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Operational transfer path analysis applied to a small gearbox test set-up

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Operational Transfer Path Analysis (OTPA) is very attractive from a practical point of view, as it not requires an expensive measurement campaign in which transfer functions are measured, as is the case in the traditional Transfer Path Analysis (TPA). Instead, transmissibilities are measured from operational data, making the method relatively cheap in terms of measurement efforts. In practice, however, a lot of difficulties have to be overcome in order to obtain reliable path estimates.

In this paper the Operational Transfer Path Analysis (OTPA) technique is applied to a small gearbox. Laboratory experiments are presented, discussing the use of singular value decomposition to determine the number of physical transmission paths involved, the development of verification checks, and finally the application of OTPA to estimate the strengths of the transmission paths. A comparison with the classical Transfer Path Analysis (TPA) will be included as well. Shortcomings of both the OTPA and TPA method are discussed and conclusions are drawn from the experiments.

1 Introduction

The experimental determination of noise and vibration transmission paths is an essential step in noise control engineering. Knowing the dominant transmission paths enables the engineer to take appropriate measures to reduce the noise and vibration problem at hand. Traditionally transfer paths are determined by means of a *synthesis* of contributions, i.e. the contributions are calculated as the product of source strengths and path sensitivities [1]. In this approach, denoted by classical TPA, the path sensitivities are determined by means of a separate measurement of the relevant transfer functions. The source strengths are estimated by means of an inversion of the measured transfer function matrices. The separate measurement of transfer function matrices makes the procedure time consuming and costly, as the machine needs to be removed from its connection points to allow the transfer function matrices to be measured.

Another approach which is gaining much interest is the Operational Transfer Path Analysis (OTPA) technique, which is based on a *decomposition* of the received signals based on transmissibility measurements. In this procedure a transmissibility matrix identified from operational measurements is used to obtain a decomposition of the received signals that should indicate the strengths of the transmission paths. Transmissibility matrices between points close to the paths to be identified and the target points, where the reduction of noise and vibration is of interest, are measured during operation of the machine. This makes the measurement technique relatively cheap. There is no need to remove the machine, which also guarantees correct boundary conditions during the measurements (think about the stiffening effects of rubber mounts, for instance).

So the advantages of OTPA as compared to TPA are evident. However, the transmissibility approach also constitutes some inherent difficulties to obtain a decomposition of the received signals into meaningful quantities that are useful for the identification of the transmission paths.

In this paper both the OTPA and the classical TPA techniques are applied to an experimental test set-up, consisting of a small gearbox. Due to the interaction of the gears vibrations are generated which are transmitted to a frame structure. The ultimate goal of the analysis is to determine the strengths of the transmission paths, i.e. determine the strength of the transmission paths at each bearing as function of frequency. It will be shown that both OTPA and TPA have their shortcomings.

2 Theory

In this section the theory of TPA and OTPA is not fully treated. We refer to an accompanying paper by Sandier, Leclerc and Roozen [3] for a more extensive discussion of the theory. In this section only some definitions and some basic equations are given, taken from [3], that are referred to in the sequel of this paper.

In OTPA, a linear system is considered between two sets of response dofs. The so-called indicator signals are denoted by y . Indicator sensors are placed on the receiving structure side as close as possible to the physical transmission paths. Each indicator signal is thus related to one excitation force acting on the structure. The target points are denoted by x . These points are distributed on the receiving structure.

The cross spectral matrix between the indicator points is denoted by S_{yy} . The cross spectral matrix between the target points is denoted by S_{xx} . The cross spectral matrix between the indicator and target points is denoted by S_{xy} . The transmissibility matrix between indicator signals y and target signals x can, in theory, be determined as follows

$$T = S_{xy} S_{yy}^{-1} \quad (1)$$

In practice, however, it is often numerically not possible to invert the cross-spectral matrix S_{yy} as obtained from a single measurement. More operational conditions are required to make the system invertible. One approach is to use spectral analysis tools like principal component analysis (PCA) [4],[5] or conditioned spectral analysis (CSA) [6] to obtain a limited number, let's say N , orthonormal response vectors X_i (for the target dof) and Y_i (for the indicator dof) for each operational condition. Collecting all response vectors X_i , ($i = 1, N$) and Y_i , ($i = 1, N$) for all operational conditions into a single matrix X_c and Y_c , respectively, gives:

$$X_c = [X_1 \ X_2 \ \dots \ X_{NM}] \quad Y_c = [Y_1 \ Y_2 \ \dots \ Y_{NM}] \quad (2)$$

where M is the number of operational conditions considered.

In classical TPA transfer functions Φ are measured which relate the cross spectral matrix at the target points S_{yy} to the cross spectral matrix of the forces at the target points S_{ff} , as

$$S_{yy} = \Phi S_{ff} \Phi' \quad (3)$$

3 Measurement test set-up

The measurement test set-up comprises an electro-motor that drives two shafts with gears mounted on them. The electro-motor can operate at various speeds, steered by an electronic controller. By means of four ball bearings the

shafts are mounted on a frame structure, as shown in Figure 1. At each bearing housing a 3D-accelerometer is mounted, 4 in total. The vibration signals thus obtained are numbered from 1 to 12 as shown in Figure 2. These 12 signals constitute the indicator signals for OTPA, which are denoted by y . At the receiving part of the structure, i.e. the frame structure, three 1D-accelerometers are mounted, numbered 13, 14 and 15, as shown in Figure 2 also. These 3 signals constitute the target signals for OTPA, denoted by x . Data acquisition is done by means of a 32 channel OROS measurement system.



Figure 1: Test set-up.

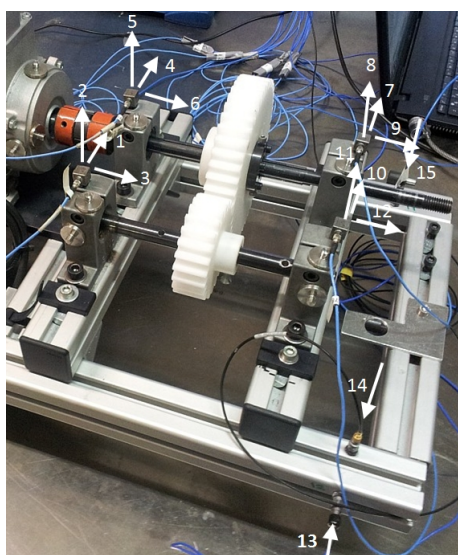


Figure 2: Test set-up numbering.

4 Transfer path analysis

In this section both the Operational Transfer Path Analysis (OTPA) technique and the classical Transfer Path Analysis (TPA) technique will be discussed. Both techniques require matrix inversions, being one of the issues to be dealt with. Another point of concern is the choice of the number and location of the input and output sensors. These issues, as well as a means to check the correctness of the analysis (OTPA or TPA), will be addressed in the following subsections.

4.1 Checking condition numbers

Matrix inversions can cause erroneous results, especially if the system is ill-conditioned. In most cases regularization

is of prime importance to this respect. A parameter which can be used to inspect the need of regularization techniques is the condition number. The condition number gives the ratio between the largest and the smallest singular value of the matrix. From the experience of the authors, there is no need for regularization if the condition number is lower than let's say 100. If the condition number is between 100 and 1000 regularization might be considered, and is it above 1000 regularization is absolutely required.

When dealing with the classical Transfer Path Analysis (TPA), an inversion of the matrix Φ (see Eq. 3 and [3]) is required, which is typically done by means of a (regularized) pseudo-inverse. For the present case the condition numbers of the matrix with measured transferfunctions Φ are shown in Figure 3. In this case the matrix Φ consists of transfer functions from different number of indicator channels to 3 target channels. The number of indicator channels vary from 4 to 12, as shown in the legend of Figure 3 ("incha"=indicator channel). From this figure it can be seen that in all cases the condition numbers are low enough, thus not requiring regularization.

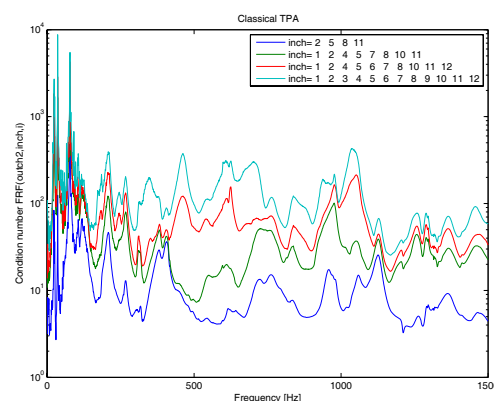


Figure 3: Condition numbers of matrix H (inch x 3 channels), classical TPA, for different number of indicator channels

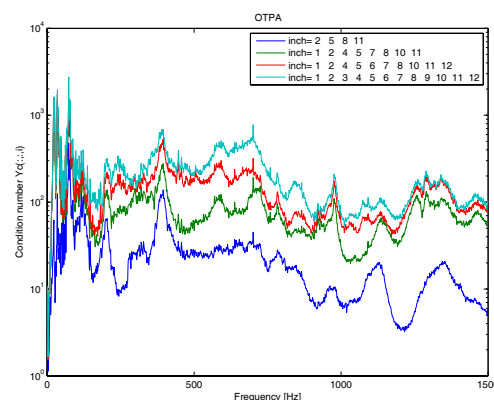


Figure 4: Condition numbers of OTPA operational response matrix, for different number of indicator channels

When dealing with the Operational Transfer Path Analysis (OTPA), an inversion of the cross-spectral matrix S_{yy} (see Eq. 1 and [3]) is required. In practice, however, it is often numerically not possible to invert the cross-spectral matrix as obtained from a single measurement. More operational conditions are required to make the system invertible. The approach taken here is to obtain measurement data from 14 dif-

ferent operational conditions (i.e. 14 different motor speeds). On each data set a principal component analysis (PCA) was performed of which the first 12 principal components were used for subsequent data processing. Thus, a matrix X_c and Y_c (see Eq. 2) of 14 x 12 columns for each matrix was obtained, which was used to determine the transmissibility. The condition number of these matrices, using different indicator channel sets is shown in Figure 4. From this figure it can also be concluded that a regularization is not very crucial in this case.

4.2 Checking for the number of relevant transmission paths: Singular value decomposition

When setting up a measurement for operational transfer path analysis, one of the biggest problems is the selection of the number of sensors and their locations. Often engineering judgement is used to make a sensible choice. In the present case the transfer paths at the bearings are to be identified, which led to the choice to use 3D-accelerometers at the bearing housings as indicator sensors. It remains to be seen, however, if all those 12 degrees of freedom are required for the present analysis. To answer this question it is useful to perform a singular value decomposition (SVD) on the measurement data in an effort to assess the number of physical transmission paths involved.

Figure 5 shows the singular values of the matrix $[X_c Y_c]$, containing the first 12 principal components of each measurement data set, containing all signal dof (i.e. both indicator signals and target signals). In addition, a separate measurement was performed in which the electro-motor was disconnected from the driving shaft. In this case, the shafts and gears were not rotating, leaving the motor as the only source of vibrations. Performing the same data processing, the singular values of this configuration was obtained as well. The thick black line in Figure 5 shows the most significant singular value, denoted by $sv_{disconnected}$, which corresponds to the transmission path from electro-motor to the frame structure. From Figure 5 it can be seen that below 500 Hz 10 singular values are more significant than $sv_{disconnected}$. These singular values thus must represent the transmission paths via the bearings. Looking at Figure 5 one could possibly say that at frequencies higher than 800 Hz there are less significant transmission paths as compared to $sv_{disconnected}$.

Summarizing, up to 500 Hz at least 10 physical transmission paths seem to be involved.

4.3 Checking for the correctness of reconstruction

The ultimate aim of TPA is to reconstruct individual contributions of each indicator channel to a target channel. Thus, when adding all individual contributions as obtained by means of TPA, it should reproduce the measured response at that specific output channel. In this manner the TPA results can be verified. Passing this check does not, however, guarantee that the results will be correct. Only in cases where there is a significant difference, one could conclude that other paths are also participating which are not 'sensed' by the indicator channels, which makes the TPA analysis incorrect.

For OTPA the situation is a little bit different. As the method is based upon a *decomposition* of the signals at the

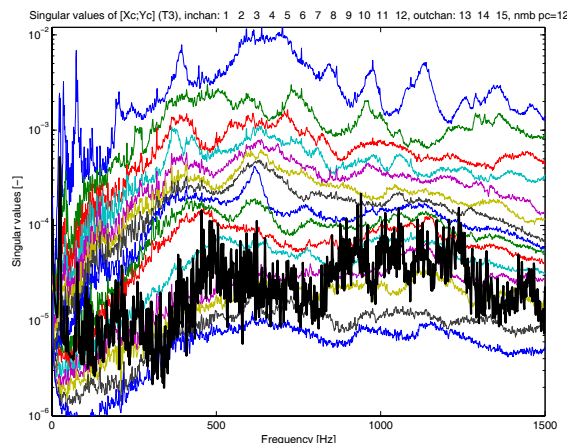


Figure 5: Singular values using 12 indicators, with shafts. Thick black line: first singular values with gears disconnected from motor

target points, the vibration contributions of the erroneously omitted transmission paths will be distributed amongst the indicator channel paths that are being considered by the OTPA analysis [2]. The result will be, in that case, that the global reconstruction at the target points is quite well, but that the partial contributions are not correct. Thus, it can be concluded that this check will be less discriminative for OTPA.

Figure 6 and Figure 7 show the reconstruction of TPA and OTPA, respectively, for output channel 15. It should be noted here that the vibration signal from channel 15 was *not* used in the TPA procedure to calculate the cross-power spectral matrix S_{FF} of (virtual) sources according to Eq. 3 (see also [3]). Thus, a reconstruction of output signal 15, based on the information contained in all other sensor signals is truly predictive. As can be seen from Figure 6 the TPA procedure is very well capable in predicting the total response at sensor 15, except for a frequency range around 700 Hz and around 1400 Hz. Later we will see that in the frequency range around 700 Hz the TPA method is indeed lacking, obviously because of some vibrations that are not captured by the sensors. It is, however, not clear which vibrations in that frequency range are responsible for this behavior.

From Figure 7 it can be seen that the OTPA method apparently does a good job. It is, however, incorrect to conclude that OTPA is without any error, in view of what has been said earlier, as this check is simply not strict enough for OTPA to make such a conclusion. On the other hand, it is not self-evident that the reconstruction will *always* be good, because in the data processing of OTPA one data set was deliberately *not* employed for the estimation of the transmissibility matrix, which is the (indicator) data-set which is used for this check.

For ease of interpretation, Figure 8 and Figure 9 show the ratio between the reconstructed data and the measured data for TPA and OTPA, respectively. Note the difference in ordinate axis magnitudes in both figures. It shows that the errors for OTPA are very small as compared to TPA, at least in terms of this reconstruction check. It should be stressed, however, that especially for OTPA this check is not a sufficient validation.

The comparison of the individual transmission path contributions allows for ranking of sources (or transmission paths) at target points. It should be noted that this source ranking makes sense, only if the individual contributions are not sig-

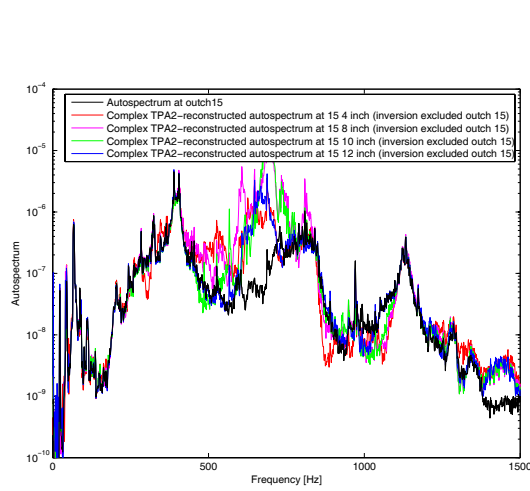


Figure 6: Classical TPA reconstructions, using different number of indicator channels

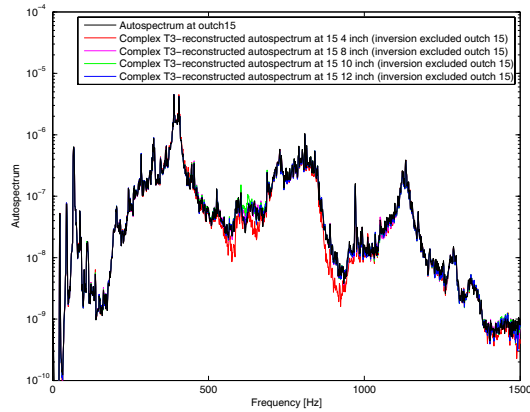


Figure 7: OTPA reconstructions, using different number of indicator channels

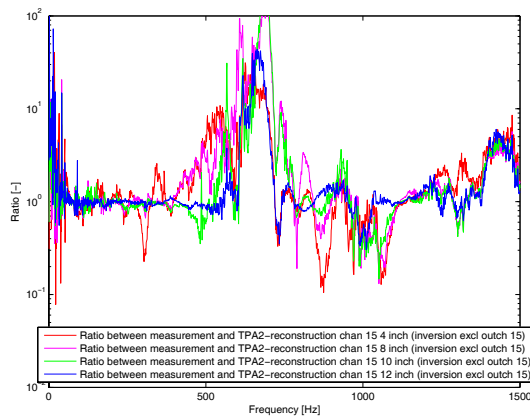


Figure 8: Ratio between Classical TPA-reconstructed and measured data, using different number of indicator channels

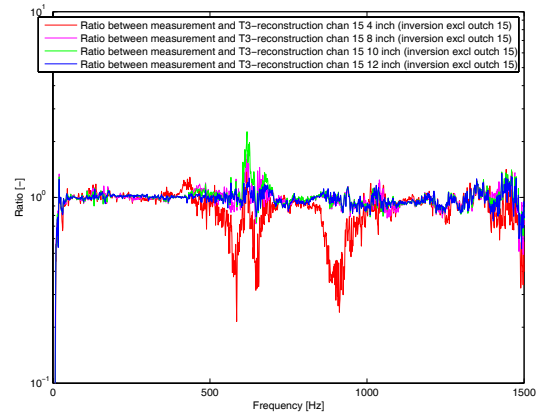


Figure 9: Ratio between Classical OTPA-reconstructed and measured data, using different number of indicator channels

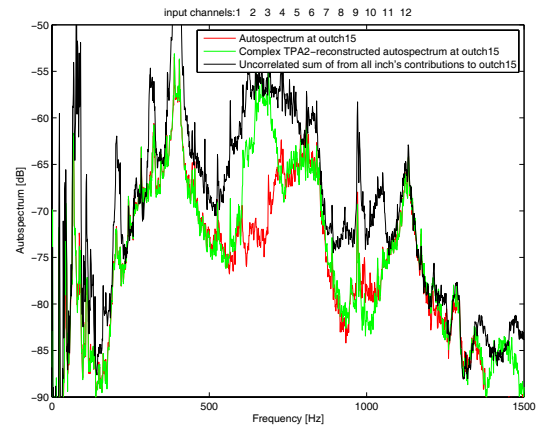


Figure 10: Classical TPA energetic sum check, using 12 indicator channels

nificantly greater than the global contribution. For instance, if sources are correlated and are canceling each other at target points (destructive interference, e.g. in case of a strong undamped modal behavior), the individual contributions will be (possibly significantly) higher than the measured vibration level at the target point. In that case, the results obtained by TPA or OTPA will not by definition be incorrect, but it will be difficult, if not impossible, to speak about a decomposition of the signal at the target point. The following criterion is proposed in [3], called the energetic sum check of partial contributions, which reads

$$\sum_j S_{xx}^{(j)} = < S_{xx} + 3dB \quad (4)$$

where $S_{xx}^{(j)}$ is the partial contribution of the j -th indicator signal. If this check is passed, a meaningful decomposition of the signal at the target point can be realized by means of TPA or OTPA.

Figure 10 and Figure 11 show a plot of the actually measured response at channel 15, the complex reconstructed response (which was already considered in Figure 6 and Figure 7), and the energetic sum check (black curve in the figures) as expressed mathematically in Eq. 4, for TPA and OTPA, respectively. From these figures it can be seen that this criterion is only met at the frequency peaks at approximately 400 Hz and 1140 Hz. In the frequency range around 700 Hz the criterion is not met by far, indicating that a meaningful decomposition by means of TPA and OTPA will be difficult.

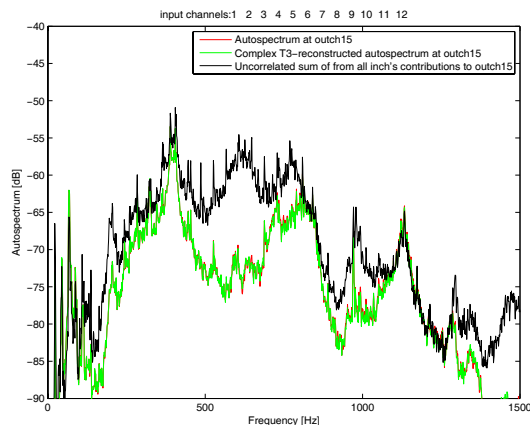


Figure 11: OTPA energetic sum check, using 12 indicator channels

4.4 Identifying dominant transmission paths

Using both TPA and OTPA an attempt is made to identify dominant transmission paths for the small gearbox test set-up. It was decided to use all 12 indicator sensor signals, as the singular values of the matrices $[X_c \ Y_c]$ indicated 10 physical transmission paths (at least up to 500 Hz). Numerical experiments were also performed using 8 or 10 indicator signals, but the path reconstruction was not much different, with a slightly better result when using 12 indicator signals.

In Figure 12 a TPA estimate of the individual contributions are given. Though in total 12 indicator dof are considered, the figure only shows the vertical vibration components at the bearing housings. The contributions related to the two horizontal dofs appeared to be less dominant as compared to the vertical dof. The difference between the global TPA reconstruction and the measured response (showing the shortcomings of TPA at certain frequencies) is filled in blue. Note that the blue filled areas correspond to the errors shown in Figure 8. For instance big errors are present in the frequency range around 700 Hz. As from the energetic sum check it was clear that only the frequency peaks at approximately 400 Hz and 1140 Hz passed, we will focus on these two frequencies. In Figure 14 and Figure 16 a detail zoom plot around these two frequencies can be found. From the TPA analysis it can be concluded that at 400 Hz the bearing at indicator channel 5 and indicator channel 8 are both very dominant. Note that these two bearings are on the same driving shaft. Also the bearing at indicator channel 11 seems to play a significant role at this frequency. At the resonance around 1140 Hz it is clear that the bearing at indicator channel 8 is by far dominant only.

The OTPA estimates of the individual contributions are given in Figure 13. Also here the difference between the global OTPA reconstruction and the measured response is filled in blue. Again, as from the OTPA energetic sum check it was clear that only the frequency peaks at approximately 400 Hz and 1140 Hz passed also, we will focus on these two frequencies. In Figure 15 and Figure 17 a detail zoom plot around these two frequencies can be found. From the OTPA analysis it can be concluded that the bearing at indicator channel 11 is only dominant, perhaps also a bit the bearing at indicator channel 5. So this is partly in correspondence with the TPA estimates. At 1125 Hz there is full agreement with TPA: the bearing at indicator channel 8 is by far dominant only also.

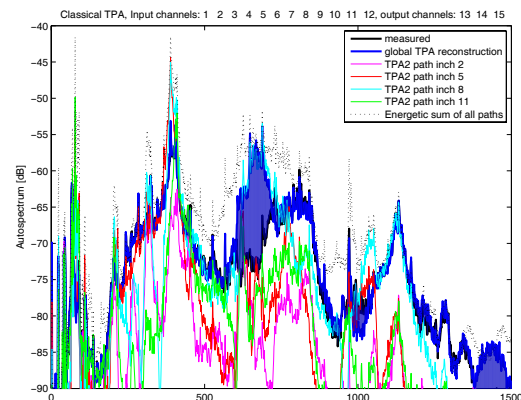


Figure 12: Classical TPA estimate of Y-component contributions, using 12 indicators

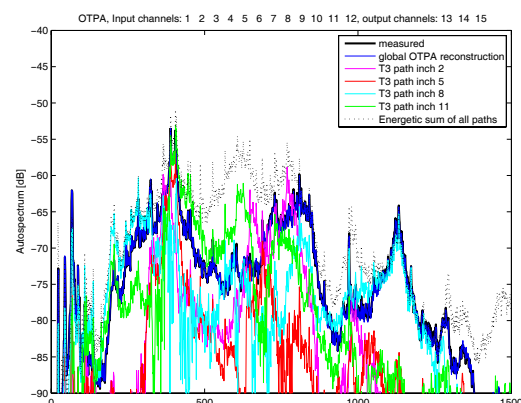


Figure 13: OTPA estimate of Y-component contributions, using 12 indicators

5 Conclusion

Both Operational Transfer Path Analysis (OTPA) and classical Transfer Path Analysis (TPA) are difficult to apply to real-life applications. For instance, one must be aware of a correct choice of the number and location of indicator sensors. A singular value decomposition (SVD) can be used posterior to estimate the number of physical transfer paths being present. In addition, the correctness of the reconstruction can be verified by comparing the complex added sum of the reconstruction with the actually measured vibration data at the target points. This check revealed that the classical TPA suffered from missing vibration data in certain frequency regions to capture all the transmission paths. To this respect, this is a very useful check, as troublesome frequency regions can be identified. OTPA suffered from the same shortcoming in this frequency region. Yet another check is the so-called energetic sum check, which allows the energetic sum of all individual paths not to be larger than 3dB as compared to the actual vibration level at the target points. Following the outcome of this check, we focussed on two resonant frequency regions, which could be identified successfully with both TPA and OTPA.

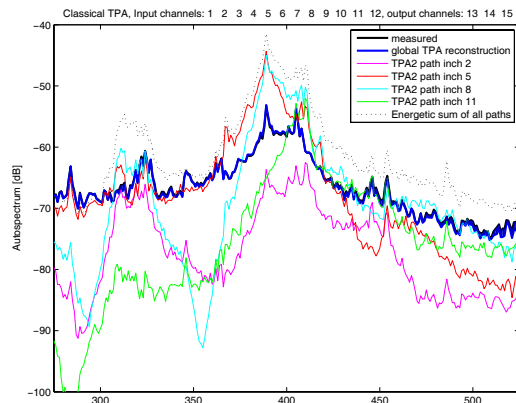


Figure 14: Classical TPA estimate of Y-component contributions, using 12 indicators, detail zoom plot around 400 Hz

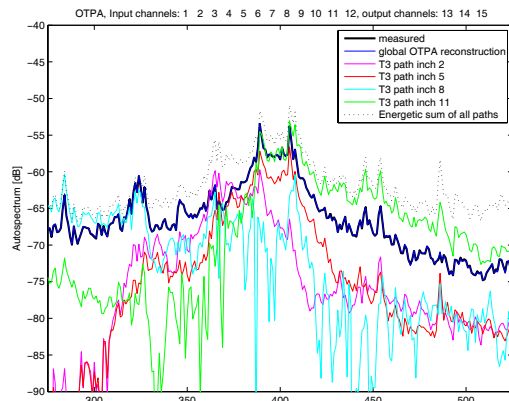


Figure 15: Classical OTPA estimate of Y-component contributions, using 12 indicators, detail zoom plot around 400 Hz

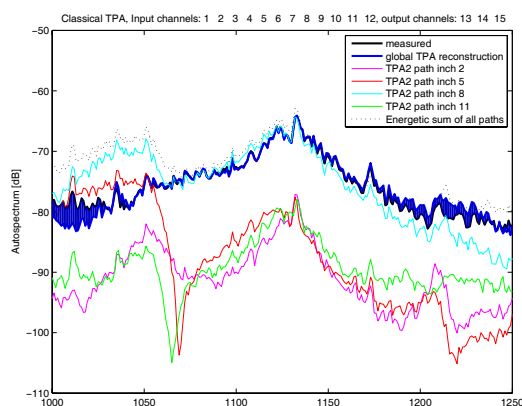


Figure 16: Classical TPA estimate of Y-component contributions, using 12 indicators, detail zoom plot around 1125 Hz

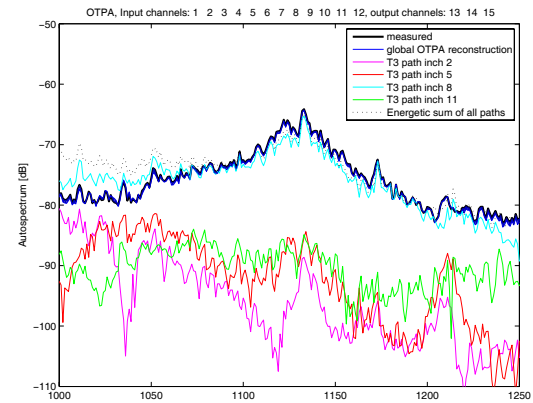


Figure 17: Classical OTPA estimate of Y-component contributions, using 12 indicators, detail zoom plot around 1125 Hz

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