Comparison of various models to compute the vibro-acoustic response of large CMUT arrays

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Capacitive Micromachined Ultrasound Transducers (CMUT) is a promising alternative to piezo-electric transducers for medical imaging. CMUT generally comprises more than 10 thousand unitary cells. When immersed in a fluid the cells are strongly coupled by the fluid, leading to cross talk effects. Accurate simulation tools that take into account properly all these coupling are very much CPU time consuming. From an engineering point of view it will be interesting to use simplified models to conduct parametric studies. We propose in this paper to evaluate various simplified models in term of accuracy. As a test case, we chose a typical 1D CMUT array in which groups of 80 cells, called elements, are electrically connected. The reference model takes into account explicitly the contribution of all the cells to compute, in the harmonic domain, the displacement field on the transducer surface and the radiated pressure. A parametric study has been conducted on the interaction radius to compute the acoustic coupling terms. Also kinematic constraints have been investigated to reduce the computational cost.

1 Introduction

The numerical simulation of the radiated pressure of a CMUT array is often conducted by approximate methods, due to the huge number of membranes that have to be taken into account. Simple CMUT design tools, used piston like analytic solutions and simplified coupling hypothesis, as described in [6]. Commercial Finite Element Software are also currently used to study the radiation of a single membrane. Some authors [3], [4] have used ANSYS to study cross talk effects on models limited to a few number of membranes. The use of periodic conditions as in [1] is exact only for infinite arrays. Other models take acoustic coupling into account with various simplification hypothesis [8], [7].

We present here a method where all the membranes are taken into account explicitly in the simulation. Computation time is reasonable, because acoustic couplings are calculated by the Rayleigh integral and membrane displacements are projected on mechanical mode shapes. We present first the model, then the 1D CMUT array that will serve as test case and then we will discuss various simplification hypothesis.

2 Model

A CMUT array is composed of many cells organized as an array. Each cell comprises a small membrane over a sealed vacuum cavity. Electrodes are deposited at the bottom of the cavity and on the membrane to permit electrostatic actuation. Electrical connections allows to apply the same electrical tension to a group of cells called element. We suppose that the membranes are coupled by the semi infinite acoustic medium but not by the mechanical substrate. The objective of our method is to predict first the displacement of all the membranes of the network. Then radiated pressures and directivity diagrams can be deduced from the velocities of all the membranes.

2.1 Cell electro-mechanical model

Each membrane, is statically deflected by bias voltage U_{dc}. We suppose that the alternating voltage U_{ac} is small compared to U_{dc}. In this case the alternating electrostatic forces F_{dyn} can be linearized around the static deflection of the membrane w_{dc}. We thus can write :

$$F_{dyn} = \varepsilon_0 U_{dc}^2 \int_S \frac{w}{(h_{gap} - w_{dc})^3} dS \qquad (1)$$

$$+ \varepsilon_0 U_{dc} U_{ac} \int_S \frac{1}{(h_{gap} - w_{dc})^2} dS \qquad (2)$$

where S is the surface of the electrode, h_{gap} the initial electric effective gap and w the dynamic membrane deflection.

2.2 Acoustic model

The CMUT is flat and each membrane can be considered as baffled. Thus the pressure $P(\mathbf{r}, \omega)$, in the frequency domain, radiated at the point M of coordinates $\mathbf{r}$, by the membrane of surface S_m, can be expressed by the Rayleigh integral :

$$P(\mathbf{r}, \omega) = -\frac{\omega^2 \rho_0}{2\pi} \int_{s_m} \frac{W(\mathbf{r}', \omega) e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} dS' \qquad (3)$$

where $W(\mathbf{r}', \omega)$ is the harmonic deflection of the membrane current point with $w(\mathbf{r}',t) = Re[W(\mathbf{r}',\omega)e^{j\omega t}]$, ρ_0 is the mass per unit volume of the fluid and $k = \frac{\omega}{c_0}$ is the wave number with c_0 the speed of sound in the fluid.

The Rayleigh integral will be use in a first step to compute the acoustic direct and mutual impedances on the membranes. Then in a second step, it will be used to compute the pressure radiated by the network.

2.3 Cell mechanical model

The dynamic membrane deflection w can be projected on the first M modes shapes φ_k of the membrane in vacuum and clamped on its edge [5].

$$w = \sum_{k=1}^{M} \varphi_k q_k \qquad (4)$$

where q_k is the generalized coordinate associated with mode number k. We will further consider an harmonic domain approach where

$$q_k(t) = Re[Q_k(\omega) e^{j\omega t}] \qquad (5)$$

In addition, for each mode k, we will introduce a mechanical modal viscous damping.

2.4 Array model

If we use M mode shapes to represent the displacements of the membrane number i, for $i = 1, N$, the unknown vector for the membrane i $is \left\{Q^i\right\}^T = \left\{Q_1^i, ..., Q_M^i\right\}$ and the unknown vector for the entire network is $\{Q\}^T = \left\{\left\{Q^1\right\}^T, ..., \left\{Q^N\right\}^T\right\}$. The momentum equation for the unknown $\{Q\}$ is obtained after projection of the forces on the modal vectors φ_k^i, for

$i = 1, N$ and $k = 1, M$. It leads to the following linear system of size $N \times M$ to be solved in the frequency domain :

$$(-\omega^2[M] + j\omega[C] + [K] - [K_{elec}] + \frac{\omega^2 \rho_0}{2\pi}[A(\omega)])\{Q\} = \{F_{elec}\} \quad (6)$$

Where :

- $[M], [C], [K]$ are the diagonal structural mass, damping and stiffness matrices
- $[K_{elec}]$ is the linearized electrostatic softening matrix evaluated from (1)
- $\{F_{elec}\}$ is the linearized electrostatic forces vector evaluated from (2)
- $[A(\omega)]$ is the acoustic coupling matrix evaluated from (3)

Most of the physics is thus included in the model for small amplitude responses. Only two parameters have to be chosen; the number of modes and the radius out of which acoustic interactions between membranes are neglected.

The model has been validated on various analytical test cases involving piston motions, and ANSYS simulations. The more complete validation test case was composed of an hexagonal array with 7 circular membranes and is described in [2]. The agreement with ANSYS simulations were excellent, both for membranes velocities and radiated pressures.

3 Test case

3.1 Description

We consider a single generic CMUT cell made of a poly silicon membrane of 1,5 μm thickness and 25 μm radius. The electrode radius is 20 μm and the air gap height is $0, 3$ μm. Collapse voltage has been calculated to 106 V and the bias voltage U_{dc} is equal to 95 V (90% of the collapse voltage). Alternating voltage U_{ac} is equal to 1 V. Water is the acoustic radiation medium. A modal viscous damping ratio of 0.5% has been taken for all the mechanical modes. The first five eigen frequencies (for the first six modes) of this membrane in vacuum are given on table1.

Table 1: First five Eigen Frequencies of the membrane in vacuum

Frequency MHz	Multiplicity	Nb. of nodal diameters	Nb. of nodal circles
9.9	1	0	0
20.7	2	1	0
33.9	2	2	0
38.7	1	0	1

The cells are arranged according to a 1D array with 16 elements shown in figure 1. Each element is composed of 4 x 20 cells. The spacing between two adjacent elements is 30 μm. As the diameter of each cell is 60 μm, the pitch between element centers is $d = 270\,\mu m$.

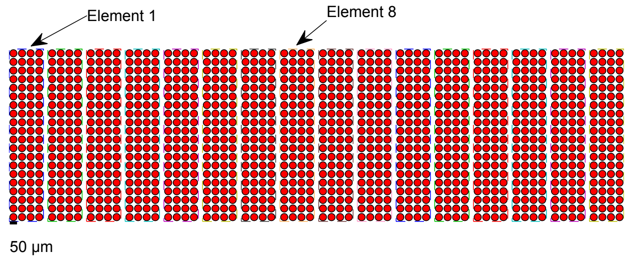

Figure 1: 1D array of 16 x 4 x 20 cells

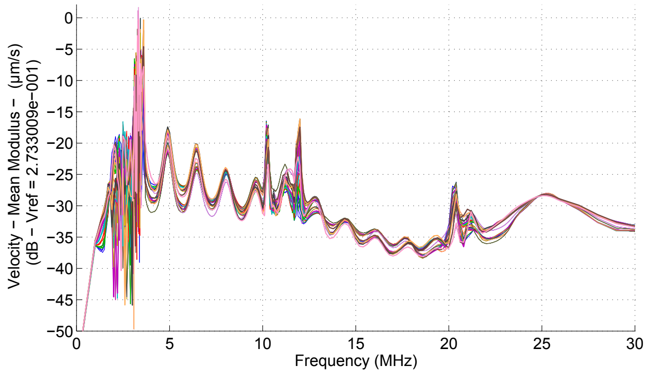

Figure 2: Velocity for all cells of element 8

3.2 Reference solution and analysis

We simulated, with the aforementioned model, the vibroacoustic response of the array subject to a 1V alternating voltage U_{ac}, applied to all elements without phase shift. We retained the first six membranes modes in the mechanical model and considered an interaction radius of 910 μm to compute the acoustic coupling terms. This means that each membrane is acoustically coupled with its fifteenth closest neighbors in each direction (in the case where the spacing between membranes is 10 μm). This leads to compute, for each frequency, $559 \times 6 \times 6$ different coupling terms to build the matrix $[A(\omega)]$. As the number of modes and the interaction radius seems reasonably large, we called this model the reference model. We will analyze the results of this reference model simulations in the rest of this paragraph.

The mean velocity modulus per cell of the 80 cells of element 8 are represented on figure 2. Between 2 MHz and 4 MHz we see a lot of sharp peaks, with no superposition of the curves, which is characteristic of underlying localized array modes. In contrast between 4MHz and 30 MHz, we can see that the velocities are much homogeneous across the element. The oscillation of the frequency response curve indicates the presence of global array modes.

The modal contributions are shown for a cell in the middle of element 8 (cell 610) on figure 3.

We can see that the localized array modes between 2 and 4 MHz involve the first membrane mode. This is confirmed by figures 4 and 5. Moreover the response between 4 and 19Mh is mainly dominated by the first mode. The smooth peaks of the response curve are due to global array modes involving the first membrane modes. This is illustrated, as an example by the velocity field for the 4.9 MHz peak represented figure 6.

The participation of the second membrane mode is very localized around 10.2 MHz and 11.8 MHz. The velocity field for 10.3 MHz is represented figures 7 and 8. The

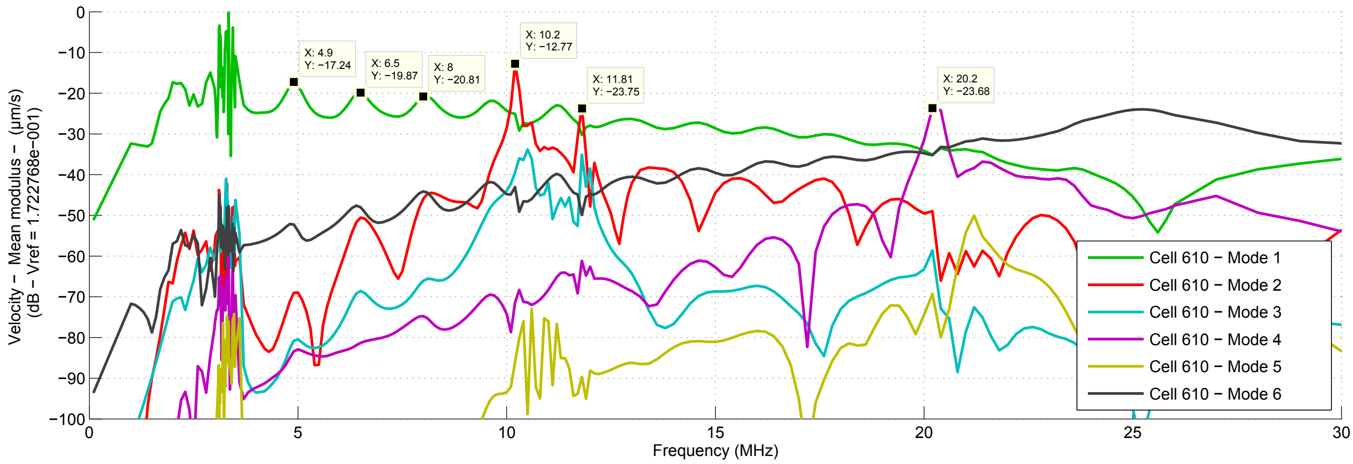

Figure 3: Modal contributions for cell 610

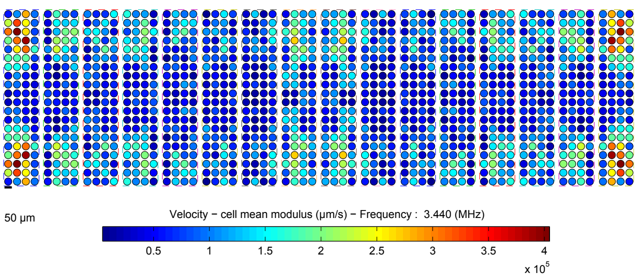

Figure 4: Velocity field at 3.44MHz - Mean velocity modulus per cell

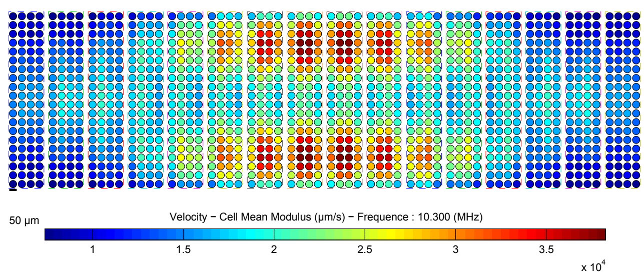

Figure 7: velocity field at 10.3 MHz - Mean velocity modulus per cell

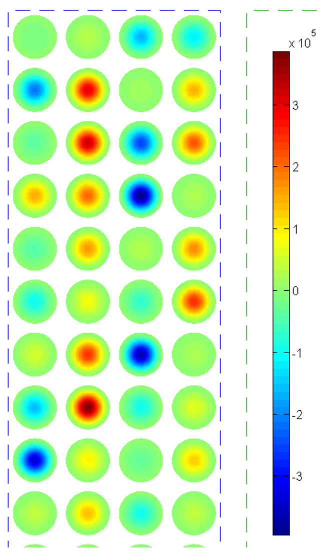

Figure 5: Zoom Velocity field (Imaginary Part) at 3.44 MHz for the upper part of element 1

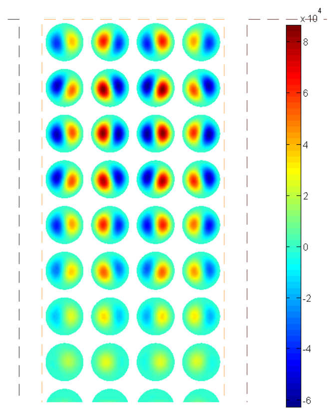

Figure 8: Zoom Velocity field (Real Part) at 10.3 MHz for the upper part of element 8

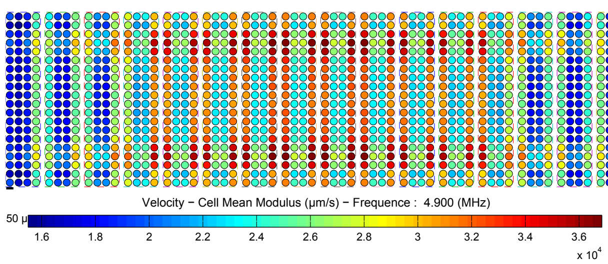

Figure 6: Velocity field at 4.9 MHz - Mean velocity modulus per cell

participation of the fourth membrane mode is also localized around 20.3 MHz. The is confirmed by figure 9. All the other modes have a negligible participation in the response.

The pressure modulus, at the distance of 3 cm perpendicular to the center of the array is presented figure 3. All the pressure peaks correspond to maximum of velocity on the array. The localized velocity patterns between 2MHz and 4 MHz do not correspond to the highest pressure peak. Instead, the maximum pressure peak for 4.9MHz, correspond to a more global velocity pattern represented figure 6. It is also not surprising to notice that the membrane modes 2 and 4 are poor acoustic radiators.

The directivity diagram at 3 cm from the array is represented for frequencies 2.44MHz and 4.9 MHz on figure 11.

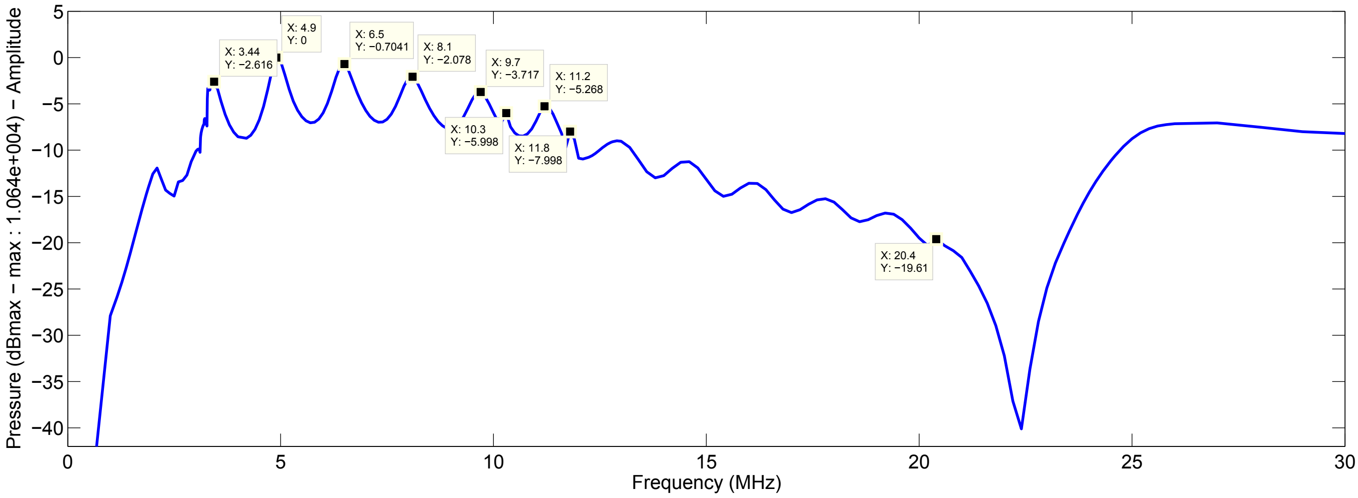

Figure 10: Pressure modulus in front of the array at 3 cm.

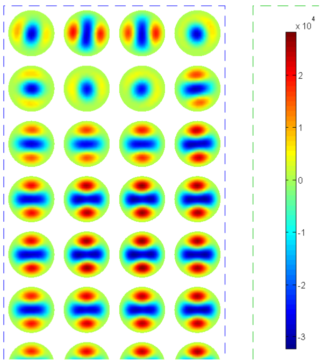

Figure 9: Zoom Velocity field (Imaginary Part) at 20.4 MHz for the upper part of element 1

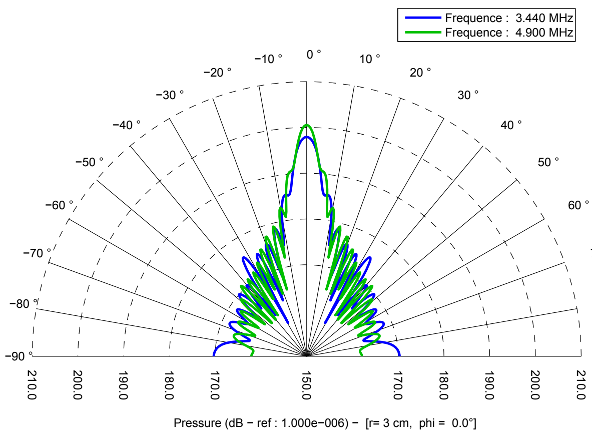

Figure 11: Directivity for 3.44MHz and 4.9 MHz

3.3 Coupling radius influence

The same simulation as with the reference model has been perform, but with interaction radius respectively equals to $10\,\mu m$, $100\,\mu m$ and $300\,\mu m$. The number of coupling terms to compute for each frequency is respectively equal to 1 (for $10\,\mu m$), 6 (for$100\,\mu m$), 63 (for $300\,\mu m$) and 559 (for $910\,\mu m$). The mean velocity modulus for cell 610 is compared for all cases to the reference model on figure 12. The pressure modulus at the distance of 3 cm perpendicular to the center of the array is also compared for all cases to the reference model, on figure 13. We can see that the interaction radius plays a very important role. It strongly affects the dynamics of the array, thus modifying the radiated acoustic pressure. The participation of the higher membrane modes as well as the localized array modes around 3 MHz can be approach with a relatively small interaction radius as $300\,\mu m$. However, the global array modes can only be correctly predicted with large interaction radius. Unfortunately, the highest level of radiated pressure are associated with the global array modes.

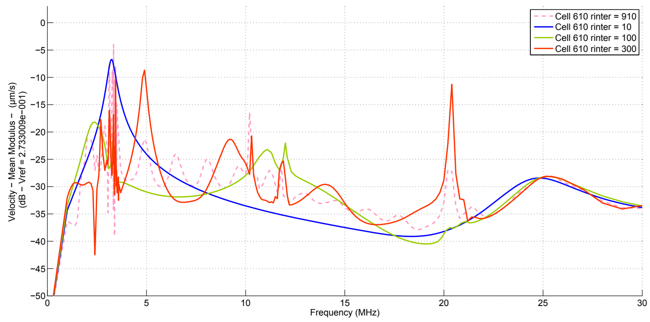

Figure 12: Cell 610 - mean velocity modulus for various interaction radius

3.4 Kinematic constraints on membranes displacements

We propose here to simplify the initial model by applying a kinematic constrain on the displacement of the cells. We forced all the membranes of an element to have the same displacement. From the computational point of view this hypothesis can reduce drastically the size of the system to

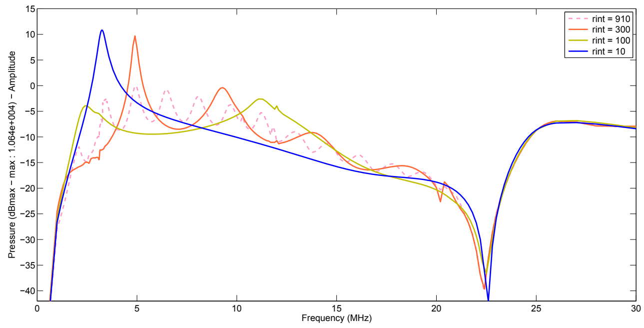

Figure 13: Pressure modulus for various interaction radius

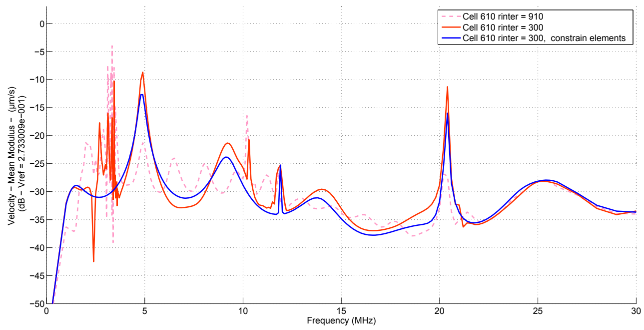

Figure 14: Cell 610 mean velocity modulus with kinematic constrain

be solved. The effect of this constrain on the cell 610 mean velocity modulus and on the radiated pressure are shown on figure 14 and 15. These constraints have more an effect on the localized modes than on the global arrays modes. They can thus provide reasonable approximation for the prediction of radiated pressure.

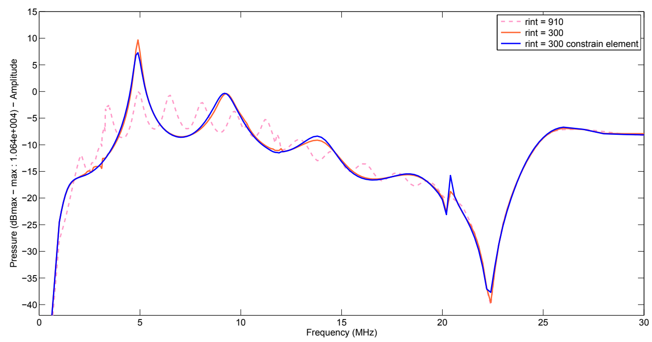

Figure 15: Pressure modulus computed with kinematic constrain

4 Conclusion

We developed a model able to predict the vibroacoustic response of CMUT arrays. The model represent explicitly all the membranes and most of the physics is taken into account. The number of membranes modes and interaction radius are the only parameters to be chosen by the user. The model has been successfully applied to a representative 1D CMUT array of 1280 membranes. A parametric study on the interaction radius has been conducted, that shows that relatively large interaction radius have to be chosen. Also kinematic constraints can be useful to approximate the radiated pressures for very large arrays.

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