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Measurements of temperature and velocity fluctuations in oscillating flows using thermal anemometry - application to thermoacoustic refrigerators

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This paper summarizes our recent work on the development of thermal anemometry to measure velocity and temperature fluctuations in oscillating flows. First, we demonstrate that velocity cannot be measured accurately by hot-wire anemometry in oscillating flows when the flow reverses its direction. Indeed, there is no unique and well-defined correlation between the flow velocity and heat transfer near flow reversal, which prevents the recovery of velocity fluctuations from the anemometer signal. Second, we detail new procedures for the measurement of temperature fluctuations in oscillating flows using cold-wire thermometry. Thermal inertia alters the response of the sensor to temperature changes. The thermal inertia of the cold-wire (operated by a constant-current or a constant-voltage anemometer) is corrected instantaneously using the same wire but in the heated mode (operated by a constant-voltage anemometer). The new procedures are validated in an acoustic standing-wave where temperature fluctuations amplitudes lower than 0.2K at approximately 1000Hz are successfully measured. Experiments near the edges of the stack in a thermoacoustic refrigerator demonstrate the nonlinearity of the temperature field.

1 Introduction

The experimental characterization of heat transport in thermoacoustic systems requires to measure the oscillating aerodynamic field as well as the instantaneous temperature field. The characterization of the aerodynamic field has drawn the attention of a number of groups in the past decade, driven by the advent of laser-based measurement techniques such as Particle Image Velocimetry (PIV) or Laser-Doppler Velocimetry (LDV). Research has focused mainly on two distinct topics 1/ the study of second-order mean flows (streaming), which was most successfully measured using LDV (e.g. [1, 2]); 2/ secondary-flows at the interface between the stack and the heat exchangers (e.g. [3]–[6]). A few attempts have been made to use hot wires for measuring the flow inside thermoacoustic refrigerators [7]. However, as we demonstrate below, this is vain as hot wires are not suitable for velocity measurements in oscillating flows.

Only a few studies have investigated the oscillating temperature field near the stack [7]–[9] and either their measurements were limited to the amplitude of the fundamental frequency or they needed artifical heating of the stack due to the lack of sensitivity of their measurement technique. In this paper, we present the recent developments we have made in cold-wire thermometry. To solve this problem, a new procedure was developed to correct the thermal inertia of the wire. The procedure is based on a constant-voltage anemometer (CVA) [10]. In unsteady flows, the “time constant” of the wire is in fact not constant as it fluctuates with velocity. When velocity fluctuations are large, as in oscillating flows, the thermal inertia of the cold wire has to be corrected at each instant. Our procedure measures the “time constant” instantaneously using a hot wire operated by a CVA to correct the thermal inertia of a cold wire operated by a constant-current anemometer (CCA). It is simple and performed during post-processing, which saves time spent in experimental facility.

The first section discusses the use of hot wire anemometry for the measurement of velocity fluctuations in oscillating flows. The second section summarizes the new procedure for temperature measurements in oscillating flows using cold-wire thermometry. Validation of this procedure and experimental characterization of the nonlinear temperature field near the stack of a thermoacoustic refrigerator are shown in the third section.

2 Measuring velocity in an oscillating flow using a hot wire

Hot wires are commonly used to measure instantaneous velocity in unsteady flows. A thin metallic wire, heated by an electric current and placed in a flow is cooled down due to convection. The convective flux can be measured and related to the velocity of the flow as was demonstrated almost a century ago by King [11]. The main advantage of hot wires is their large bandwidth. Measurements up to hundreds of kilohertz are possible, hence the routine use of hot wires in turbulence studies. It is therefore tempting to use hot wires to measure velocity fluctuations in an oscillating flow as they are an inexpensive alternative to laser-based techniques. In addition, they do not require any optical access and can be used in industrial-type systems. However, as we demonstrated previously [12], *hot wires cannot measure velocity accurately in oscillating œws.*

Procedures for the post-processing of hot-wire data classicaly rely on a small-fluctuations approximation. However, oscillating flows usually experience large-amplitude velocity fluctuations around their means. In essence, a purely oscillating flow even experiences infinitely large velocity fluctuations since its mean velocity is nil. The small-fluctuations appproximation breaks down when the amplitude of velocity fluctuations is large, i.e. greater than 20% approximately. At large amplitude and for all three main types of anemometers (constant-current (CCA), constant-temperature (CTA) and constant-voltage (CVA) anemometers) the system becomes nonlinear and adequate post-processing is necessary. The CCA is hardly ever used nowadays for velocity measurements. The nonlinearity of the CTA at large-amplitude fluctuations was first demonstrated by Freymuth [13]. We have recently confirmed Freymuth's results using modern computational methods [14] and, based on these results, we are working on a post-processing procedure to correct the nonlinearities of the CTA. However, to date, the nonlinearities of the anemometer can only be corrected for the CVA [15]. The procedure described in Ref. [15] consists in inverting numerically all the governing equations of the system, *without any linearization*. It was successfully tested on simulations of an oscillating flow with a large mean velocity and of a high-speed jet flow.

However, the main impediment to measuring velocity in oscillating flows is that heat transfer around the wire is not well defined when the flow reverses. Indeed, instantaneous velocity is deduced from the amount of heat convected away from the wire at each instant. In a steady flow, there is a well-defined correlation between the velocity (or the Reynolds number, Re) and the heat transfer around the wire (or the

Nusselt number, Nu). This correlation was first given by King [11], Nu= $A + B\mathrm{Re}^{0.5}$ where A and B are constants. In practice, it is determined during calibration in a stationnary flow such as the potential core of a jet flow [16].

The situation is different when the flow reverses. Figure 1 shows the Nusselt number as function of the Reynolds number during one oscillation period obtained from simulations of the heat transfer around a heated cylinder in a sinusoidal flow with no mean velocity (details of the simulations are available in [17, 12]). The relationship between the Nusselt number and the Reynolds number is not unique because the thermal boundary layer around the wire reverses with a slight delay compared to the flow far from the wire. The thermal boundary layer is also thinner near flow reversal than it would be if the flow were steady. In addition, both the thickness and the lag of the thermal boundary layer near flow reversal depend on the frequency and the amplitude of oscillations. Because heat transfer near flow reversal does not depend only on velocity, a calibration is impossible, which prevents the use of hot-wire for measuring velocity in flows with flow reversal.

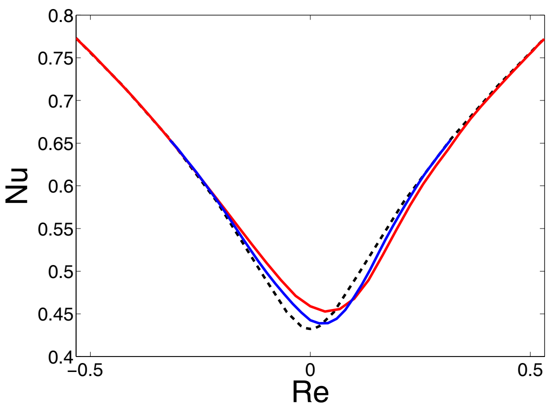

Figure 1: Nusselt number as a function of the Reynolds number during one period of oscillation of the flow. Dashed line: quasi-stationnary simulation. The heat transfer relationship is obtained from simulations in stationnary flows at various Reynolds numbers. Blue line: simulation of a sinusoidal flow with zero mean velocity, velocity amplitude of 3 m.s^{-1} and frequency of 210 Hz. Red line: simulation of a sinusoidal flow with zero mean velocity, velocity amplitude of 5 m.s^{-1} and frequency of 210 Hz.

3 Measuring temperature in an oscillating flow using a cold wire

A resistance thermometer relies on the dependence of the resistance of a “cold wire”, i.e. a thin metallic wire, with the temperature of the surrounding fluid. The resistance of the wire, $R_w^*(t)$ is measured using a very low current in order to keep the wire “cold”. It is related to the temperature of the fluid $T_a(t)$ according to the following equation:

$$T_a(t) - T_0 = \frac{R_w^*(t) - R_0}{R_0 \chi}, \quad (1)$$

where T_0 is a reference temperature, R_0 is the resistance of the wire at T_0 and χ is the thermal coefficient of resistance of the wire material. Equation 1 is valid for an ideal wire that does not undergo thermal inertia. Contrary to the ideal wire, a real wire does not respond instantaneously to a change in the temperature of the fluid. The response of the real wire obeys a first-order system where the resistance of the ideal wire $R_w^*(t)$ is related to the resistance of the real wire $R_w(t)$ by

$$\mathcal{M}(t)\frac{dR_w(t)}{dt} + R_w(t) = R_w^*(t), \quad (2)$$

where $\mathcal{M}(t)$ is the time lag of the cold wire, which is given by:

$$\mathcal{M}(t) = \frac{m_w c_w}{\chi R_0}\frac{1}{f[U(t)]}. \quad (3)$$

$\mathcal{M}(t)$ can be separated into two terms. The first one, $m_w c_w/\chi R_0$, depends on the wire properties, with m_w the mass of the wire and c_w its specific heat. The second term, $1/f[U(t)]$, is a function of the flow velocity $U(t)$. Note that, for the sake of simplicity, the resistance of the connection cable has been neglected. Equation 3 is valid both for the CCA and the CVA. The CTA is unstable at low overheat and cannot be used to measure temperature.

To date, most studies have either neglected the effect of thermal inertia on temperature measurements (e.g. [7]), which may be justified for low-frequency fluctuations, or they have assumed that the thermal lag is constant, i.e. $\mathcal{M}(t) = \mathcal{M}$. In the latter case, $\mathcal{M}$ has to be calibrated as a function of the flow velocity first, using, for instance, a square-wave current injection technique [18, 19]. The temperature fluctuations are subsequently measured using a cold wire in the CCA mode and the time-averaged velocity are obtained from a hot wire operated by a CTA. The method is tedious and is valid only if velocity fluctuations are small, which is not the case in a purely oscillating flow.

A variant of this method was proposed in Refs. [20, 21, 22] to measure $\mathcal{M}(t)$ in flows with large-amplitude velocity fluctuations. Again, the method relies on velocity, and therefore the instantaneous time lag of the wire, being measured by a hot wire operated by a CTA. It requires calibrations of the time lag as a function of velocity. Two problems arise in purely oscillating flows. First, as discussed earlier, even the CTA has a nonlinear response to large-amplitude velocity fluctuations, although this effect may not be very important at low and moderate frequencies. Second and foremost, as heat transfer is not well defined near flow reversal, calibrations of the hot wire are impossible and $\mathcal{M}(t)$ cannot be measured.

These problems were resolved by using a CVA to operate the hot-wire and following the post-processing procedure summarized below. Complete details of the post-processing procedure are given in Ref. [23]. In Ref. [23], temperature is measured by a cold wire operated by a CCA while the correction of the thermal inertia relies on a hot wire operated by a CVA. Adapting the procedure to the use of a CVA for both temperature and time lag measurements is straightforward and we hope to present our first results using only a CVA during the conference.

The calibration procedure is simple and does not require velocity calibration. In addition, the thermal-inertia correction is performed during post-processing, which reduces the time spent using the experimental facility. The main steps of the procedure are:

- *Prior to the test*, the cold resistance (R_0) of the wire is measured and the anemometer channels are set up to the desired overheat ratio (or voltage across the wire). The resistance of the wire is calibrated as a function of temperature in a suitable facility (e.g. thermostated oven). The coefficient $m_w c_w/\chi R_0$ is measured using the square-wave test function available on the CVA unit.

- *During the test*, the output voltages of the wire in both the hot and the cold modes are recorded. Note that measurements are synchronised with the oscillating flow, which allows us to use only one wire.
- *After the test*, the factor $1/f[U(t)]$ is obtained from the signal of the hot wire using the procedure described in Ref. [15]. This does not require to know the velocity explicitly. $\mathcal{M}(t)$ is subsequently computed using Equation 3. $\mathcal{M}(t)$ is used to correct the output signal from the cold wire by solving Equation 2. Finally, using the temperature calibration curve, temperature fluctuations are readily obtained from the corrected output signal of the cold wire.

4 Results

4.1 Validation in a standing-wave acoustic resonator

The procedure has been successfully validated in a standing-wave acoustic resonator [23]. In an acoustic resonator, temperature fluctuations can be deduced from pressure measurements by assuming adiabatic compression and expansion cycles driven by the acoustic wave. Temperature fluctuations measured by a cold wire with and without thermal inertia correction are compared to temperature fluctuations deduced from pressure measurements in Figure 2. The setup consists of a quarter-wavelength standing-wave acoustic resonator operating in air at atmospheric pressure. The resonator is operated at its resonance frequency of 464 Hz. The wire has a rather large diameter of 2.5μ m. It is operated successively as hot wire using a CVA and as a cold wire using a CCA. Full details about the experimental setup are given in Ref. [23]. The figure shows the spatial distribution along the axis of the resonator of the amplitude of temperature fluctuations at both the fundamental frequency and the second harmonic. Uncorrected data significantly underestimate the amplitude of temperature fluctuations. With thermal inertia correction, temperatures measured with the cold wire are in good agreement with those predicted from pressure measurements. Amplitudes of temperature fluctuations smaller than 0.2K at a frequency near 1000Hz are successfully measured, despite a rather bulky wire.

4.2 Nonlinear temperature field behind the stack of a standing-wave thermoacoustic refrigerator

The new procedure allows the measurements of the instantaneous temperature field near the stack of a standing-wave thermoacoustic refrigerator. Analytical models and numerical simulations ([24, 25, 26, 27]) predict a nonlinear temperature field near the end of the stack plates. Thermal harmonics are generated near the stack edges that affect heat transport between the stack and the heat exchangers. The

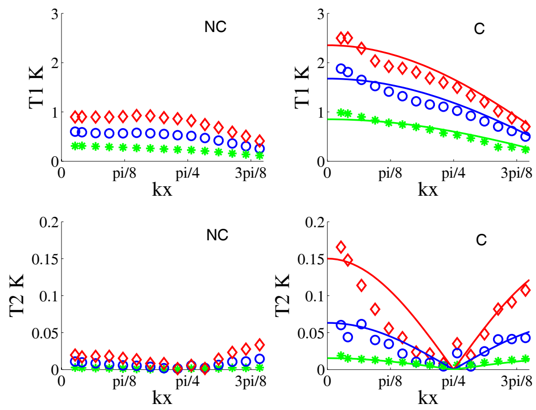

Figure 2: Amplitudes of temperature fluctuations along the axis of the resonator at the fundamental frequency (top) and for the second harmonic (bottom). Plots on the left are for uncorrected data whereas those on the right are for corrected data. Acoustic pressure at the closed end of the resonator is 1000 Pa (green), 2000 Pa (blue) and 3000 Pa (red). Solid lines indicate the amplitude of temperature fluctuations calculated from pressure measurements.

nonlinearity of the temperature field can have important implications in the design of the stack-heat exchangers couple. For instance, models predict an increase in thermal performances when there is a gap between the stack and the heat exchangers, which is contrary to the conclusions of the linear theory. The first measurements of the nonlinear temperature field near the stack are briefly presented in this section. More details are available in Ref. [27].

The model thermoacoustic refrigerator consists of the same acoustic resonator as described above but with a stack of parallel glass plates inside. The stack is made of parallel glass plates of thickness 0.17 mm of length 18 mm and separated by 0.41 mm. The hot side of the stack is located 37 mm away from the closed end of the resonator. In this configuration, a mean temperature gradient of approximately 9 K develops along the stack at the acoustic pressure of $P_{ac} = 3000$ Pa. Figures 3 and 4 show the spatial distributions of the amplitudes of the fundamental and the second harmonic behind the hot side of the stack. In these figures, the distance ξ to the stack edge is normalised by the acoustic displacement and the amplitude of temperature fluctuations is normalised by the amplitude of adiabatic temperature fluctuations that would have occurred without the stack. Figure 3 shows an increase in the amplitude of temperature fluctuations at the fundamental frequency behind the stack. There is good qualitative agreement with the predictions of a simple analytical model [27]. Figure 4 shows the generation of temperature harmonics behind the stack, in good agreement with the model.

5 Conclusion

For the first time, the nonlinear temperature field near the stack of a standing-wave thermoacoustic refrigerator has been successfully characterized using cold-wire thermometry. This was made possible by the development of a new procedure for the measurement of temperature fluctuations in oscillating flows. The thermal inertia of the cold wire is cor-

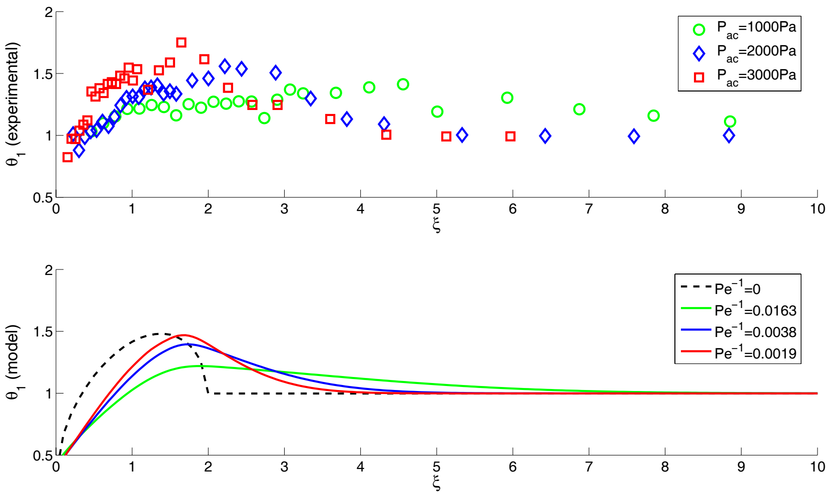

Figure 3: Amplitudes of temperature fluctuations behind the stack at the fundamental frequency. Top: experimental. Bottom: model. Acoustic pressure at the closed end of the resonator is 1000 Pa (green), 2000 Pa (blue) and 3000 Pa (red). The hot end of the stack is located at $\xi = 0$.

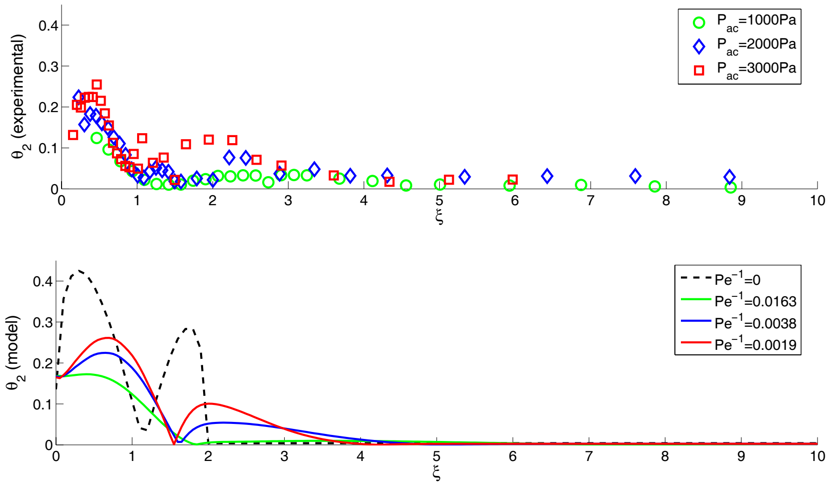

Figure 4: Amplitudes of temperature fluctuations behind the stack at the second harmonic. Top: experimental. Bottom: model. Acoustic pressure at the closed end of the resonator is 1000 Pa (green), 2000 Pa (blue) and 3000 Pa (red). The hot end of the stack is located at $\xi = 0$.

rected by measuring the instantaneous time lag of the wire. The procedure relies on the constant-voltage anemometer (CVA). It is simple and does not require tedious calibrations and settings of the anemometer. Signal correction is performed during post-processing, which limits time spent using the experimental facilities.

Unfortunately, velocity fluctuations could not be measured using thermal anemometry. Indeed, in oscillating flows, heat transfer is not well defined near flow reversal and establishing calibration curves relating the anemometer output to the instantaneous velocity of the flow is impossible.

Further developments of the research summarized in this paper is going on. We are currently validating the use of the procedure described in this paper using only a CVA, both for temperature and velocity measurements. In addition, tests are in progress to demonstrate the technique in more complex flows.

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