Characterization and modelling tools for bow making

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The aim of this work is to supply bow makers with dedicated characterization and simulation tools. As the possibility to tighten the bow and evaluate its behavior comes at the end of the making process, a tool helping the maker to validate conception choices earlier would be useful. To this purpose, a model of bow taking into account prestress and geometric nonlinearity is developed. A non-destructive method for determining mechanical properties of the stick and hair is proposed. The needed equipment is affordable and easily transferable in a workshop. Once the bow properties determined, the model is able to predict the static behavior of the tightened bow and allows to calculate bow properties that are difficult to measure directly. The interest of the developed model and experimental method with regards to bow making are discussed.

1 Introduction

The project PAFI (“Plateforme d’Aide à la Facture Instrumentale”), supported by French National Research Agency, aims at supplying instrument makers with dedicated characterization and simulation tools. For string instruments, the tools currently developed are based on dynamic measurement, using an impact hammer and an accelerometer. In the case of the bow (Fig. 1), however, static properties such as hair tension or bow compliance seem to be of primary importance with regards to the adjustment of playing qualities by the maker.

The aim of our study is to develop a tool capable to determine bow properties which are difficult to measure directly (Young’s modulus of the wood, hair tension...) and to simulate the behavior of a tightened bow, in order to anticipate the consequences of conception choices early in the making process.

2 Model

A model of bow aimed at predicting the static behavior has been developed. The main assumptions express as follows:

- the stick is modeled as an Euler-Bernoulli beam,
- the stick is assumed to be oriented along the grain of the wood (only longitudinal Young’s modulus must be considered),
- geometric nonlinearity is taken into account (a corotational finite-element formulation is chosen),
- the material is homogeneous and elastic,
- the hair has longitudinal stiffness.

Two models have been developed. One is a 2D model [1]. It offers a very short computation time, which is advantageous in the inverse method presented further (Section 3.2), where numerous simulations have to be made inside an optimization loop. In this model, only the stick is discretized into 2D beam elements (Fig. 2(a)) while the hair is described by analytical equations (Fig. 2(b)). The model allows to simulate the tightening of the bow and its response to a vertical force exerted on the hair.

The second model (3D) takes into account lateral bending of the stick which occurs during actual playing [2]. It is useful to anticipate the effect of hair tension on lateral compliance (Section 4.2). In this model, the stick as well as the hair are discretized into 3D beam elements (Fig. 3). It is possible to simulate the response of the bow to an inclined force, as it is often the case in actual playing.

3 Determination of bow properties

3.1 Geometry: taper and camber

Taper denotes the gradually decreasing thickness of the stick along its length. On modern bows, the cross section of the stick is described either as round or octagonal. However, many bows are not perfectly round (or octagonal). This can...
be due to the making process (the taper is obtained by plan-
ing down the stick progressively), or voluntarily decided by the maker. To take this into account, it is necessary to mea-
sure the thickness in both vertical and lateral directions of the stick. Figure 4 shows vertical and lateral diameters measured on an apparently round stick. In this case, an oval shape can be considered a good approximation.

The probably most convenient and accurate way to mea-
sure diameter is by using a digital caliper, as bow makers usually do it. Measurements can be carried out at discrete, possibly unevenly spaced abscissas along the stick. On the condition that a sufficient number of abscissas is chosen, a fine continuous description can then be obtain by piecewise polynomial interpolation (see Fig. 4).

Camber denotes the initial curvature of the bow. It can be determined by means of image processing. For the purpose of the model, only the portion of the stick above the hair has to be determined, since the rest of the stick (portions above and behind the frog, as well as the head) are assumed rigid. The starting point of the procedure is a picture of the loosened bow reposing on two supports, as represented on Figure 5. The algorithm used to determine camber is based on the detection of the lower and upper outlines of the stick. For this reason, backlighting is prescribed to reinforce contrast and to avoid reflection on the stick due to varnish.

The procedure starts by determining manually the two endpoints of the hair, on magnified views near the frog and the head (Fig. 5(a)). This allows to define the reference frame of the bow. If necessary, the picture is rotated such as to make the direction of the hair horizontal. Then, the lower and upper outlines of the stick are detected at each abscissa in the region highlighted on Figure 5(b) (white dashed lines). As the curvature is small, the vertical difference between the two outlines provides a fair approximation of the neutral axis. A smooth, continuous description of camber is then obtained by fitting the points of the neutral axis by a polynom. Absurd points resulting from image processing, or points likely to impair the polynom fit (e.g., just behind the head), are detected and disregarded for the fit. To automatically find the appropriate polynom order, a criterium based on the residual error is implemented.

In order to minimize the number of actions to be done by the maker, an effort has been made to automatize the procedure as much as possible. However, the procedure just described may rise some difficulties for some bows. On bows which are much cambered, the stick can be very close to the hair at its lowest point (it can even slightly pass through the hair). This prevents the detection of the lower outline, as the stick and hair cannot be distinguished from each other over a certain length. To overcome this problem, the simplest solution consists in move the hair away from the stick by means of a small, light cylinder, as showed by Figure 6. Another difficulty arises when measuring the camber of Renaissance and Baroque bows. There is generally no screw mechanism to tighten the hair on these bows. The hair is attached to the stick at its both ends. The frog, made as a one-block wooden part, iswedged between the hair and the stick to tighten the bow. To determine the “effective” endpoint of the hair (i.e., the point at the front end of the frog) without hair tension on such a bow, the frog has to be maintained at the appropriate place before taking the picture.

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Each parameter is determined by means of an inverse method. The deformation of the stick caused by a specific loading is measured with the same method as for the determination of camber. Then, simulations of the same load case are performed inside an optimization loop, the purpose of which is to find the value of the parameter that gives the best fit between measured and simulated deformed shape of the stick.

The successive load cases used to determine the three parameters is illustrated by Figure 8. At the frog, a clamped boundary condition is realized by two metal fingers grasping the bow, one above the stick at the front end of the frog (abscissa \( x = 0 \text{ mm} \) in the model), the other under the frog at its rear end. It should be noted that the metal fingers can be covered by a soft material to avoid any damage to the stick or the frog, since a compliant boundary condition is tolerated, as will be explained further.

![Figure 8: Successive load cases for the determination of bow parameters](image1)

The Young’s modulus of the stick is found by loading the bow at the tip by a vertical force at the tip \( F_z \) (Fig. 8(b)). At this step, the hair has to be loosened at the most, so that it does not impair the motion of the stick. On some bows, such as the one used in this experiment, the length of the hair is too short to satisfy this condition. A solution consists in replacing the frog by a similar one from another bow, as was done here (see Fig. 8(a) and 8(b)). Then, the hair tension is determined from the picture of the tightened bow (Fig. 8(c)). Finally, loading the tightened bow by a vertical force \( F_z \) (Fig. 8(d)) allows to determine the stiffness of the hair. This is possible due to the fact that the tension, increasing from \( T_0 \) to \( T \) (unknown) under loading, significantly affects the bending of the stick.

The solution found by the optimization loop for each parameter is shown on Figure 9. The initial state at each step is also plotted (gray dashed lines) to give an idea of the deformation caused by loading. As seen on the figure, the simulated and measured shape are compared in the reference frame of the bow and over the whole length of the stick. This has two notable advantages. It the clamped boundary condition is actually slightly compliant, the deflection measured along the bow is the sum of two contributions: one caused by deformation, the other due to an unknown rigid body rotation. Even a small rotation may have a non-negligible contribution in the deflection measured away from the frog. Thus, it is essential to eliminate this effect, which is actually done by using the reference frame of the bow for the comparison. Another advantage, compared to a procedure that would repose on the measurement of deflection at a single abscissa (e.g. at the tip), is that one may qualitatively judge the quality of the model by observing the agreement between measurement and simulation along the bow. Inhomogeneities in wood elasticity, or a mistake when applying the procedure (e.g. entering a wrong diameter, measuring the wrong bow), are likely to increase of the residual error.

4 Possible applications

4.1 Characterization of bows

Bow makers generally keep trace of the characteristics of bows that they have made, or bows that they have repaired. The physical characteristics usually considered are relatively easy to measure: mass, position of center of inertia, stiffness of the stick, taper. On the contrary, characteristics like elasticity of the wood and range in hair tension are difficult to obtain. Yet, they could interest the maker, for instance when making a copy of bow. Moreover, the measurement of camber from image processing offers a convenient means to

![Figure 9: Comparison between simulated and measured deflected shape for the determination of bow parameters](image2)
compare the geometry of two bows (e.g. the original bow and its copy) or to quantify successive modifications brought to camber when adjusting a bow.

For this purpose, the experimental procedure described in Section 3 is currently implemented within the PAFI project.

### 4.2 Prediction of compliance

The compliance of a bow in playing situation is generally considered of great importance in the control of bowing force by the player [3]. The compliance that will be felt by the player on the tightened bow is determined by the elasticity of the stick and that of the hair in the transverse direction. The elasticity of the stick basically depends on the Young’s modulus and taper. Yet, once the bow tightened, vertical and lateral compliance of the stick also depend on camber and hair tension. This is a consequence of geometric nonlinearity, which is strongly involved in the behavior of the bow. Thus, predicting compliance during the making process is not straightforward, especially when the bow has not yet been haired. For this reason, we discuss hereafter the ability of the model to predict the compliance of a tightened bow.

The compliance may be determined by measuring the deflection $u$ caused by a force $F$ exerted on the hair. The experimental setup is shown on Figure 10. The signals from a displacement sensor and from a force sensor are simultaneously acquired during (manual loading and unloading at several abscissas along the hair. Figure 11 shows typical force-deflection curves obtained in the middle and at the tip. Due to geometric nonlinearity, the bow exhibit a softening behavior near the tip, a stiffening behavior near the middle. From the approximation of the measured force-deflection curves by 2nd order polynoms, it is possible to calculate compliance, $c = \frac{du}{F}$, for a given force.

![Figure 10: Experimental setup used to measure compliance.](image)

The compliance at a typical bow force of 1 N, measured at several abscissas along the bow, is plotted on Figure 12 against relative abscissa $\gamma$. A measurement with the bow oriented at 90° relative to the direction of the sensors, allowing to calculate lateral compliance, has been carried out as well. The higher lateral compliance is due to prestress (it has been verified that the compliance of the bow without hair tension is the same in both directions).

![Figure 11: Measured force-deflection curves in the middle ($\gamma = 0.5$) and at the tip ($\gamma = 1$).](image)

![Figure 12: Comparison of measured and simulated compliance along the bow for a typical force of 1 N.](image)

The geometry and mechanical properties of the same bow was determined from the procedure described in Section 3. Then, the compliance of the bow at the tension found by the procedure was obtained from simulations, for both vertical and lateral directions. It is also plotted on figure 12. A good agreement between numerical and experimental results is observed. Since all mechanical parameters were determined using 2D load cases, the agreement on lateral compliance is particularly satisfying. The model, which prove here to be predictive, could be used for instance to anticipate the consequences of modifications brought to taper during the process. It may also be useful to help the maker adapting the geometry with regard to given wood properties. For this purpose, the model could be used in an optimization procedure [4].

### 5 Conclusion and further work

A numerical model capable of predicting the static behavior of a tightened bow has been developed, as well as
a specific procedure to calculate bow properties from measurements. The finite-element model and the experimental procedure have been until now implemented into MATLAB. The code has been designed to be understandable and usable by everyone having experience with MATLAB and programming in general. In the context of PAFI project, a software framework accompanying the measurement equipment is currently developed, using Python programming language. The aim is to provide makers with free software, which is easy to use and robust relative to predefined scenarios (concerning either characterization or simulation).

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References


