the x-axis is parallel to the walls of the guide.

In order to use the finite element method, it is necessary to truncate the infinite domain. Artificial boundaries should be introduced in either side of the source at $x = 0$ ($\Sigma_- = \Sigma_{1-} \cup \Sigma_{2-}$) and $x = L$ ($\Sigma_+ = \Sigma_{1+} \cup \Sigma_{2+}$) on which transparent boundary conditions will be defined (see Figure 2). In this paper, the transparent boundary conditions is expressed by a Dirichlet-to-Neumann (DtN) operator.

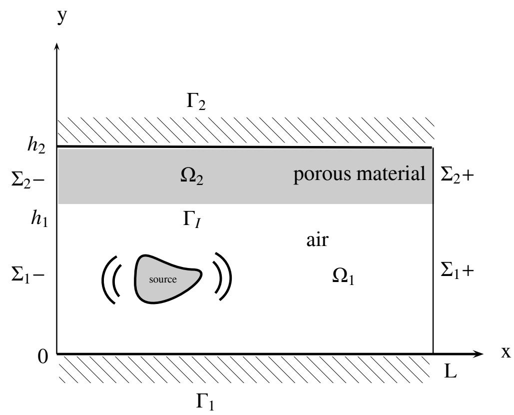

Figure 2: Geometry of the computational domain

3 Governing equations

3.1 Governing equation in air

The duct (Ω_1) contains air considered as a perfect, compressible, and inviscid fluid at rest. According to the classical linear acoustics hypothesis, the time-harmonic acoustic wave propagation ($e^{-i\omega t}$) is described by the homogeneous Helmholtz equation :

$$\Delta p_1 + \frac{\omega^2}{c_0^2} p_1 = 0 \tag{1}$$

where c_0 denotes the speed of sound in air, p_1 is the sound pressure in Ω_1 and Δ is the Laplacian in cartesian coordinates.

3.2 Governing equation in the porous material

The bulk-reacting liner (domain Ω_2) is a porous material whose pores contain air at rest. A porous absorbing material is an heterogeneous media consisting of a solid phase or skeleton and a fluid phase. The skeleton can be continuous (foams, ceramics) or not (fibrous, granular). The fluid is generally composed of air that saturates the pores. The most general approach to study acoustic propagation in porous media is to consider specific movements of waves through the solid skeleton and the saturating fluid. However, in some cases, the fluid is more lightweight and compressible than the solid skeleton. It is often the case for air-saturated porous materials (materials in acoustic engineering) in the frequency range of interest. This corresponds to the approximation of rigid skeleton in which the material is considered as an " equivalent" fluid with effective properties. In the equivalent fluid approximation, several models like Zwikker and Kösten [2] model, Delany-Bazley [3] model or recently the Miki [4] model, have been developed over the last decade. Most elaborated models exist, they are based on the original theory of Biot [5] and can be used in the equivalent fluid approximation or in the general case of the poroelasticity proposed by Biot.

With heavy and rigid skeleton assumptions, only the movement of the fluid phase is considered. This model is retained in this work, because it is a good approximation for many porous materials and because the physical model is on same form that the fluid model (domain Ω_1). This leads to the equivalent fluid model where the pressure p_2 is the unique variable:

$$\Delta p_2 + \frac{\omega^2 \rho_2}{K_2} p_2 = 0 \qquad (2)$$

where the dynamic density ρ_2 and the dynamic bulk modulus K_2 are given by the Johnson-Allard [6] model of five parameters : $\Phi,\ \sigma,\ \Lambda, \Lambda',\ \alpha_\infty$.

$$\tilde{\rho_2} = \rho_0 \alpha_\infty - \frac{\phi}{i\omega} \sigma \tilde{G}(\omega)\ ; \quad \tilde{G}(\omega) = \sqrt{1 + \frac{4\alpha_\infty^2 \eta \rho_0 \omega}{i\sigma^2 \Lambda^2 \phi^2}} \qquad (3)$$

where ϕ is the porosity, η the air viscosity and Λ is the viscous characteristic length introduced by Johnson [7].

$$\tilde{K_2} = \frac{\gamma P_0}{\gamma - (\gamma - 1)\left[1 + \frac{i8\eta}{\Lambda'^2 \omega B^2 \rho_0} \tilde{G}'\right]^{-1}}\ ; \quad \tilde{G}'(\omega) = \sqrt{1 + \frac{\omega \Lambda'^2 B^2 \rho_0}{i16\eta}} \qquad (4)$$

where Λ' is the thermal characteristic length introduced by Champoux and Allard [6], B is the Prandt number, γ is the specific heat ratio and P_0 is the atmospheric pressure.

3.3 Boundary Conditions

The boundary conditions on the rigid walls Γ_1 and Γ_2 are :

$$\frac{\partial p_1}{\partial n} = \frac{\partial p_2}{\partial n} = 0 \quad \text{on} \quad \Gamma_1 \text{and} \ \Gamma_2 \qquad (5)$$

Unlike the case of locally reacting liners, for which an impedance boundary condition is needed [1], continuity conditions at the interface Γ_I for the pressure and the normal displacement must be verified :

$$\begin{aligned} p_1 &= p_2 \quad \text{on } \Gamma_I & (6) \\ \xi_1.n &= \phi\, \xi_2.n \quad \text{on } \Gamma_I & (7) \end{aligned}$$

where ξ is the particle displacement vector and n is the exterior unit normal vector to the boundary. For time-harmonic fluctuations, momentum equations in both media give:

$$ik_1 c_1 \rho_1 v_1 = \nabla p_1 \ \text{ and } \ ik_1 c_1 \rho_2 v_2 = \nabla p_2 \qquad (8)$$

where ∇ is the Nabla operator, the velocities being related to the displacement vectors by:

$$-ik_1 c_1 \xi_1 = v_1 \ \text{ and } \ -ik_1 c_1 \xi_2 = v_2 \qquad (9)$$

By using the two equations (8) and (9) in (7), one obtains the condition of continuity in terms of pressure at the interface:

$$\frac{1}{\rho_1}\frac{\partial p_1}{\partial n} = \frac{\phi}{\rho_2}\frac{\partial p_2}{\partial n} \quad \text{on } \Gamma_I \qquad (10)$$

The transparent boundary condition on the artificial boundaries $\Sigma-$ and $\Sigma+$ are expressed with:

$$\frac{\partial p}{\partial n} = -T^{\pm}(p) \ \ on \ \ \Sigma\pm \qquad (11)$$

where $T^-(p)$ and $T^+(p)$ are the "Dirichlet to Neumann" (DtN) operators. A classical approach is to determine a modal expression of the DtN operators. Such expression can be deduced from an expansion of the pressure on the guide modes. Suppose for instance that:

$$p(x,y) = \sum_{n=0}^{\infty} A_n(p)\theta_n(y)e^{i\beta_n(x-L)} \qquad \text{for } x > L, \qquad (12)$$

then a simple derivation leads to:

$$T^+(p) = -\sum_{n=0}^{\infty} i\beta_n A_n(p)\theta_n. \qquad (13)$$

A main difficulty to justify such an expansion is to prove the completeness of the modes. Then, an orthogonality relation is needed to provide the modal amplitudes $A_n(p)$. An important part of this work is then devoted to the determination of the modes which are solutions of (1) and (2).

4 Modes of the lined guide

4.1 Modes calculation

The modes in the cross section of the guide are the classical solutions obtained by the method of separation of variables. Due to the presence of two media, the solution is expressed in two parts according to the transverse Oy direction. There is whereas the same propagation constant in both the porous and the fluid media : $\beta_1 = \beta_2 = \beta$:

$$p(x,y) = \begin{cases} \theta_1(y)e^{i\beta x} & 0 \le y \le h_1 \\ \theta_2(y)e^{i\beta x} & h_1 \le y \le h_2 \end{cases} \qquad (14)$$

This leads to the following eigenvalue problem :

$$\begin{aligned} \frac{1}{\rho_1}\frac{d^2\theta_1}{dy^2} &= -\Big(\frac{k_1^2}{\rho_1} - \frac{1}{\rho_1}\beta^2\Big)\,\theta_1 & 0 \le y \le h_1 \quad (15) \\ \frac{1}{\rho_2}\frac{d^2\theta_2}{dy^2} &= -\Big(\frac{k_2^2}{\rho_2} - \frac{1}{\rho_2}\beta^2\Big)\,\theta_2 & h_1 \le y \le h_2 \quad (16) \end{aligned}$$

and the boundary conditions become:

$$\frac{d\theta_1}{dy} = \frac{d\theta_2}{dy} = 0 \quad \text{at } y = 0 \ \text{ and } \ y = h_2 \qquad (17)$$

The solution is looking for in the form :

$$\begin{aligned} \theta_1(y) &= a\cos(\alpha_1 y) & (18) \\ \theta_2(y) &= b\cos(\alpha_2(y - h_2)) \end{aligned}$$

This leads to two dispersion equations :

$$\frac{\omega^2}{c_0^2} = k_1^2 = \alpha_{1n}^2 + \beta_n^2 \qquad (19)$$

$$\frac{\omega^2}{c_2^2} = k_2^2 = \alpha_{2n}^2 + \beta_n^2 \qquad (20)$$

where α_{1n} and α_{2n} are the eigenvalues for the fluid and the porous media respectively and $c_2 = \sqrt{K_2/\rho_2}$.

On the other hand, the continuity relations at the interface of both media (10) lead to the transcendent relationship :

$$\frac{\rho_2\alpha_{1n}}{\rho_1\phi\alpha_{2n}}\tan(\alpha_{1n}h_1)[\tan(\alpha_{2n}h_2)\tan(\alpha_{2n}h_1)+1] + \tan(\alpha_{2n}h_2) - tg(\alpha_{2n}h_1) = 0 \quad (21)$$

The eigenvalues α_{1n} and α_{2n} are determined with the transcendent equation (21) associated to the combination of (19) and (20):

$$\alpha_{1n}^2 - \alpha_{2n}^2 = k_2^2 - k_1^2 \quad (22)$$

$$\alpha_{1n}^2 + \alpha_{2n}^2 + 2\beta_n^2 = k_2^2 + k_1^2 \quad (23)$$

The method of Newton-Raphson is used to determine the eigenvalues α_{1n} and α_{2n} with equations (21) and (22) and the corresponding constant of propagation β_n is deduced from (23). Initial values needed by the Newton-Raphson method are sought with a finite element discretization in the section of the guide.

4.2 Some examples

Modes and constants of propagation for a guide lined with a non locally reacting absorbent material are presented. The guide of height $h_1 = 1$ contains air with $c_0 = 344\ m/s$, $\rho_0 = 1,2\ kg/m^3$. The absorbent material of thickness $h = h_2 - h_1 = 0.5$ contains a glass wool with complex and frequency-dependent physical properties. The parameters ρ_2 and K_2 are determined from Johnson-Allard model. Table 1 shows the characteristics of the glass wool used in this study.

σ (N.s.m-4)	ϕ	Λ (m)	Λ' (m)	α_∞
13000	0.97	110E-06	173E-06	10.17

Table 1: Characteristics of glass wool

n	α_{1n}	α_{2n}	α_n *(rigid)*
1	1.65 - 0.52i	13.80 + 11.30i	0.00 + 0.00i
2	3.34 - 1.72i	13.76 + 10.98i	3.14 + 0.00i
3	5.97 - 0.57i	14.44 + 10.62i	6.28 + 0.00i
4	9.20 - 0.35i	15.57 + 9.86i	9.42 + 0.00i
5	12.37 - 0.25i	17.14 + 8.96i	12.56 + 0.00i
6	11.54 - 13.56i	3.16 + 0.07i	15.70 + 0.00i

Table 2: Propagation constants β_n for a guide lined with glass wool : $f = 400$ Hz, $h_1 = 1$ et $h_2 = 1.5$.

n	β_n^+ *(lined)*	β_n^+ *(rigid)*
1	7.14 + 0.12i	7.31 + 0.00i
2	6.77 + 0.85i	6.60 + 0.00i
3	4.32 + 0.79i	3.73 + 0.00i
4	0.57 + 5.61i	0.00 + 5.95i
5	0.31 + 9.99i	0.00 + 10.23i
6	14.72 + 10.63i	0.00 + 13.91i
7	12.93 + 11.98i	0.00 + 17.38i

Table 3: Propagation constants β_n for a guide lined with glass wool : $f = 400$ Hz, $h_1 = 1$ et $h_2 = 1.5$.

Figure 3, shows the transverse distribution of the first eleven modes in the guide for $f = 400$ Hz ($k_1 = 7.306$, $k_2 = 14.957 + 10.482i$). One can observe two types of modes for the lined guide (see Figure 3-bottom) contrary to those of the rigid guide (Figure 3-top).

The first type (1st, 2nd, 3rd, 4th, 5th, 8th, 10th and 11th modes) are modes whose repartition are mainly located in the air while a rapid decrease appears in the porous material. These modes have the same appearance as 1st, 2nd, 3rd, 4th, 5th, 6th, 7th and 8th modes of the rigid guide (see Tables 2 and 3 and Figure 3-top). The second type of modes (6th, 7th and 9th modes) are modes whose repartition are mainly located in the porous material. It is easy to distinguish these two types of modes on the curve of β_n in the complex plane (Figure 4-bottom): the modes in air (red o and blue *, which propagate in positive and negative x direction respectively) and modes in the porous (red . and blue +, which propagate in positive and negative x direction respectively).

Regarding the modes in the fluid, for a rigid guide it is recalled that there is an infinite number of evanescent modes (β_n pure imaginary) and a finite number of propagative modes (β_n pure real) (see Figure 4-top). As expected with the porous material, all modes are attenuated (Figure 4-bottom), but the first three modes are almost propagative (red o and blue *). We still have a finite number of (almost) propagative modes and an infinite number of (strongly) evanescent modes which correspond to pseudo cut-off phenomenon.

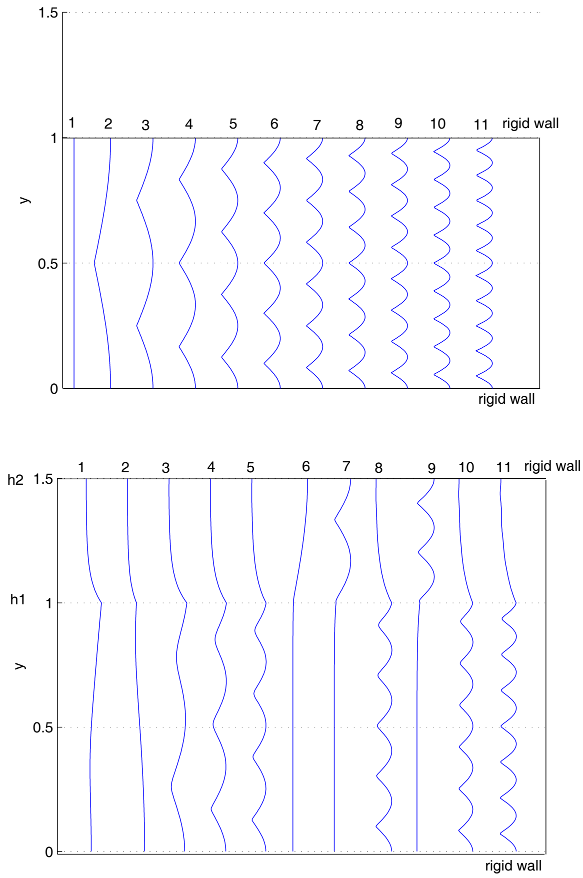

Figure 3: Module of eigenmodes for $f = 400$ Hz - top: rigid guide, bottom : guide lined with glass wool

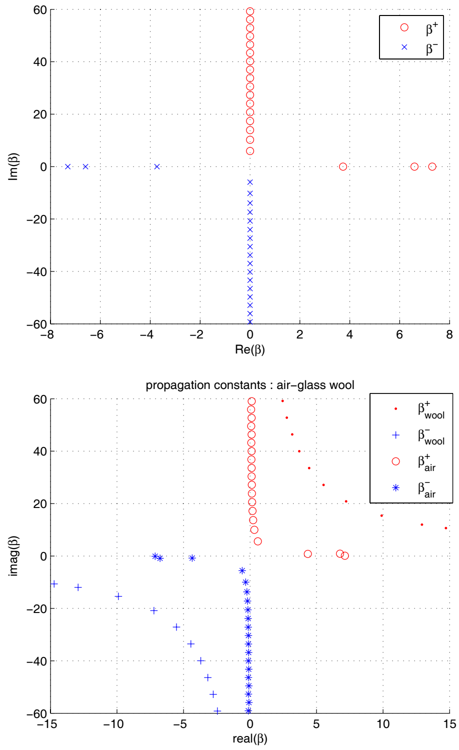

Figure 4: Constants of propagation for $f = 400$ Hz - top: rigid guide, bottom : guide lined with glass wool

5 Orthogonality relation and DtN operator

Knowledge of the modes permits to write the modal expansion of the pressure, but writing the DtN operator requires also the definition of an orthogonality relation to the calculation of the modal coefficients A_n [1] [8].
Multiplying the equations (15) and (16) by $\overline{\theta}_{1m}$ and $\overline{\theta}_{2m}$ and integrating respectively over $[0, h_1]$ and $[h_1, h_2]$, yields to:

$$-\frac{1}{\rho_1}\int_0^{h_1}\frac{d\theta_{1n}}{dy}\frac{d\overline{\theta}_{1m}}{dy}dy+\frac{1}{\rho_1}\frac{d\theta_{1n}}{dy}(h_1)\overline{\theta}_{1m}(h_1)$$
$$=-\left(\frac{k_1^2}{\rho_1}-\frac{\beta_n^2}{\rho_1}\right)\int_0^{h_1}\theta_{1n}\overline{\theta}_{1m}dy \quad (24)$$

and

$$-\frac{\phi}{\rho_2}\int_{h_1}^{h_2}\frac{d\theta_{2n}}{dy}\frac{d\overline{\theta}_{2m}}{dy}dy-\frac{\phi}{\rho_2}\frac{d\theta_{2n}}{dy}(h_1)\overline{\theta}_{2m}(h_1)$$
$$=-\phi\left(\frac{k_2^2}{\rho_2}-\frac{\beta_n^2}{\rho_2}\right)\int_{h_1}^{h_2}\theta_{2n}\overline{\theta}_{2m}dy \quad (25)$$

Adding (24) and (25), and taking into account the continuity relation (10) at the interface between the fluid and the porous material gives:

$$-\frac{1}{\rho_1}\int_0^{h_1}\frac{d\theta_{1n}}{dy}\frac{d\overline{\theta}_{1m}}{dy}dy-\frac{\phi}{\rho_2}\int_{h_1}^{h_2}\frac{d\theta_{2n}}{dy}\frac{d\overline{\theta}_{2m}}{dy}dy$$
$$=-\left(\frac{k_1^2}{\rho_1}-\frac{\beta_n^2}{\rho_1}\right)\int_0^{h_1}\theta_{1n}\overline{\theta}_{1m}dy-\phi\left(\frac{k_2^2}{\rho_2}-\frac{\beta_n^2}{\rho_2}\right)\int_{h_1}^{h_2}\theta_{2n}\overline{\theta}_{2m}dy \quad (26)$$

Exchanging the roles of m and n and conjugating one obtains after subtraction:

$$\phi\left(\frac{1}{\overline{\rho}_2}-\frac{1}{\rho_2}\right)\int_{h_1}^{h_2}\frac{d\overline{\theta}_{2m}}{dy}\frac{d\theta_{2n}}{dy}dy=\phi\left(\frac{\overline{k}_2^2}{\overline{\rho}_2}-\frac{k_2^2}{\rho_2}\right)\int_{h_1}^{h_2}\overline{\theta}_{2m}\theta_{2n}dy$$
$$+\left(\frac{\beta_n^2}{\rho_1}-\frac{\overline{\beta}_m^2}{\rho_1}\right)\int_0^{h_1}\overline{\theta}_{1m}\theta_{1n}dy+\phi\left(\frac{\beta_n^2}{\rho_2}-\frac{\overline{\beta}_m^2}{\overline{\rho}_2}\right)\int_{h_1}^{h_2}\overline{\theta}_{2m}\theta_{2n}dy \quad (27)$$

As observed for a guide with locally reacting lining [1], the modal eigenvalue problem is no longer self-adjoint and the modes are no longer orthogonal for the usual scalar product because the parameters ρ_2 and k_2 are complex. However, exchanging the roles of m and n in (26) without conjugating leads to (with $\overline{\rho}_1 = \rho_1$ and $\overline{k}_1 = k_1$) the orthogonality relation:

$$\frac{1}{\rho_1}\int_0^{h_1}\theta_{1m}\theta_{1n}dy+\frac{\phi}{\rho_2}\int_{h_1}^{h_2}\theta_{2m}\theta_{2n}dy=0 \quad (28)$$

This relation correspond to a bi-orthogonality relation [1]:

$$(\theta_n,\theta_m)^*=\int_0^{h_1}\frac{1}{\rho_1}\theta_{1n}\theta_{1m}dy+\int_{h_1}^{h_2}\frac{\phi}{\rho_2}\theta_{2n}\theta_{2m}dy \quad (29)$$

The modes can be normalized using the bi-orthogonality relation (29), the amplitude of modes (19) becomes:

$$b_n=a_n\frac{\cos(\alpha_{1n}h_1)}{\cos(\alpha_{2n}(h_1-h_2))}=\sqrt{\frac{1}{f+g}}\frac{\cos(\alpha_{1n}h_1)}{\cos(\alpha_{2n}(h_1-h_2))} \quad (30)$$

where

$$f=\frac{2\alpha_{1n}h_1+\sin(2\alpha_{1n}h_1)}{4\alpha_{1n}\rho_1}$$
$$g=\frac{\phi\cos^2(\alpha_{1n}h_1)}{4\alpha_{2n}\rho_2\cos^2(\alpha_{2n}(h_1-h_2))}[2\alpha_{1n}(h_2-h_1)+\sin(2\alpha_{2n}(h_2-h_1))]$$

Using the scalar product defined above (29) at $x = L$, the coefficients A_n^+ introduced in (12) are given by $A_n^+ = (p^+, \theta_n)^*$. By derivation, the DtN operator on the boundary Σ_+ (resp. Σ_-) is expressed by:

$$T^+(p)=-\sum_{n\geq 0}i\beta_n^+(p^+,\theta_n)^*\theta_n(y) \quad (31)$$

with:

$$\theta_n(y)=\begin{cases}\theta_{1n}(y) & 0\leq y\leq h_1\\ \theta_{2n}(y) & h_1\leq y\leq h_2\end{cases} \quad (32)$$

The variational formulation associated to equation (1) and (2) and boundary conditions (5), (10) and (11) is obtained by multiplying equation (1) and (2) by a test function ψ, integrating over Ω_1 and Ω_2, and applying Green's theorem:

$$\int_{\Omega_1}\left(\frac{1}{\rho_1}\nabla p_1\nabla\psi-\frac{k_1^2}{\rho_1}p_1\psi\right)d\Omega$$
$$+\int_{\Omega_2}\left(\frac{\phi}{\rho_2}\nabla p_2\nabla\psi-\frac{\phi k_2^2}{\rho_2}p_2\psi\right)d\Omega$$
$$-\int_{\Sigma_+}T^+(p)\psi\,d\Sigma+\int_{\Sigma_-}T^-(p)psi\,d\Sigma=0 \quad (33)$$

where $T^{\pm}(p)=\mp\sum_{n\geq 0}i\beta_n^{\pm}(p^{\pm},\theta_n)^*\theta_n(y)$.

6 Numerical results

6.1 Propagation in a semi infinite Guide

Knowledge of the transverse modes and the associated propagation constants give an analytic solution to test the transparent boundary condition developed in this study.

Thus, the distribution of pressure (isosurfaces) in a semi infinite guide troncated by an artificial boundary Σ_+ located at $x = L = 2$ is presented. A mode is imposed on the boundary Σ_- ($x = 0$). The computational domain is composed of 740 Lagrange elements of second order. Simulations are performed using the finite element library MELINA [9]. The DtN operator is calculated with the first twenty modes.

Figure 5 presents the pressure in the guide when the second mode (mode variation in air) is imposed on Σ_- for $f = 400$ Hz. A rapid decrease in the glass wool is observe. For this example the solution with the DtN operator is in a good agreement with the analytical solution. The relative error is 0.77%.

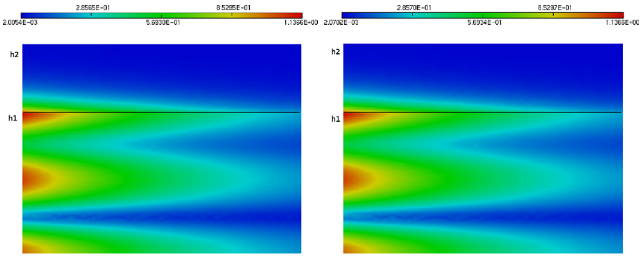

Figure 5: Module of the pressure $f = 400$ Hz ($h_1 = 1$, $k_1 = 7.306$; $h_2 = 1.5$, $k_2 = 14.957 + 10.482i$ (a) with DtN (b) analytical solution

6.2 Radiation of a source in an infinite guide

We now consider a circular source $f = 1$ of radius $R = 0.15$, located in the fluid medium of an infinite lined guide. The guide is truncated with artificial boundaries Σ_- at $x = 0$ and Σ_+ at $x = 1$ where DtN operators are used. Solutions found with DtN operators are compared to those from the utilization of Perfectly Matched Layer (PML) of thickness $e = 0.4$ placed on both sides of Σ_- and Σ_+. The computational domain consists of 2682 Lagrangian elements of second order. The DtN operator is again calculated with the first twenty modes.

Figure 6 shows the radiation of a source placed in the fluid medium and the influence of the porous material. The solutions with the DtN operator and the PML are very similar, the relative error is 0.024%.

7 Conclusion

The effects of non locally reacting absorbing material at a wall of an infinite waveguide are taken into account in order to express a transparent boundary condition based on a DtN map. This new transparent boundary condition is a generalization of the transparent boundary condition developed for guides with locally reacting liner. Modes have been calculated in a guide with air and a porous material described by an equivalent fluid model. By examining the modes, it was found that there are some modes for which amplitude varied in the porous medium and some other modes for which

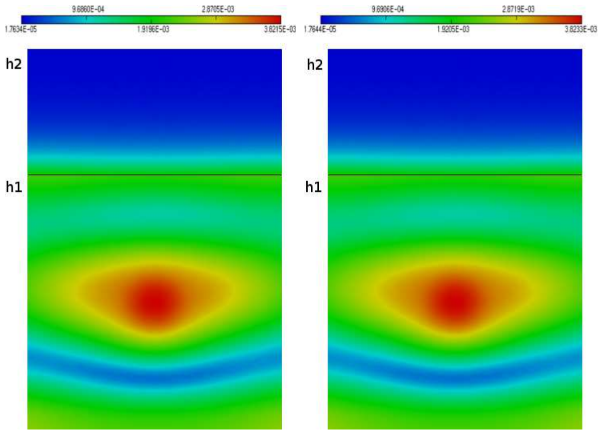

Figure 6: Module of the pressure (a) with DtN (b) pressure module with PML for the two media. $f = 400$ Hz ($h_1 = 1$, $k_1 = 7.306$; $h_2 = 1.5$, $k_2 = 14.957 + 10.482i$)

amplitude varied in the fluid medium. The generalization of the DtN operator for bulk reacting material gives results in good agreement with analytical and PML solutions. As for the locally reacting lining, the main difficulty is related to the fact that the modes are no longer orthogonal for the usual scalar product, nevertheless a bi-orthogonality condition can be used again.

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