Decay transients in single reed wind instruments

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A classical and dimensionless model of the functioning of single reed instruments such as the clarinet and the saxophone is considered. In this model, the nonlinearity is controlled by two parameters: One linked to the blowing pressure, the other one to the reed opening at rest. Transient commands are applied on this two parameters. In particular, It is shown that a transient decay of the blowing pressure still induces a nonlinear functioning of the system, while a transient decay of the reed opening leads to a free oscillation of the resonator, hence giving a way to estimate the complex frequencies of the impedance peaks. The results are compared to those obtained on natural sounds thanks to a monitored musician, both in pressure and reed opening. The meaning of the dimensionless control parameters are discussed with respect to the actual command of a musician.

1 Introduction

The study of musical instruments seen as nonlinear dynamical systems under non stationary commands has recently become an important topic of investigations in the past few years. Though few results have been published (see e. g. [1]), the understanding of the behavior of such systems is very important. It is particularly important in the field of sound synthesis [2]. Indeed, it is a priori difficult to know how the continuous control parameters should be varied to control naturally a synthesis model during attacks and extinctions. Moreover, classical wind controllers incite to use a command related to the blowing pressure to give birth or death to the sound. Is it relevant? A first goal of this paper is to highlight the difference between transients, depending on the control parameter which is varied. Two cases are considered: A blowing pressure transient and an incoming flow transient. Another goal of this paper is to compare the simulation results with measurements performed on a musician equipped with an instrumented mouthpiece. After a brief recall of the classical sound production model used, the paper focuses on the two types of possible transients commands in the model and their effects on the attack and extinction transients. The results are then compared to those obtained on a musician playing the same note.

2 Dimensionless functioning model

2.1 Input impedance model

The impedance model used here corresponds to that discussed e.g. by Dalmont et al. [3]:

$$Z_e(\omega) = \frac{1}{\text{j} \tan(kL_e) + \frac{1}{k x_e} + \frac{\alpha}{3}}$$  \hspace{1cm} (1)

In this model, a simple volume is mounted in parallel with a truncated cone. $L$ represents the length of the cone, $x_e$ represents the length of the missing part of the cone and $k = k(\omega)$ is the wavenumber including viscothermal losses.

The term $\text{j} k x_e/3$ models the mouthpiece whose volume corresponds exactly to that of the missing part of the cone and ideally compensates for the inharmonicity of the impedance peaks of the truncated cone at low frequency. This particular mouthpiece volume is derived from the continuous fraction expansion of $1/\text{j} \tan(k x_e) \approx 1/\text{j} k x_e + \text{j} k x_e/3$ for small values of $k x_e$.

In figure 1, the simulated input impedance is fully defined with the following geometrical parameters: The input radius of the cone is $r = 6\text{mm}$. The top angle is $\theta = 3.36$. The length is $L = 0.75\text{m}$. These parameters lead to a maximum of the first impedance peak at 174Hz.

![Figure 1: Top: impedance modulus. Middle: impedance phase. Bottom: impulse response](image)

Figure 2 shows, on top the variations in a log scale of the amplitudes with respect to the time of the first four partials of the inverse Fourier transform of the impedance (impulse response), in the bottom the instantaneous frequency of the partials divided by their rank.

![Figure 2: Top: amplitudes of the first four partials of the impulse response. Bottom: frequencies of the first four partials divided by their rank](image)

2.2 Reed dynamics

The reed is modelled as a linear single degree of freedom system and its dimensionless displacement $x(t)$ (defined by the ratio between the physical reed position and the reed...
position at rest) is given by the following dynamic equation:
\[ \frac{1}{\omega_r^2} \frac{d^2 x(t)}{dt^2} + \frac{q_r}{\omega_r} \frac{dx(t)}{dt} + x(t) = p_e(t) - \gamma(t) \]

where \( \omega_r = 2\pi f_r \) and \( q_r \) are respectively the circular frequency and the quality factor of the reed.

The parameter \( \gamma \) is the ratio between the pressure \( p_m \) inside the player’s mouth (assumed to be constant in the steady-state regime) and the static beating reed pressure. In a lossless bore and a massless reed model, \( \gamma \) typically evolves from 0 to 1, \( \gamma = \frac{1}{2} \) being the oscillation threshold and \( \gamma = \frac{1}{4} \) corresponding to the value above which the reed starts beating.

### 2.3 Nonlinear characteristics

The classical nonlinear characteristics used here is based on the stationary Bernoulli equation and links the acoustic flow (the product between the opening of the reed channel and the acoustic velocity) to the pressure difference between the bore and the mouth of the player. The opening of the reed channel \( S(t) \) is expressed from the reed displacement by:

\[ S(t) = \zeta \times \Theta(1 + x(t))(1 + x(t)) \]

where \( \Theta \) denotes the Heaviside function the role of which is to keep the opening of the reed channel positive by cancelling it when \( 1 + x(t) < 0 \). The parameter \( \zeta \) characterizes the whole embouchure and takes into account the lip position and the section ratio between the mouthpiece opening and the resonator. It is proportional to the square root of the reed position \( H \) at equilibrium.

The acoustic flow is finally given by:

\[ u_e(t) = S(t) \text{sign}(\gamma(t) - p_e(t)) \sqrt{|\gamma(t) - p_e(t)|} \]

### 3 Numerical study

We investigate the behavior of the attack and extinction transients on simulated sounds using the synthesis scheme proposed in [4]. The aim of this section is to compare the transients produced by Heaviside functions applied on the blowing pressure \( \gamma \) and on the admissible volume flow (controlled by \( \zeta \)). The signal under consideration is the mouthpiece pressure \( p_e(t) \).

A transient on \( \zeta \) can be related to either an attack or an extinction produced by the tongue that obturate \( (\zeta = 0) \) or liberate \( (\zeta \neq 0) \) the entrance of the reed channel.

A transient on \( \gamma \) can be related to a throat attack or extinction with no modification of the embouchure or the tongue position.

#### 3.1 Attack transient

When the transient (Heaviside function) is applied on \( \zeta \) (figures 3 and 4), the attack transient depends on the steady-state value of \( \zeta \). A high value, corresponding to a weak force applied by the lip on the reed, leads to a fast jump to the permanent regime (figure 4), while a low value leads to an unstable, non periodic regime until \( t = 0.5s \), as it can be seen on the bottom of figure 3.

Figures 3 and 5 reveal, for the same values of \( \zeta \) and \( \gamma \) in the steady-state an identical behavior of the attack transient. This can be explained by the fact that before the transient,

![Figure 3: Transient on \( \zeta \). Top: amplitudes of the first four partials of the mouthpiece pressure. Bottom: frequencies of the first four partials divided by their rank. \( \zeta = 0.6, \gamma = 0.45 \)](image)

![Figure 4: Transient on \( \zeta \). Top: amplitudes of the first four partials of the mouthpiece pressure. Bottom: frequencies of the first four partials divided by their rank. \( \zeta = 1.2, \gamma = 0.35 \)](image)

in both cases, the system is totally at rest with null initial conditions \( p_e(t) = u_e(t) = 0 \) for \( t < 0.1s \).

#### 3.2 Extinction transient

The top pictures of figures 3 and 4 reveal, from \( t = 1s \), a linear decrease of the amplitudes \( P_i \) (in dB) with respect to time. This corresponds to an exponential decrease of the \( P_i \) in a linear scale, as expected for a damped linear system. The slopes of decrease for each \( P_i \) during the transient do not depend on the control parameters \( \zeta \) and \( \gamma \) since in this case the bore is closed at its entrance when the transient on \( \zeta \) has started: this is confirmed by the comparison of figures 3 and 4 which shows same slopes, but a different duration for the system to return at rest since amplitudes of the steady state (initial condition of the extinction transient) are different. Moreover, as expected since \( \zeta = 0 \) cancels all the non-linear effects of the coupling with the exciter, these slopes are exactly the same as the one measured in figure 2.

As far as the instantaneous frequencies \( F_i \) are concerned,
shortly after the beginning of the extinction transient, they jump exactly to the resonance frequencies of the resonator. Here again, this is expected.

On the other hand, figure 5 shows a very different scenario at the extinction. First of all, the extinction transient is far more rapid than in the previous cases. Moreover, a closer look at the top picture reveals that the decrease of the instantaneous amplitudes $P_i$ is no more linear. This is the sign that the nonlinear coupling between the exciter and the resonator is still active. Indeed, even if the blowing pressure is set to $\gamma = 0$ at the beginning of the transient, the volume flow at the input of the bore does not vanish according to equation 3.

The evolution of the instantaneous frequencies is hardly distinguishable on the bottom picture of figure 5 because of the very short transient, but $F_1$ seems to increase rapidly, whereas the first natural resonance frequency of the bore is below the playing frequency.

Moreover, a complementary set of simulations has shown the duration of the transient does not depend on the reed resonance frequency and quality factor and does not depend on the values of $\zeta$ and $\gamma$ during the steady-state, hence the slope changes, depending on the values of these parameters, contrary to the case of an extinction transient applied on $\zeta$ where the slope is constant.

4 Experimental study

Figure 6 shows, from top to bottom, the pressure measured in the mouth of a musician, the pressure measured in the mouthpiece of an alto saxophone and the force applied by the lower lip of the musician on the reed. The musician played the same note as the one used for the simulations. The device used to collect these signals is described in [5].

Figure 7 shows, on top the variations in a log scale of the amplitudes with respect to the time of the first four partials of the mouthpiece pressure, in the bottom their instantaneous frequencies divided by their rank.

Many clues on figures 6 and 7 indicate that the attack and extinction transients are caused by the tongue obturating the reed channel entrance.

- The mean mouth pressure (figure 6, top) does not vary much before and after the high frequency component vanishes.
- On the FSR signal (figure 6, bottom), concerning the extinction transient, the high frequency component disappears soon after the transient has begun (no more oscillations distinguishable after $t \approx 0.15s$. Note that if it is admitted that the tongue blocks the flow, the analysis of the FSR signal after the beginning of the extinction transient is useless since the force applied on the reed by the lower lip does not control anymore the reed opening cross-section. Concerning the attack transient, it can be observed that the force applied on the reed is first close to zero, hence corresponding to a high value of $\zeta$, and increase slowly until the right pitch is obtained. As it been observed on the simulations, such a high value of $\zeta$ at the onset allows to reach the periodic regime very quickly.
- The comparison of the duration of the extinction transient between the mouth pressure (figure 6, top) and the mouthpiece pressure (figure 6, middle) reveals a fare more rapid decrease of the oscillations in the mouth.
This suggests that for $t > 0.14$, the two air-columns are disconnected: on one side the vocal tract with damped resonance peaks (hence a brutal decrease of the acoustic component), and on the other side the air column in the bore of the saxophone with sharper resonance peaks (hence an extinction ten times longer). Concerning the attack transient, it can be observed that the raising of the oscillating component of the mouth pressure is much faster than that of the mouthpiece pressure, as it has been shown in [6].

- The top picture of figure 7 reveals a linear decrease of the amplitude $P_i$ (in dB) with respect to time (when the signal to noise ratio is sufficiently high). This corresponds to an exponential decrease of the $P_i$ in a linear scale, as expected for a linear system. However the decreasing rates are not the same as those observed on figure 2 (although they are of the same order). This discrepancy should be imputed to the input impedance, where neither the losses nor the resonance frequencies are exactly fitted on the instrument played by the saxophonist. Indeed, comparing the bottom panels of figures 7 and 4 shows that in the experiment, the playing frequency is lower than that of the first impedance peak while it is higher in the simulation (this is partly due to a strong coupling with the vocal tract since the oscillating component in the mouth is of the same order than that in the mouthpiece) and that inharmonicity of the first two impedance peaks is much higher in the experiment than in the simulation.

As a conclusion, we consider that the transients performed by the musician could be modelled by a transient in the control parameter $\zeta$. This does not mean that the transient is as abrupt as the Heaviside function used in the numerical study. In the experiment, it seems that $\zeta$ is not used to modify the reed channel opening at rest but to cancel or allow the acoustic flow with the tongue without changing the lip force exerted on the reed. Moreover, the analysis of the extinction transients could be used to extract modal parameters: modal frequencies and damping, without impedance measurements.

5 Conclusion

From simulated signals, it has been shown that the behavior of the attack transient seems to be independent of the command used, at least when Heaviside profiles are considered. On extinction transients, the type of command plays a very important role. A pressure transient leads to a very fast extinction of the sound. A reed channel closing transient leads to a decrease of the partials directly related to the quality factor of the impedance peaks. It can be noted that both transients sound unrealistic from a perceptual point of view, due to the rudimentary impedance model and commands used. On the natural sound studied, it seems that both attack and extinction transients correspond to a reed channel opening/closing transient, hence to the flow cancellation.

In the model used, the dimensionless parameters $\gamma$ and $\zeta$ were primary defined from stationary hypotheses that allow to relate them with physical controls used by the musician. Though $\zeta$ is mostly related to the reed channel opening at rest, it has been highlighted that it can be used as a way to simulate for sound synthesis purposes a tonguing effect that rapidly liberates or obturates the reed channel opening.

From a “system” point of view, an hypothesis is that the behavior of the transients could be related to the past values of the acoustic flow: Whatever the attack transient applied, the behavior is the same since the acoustic flow is null before the transient. On the extinction transient, the acoustic flow is abruptly cancelled (transient on $\zeta$), or not (transient on $\gamma$), leading to very different behaviors. However, in this study, we have not considered the behavior of the acoustic flow during the transients since it would have been difficult to compare the results obtained to measurements.

Acknowledgments

The authors thank Jean Kergomard for useful remarks. This work is supported by the french Agence Nationale de la Recherche in the framework of the project SDNS-AIMV.

References


