# Coupled lumped and boundary element simulation for electro-acoustics

J. Panzer

R&D Team Joerg Panzer, Raiffeisenstr 5, 87775 Salgen, Germany
j.panzer@randteam.de

This paper gives a technical report on a strategy to couple Lumped and Boundary Element Methods for acoustics. The solution makes use of the superposition-principle and the use of self and mutual radiation impedance components. By this, the coupling approach features the convenient property of a high degree of independence of both domains. For example, one can modify parameters and even, to some extent, change the structure of the lumped-element network without the necessity to resolve the boundary element system. This paper gives the mathematical derivation and a demonstration-example, which compares calculation results versus measurement. In this example electronics and mechanics of the three involved loudspeakers are modelled with the help of the lumped element method. Waveguide, enclosure and radiation is modelled with the boundary element method.

# 1  Introduction

For electro-acoustic design work the combination of lumped and boundary element methods proves advantageous. This is so, because simulation of lumped networks is quick, and its components usually reflect its dominant physical properties. On the other hand, the boundary element method provides an easy way of modelling acoustic fields at complex boundary conditions.
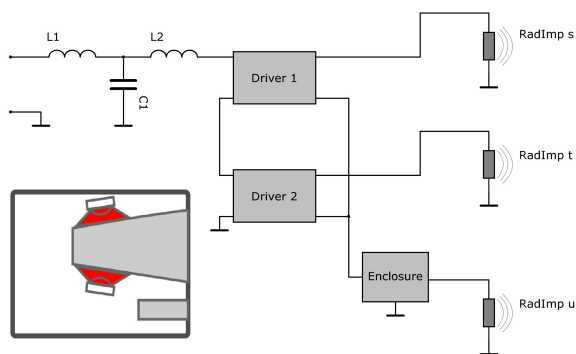


Figure 1: Example horn loaded vented loudspeaker cabinet.

The above sketch shall help introduce the problem by example. Displayed is a complicated device where two loudspeaker-drivers are radiating into a horn. The rear of these drivers share the volume of a vented cabinet, which port also radiates into the open. In this example, the horn and the exterior radiation problem is modelled with the boundary element method, whereas the interior-acoustics of the cabinet is modelled with the help of lumped elements. Additionally, there are electrical and mechanical components of the driver and electrical components of a cross-over filter.

Devices like this usually provide strong acoustical coupling. In this example, coupling is expected at the Helmholtz resonance and at higher modes of the enclosure, via the horn and its modes and between the horn and the port.

For swift simulation-work, the coupling of the lumped and boundary element network should be designed in such a way that structure and parameters of the lumped element network can be modified without the necessity of resolving the boundary element system because the latter is usually time-consuming. The following strategy of coupling meets this requirement.

This documentation assumes familiarity with both the lumped and boundary element methods. Further readings and ample details can be found in the available literature, some are listed in chapter References. The following coupling-approach is rather general and can readily be adopted to various schemata of the boundary element method.

In a way, the lumped and boundary element methods are similar. Typically there are three stages: 1) Defining the elements. 2) Assembling and solving the system-matrix for potentials. 3) Calculation of observation parameters.

The lumped element method usually uses pre-calculated elements, which reflect the dominant features of the component at its ports. All elements are assembled into a system-matrix, which, if we use the Nodal Formulation [3], favours admittance-functions of an element. The Nodal Formulation can be extended to handle the electrical, mechanical and acoustical domain if potentials are voltage, force and (sound) pressure, and if flows are current, velocity and the volume-velocity [4]. The product of potential and flow should yield a value of power. The system-matrix needs to be solved for nodal-potentials, whilst typically driven by a voltage source. Finally, observation-parameters can be calculated with the help of these nodal potentials and the admittance-functions of the components. Examples for observation parameters are the excursion of the loudspeaker-voice-coil, the current through the coil and so on.

The boundary element system usually creates its elements by a dedicated mesh-utility in the first place. This tool is discretising the surface-area of the boundaries into elements of small size, so small that the variations of the acoustic field within can be regarded constant. In this way the integrals of the Helmholtz Integral can be calculated and its results assembled into the system matrix of the boundary element part. We solve for potentials, i.e. sound pressures, at the surface of each element. Here, the observation-stage typically is concerned with sound-pressure-values away from the boundary. This can be achieved by calculation of the Helmholtz Integral for receiving points in the field and source-points taken from step (2).

# 2  Driving

In this documentation driving is understood by means of a diaphragm, for example a loudspeaker cone. There can be multiple diaphragms in a system. All boundary-elements, which belong to a certain diaphragm, are organised with the help of driving-groups with indices k or s, t, u.
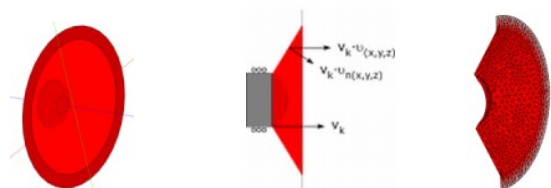


Figure 2

In the context of this documentation the motion of the diaphragm is associated to the principal rigid-body mode of

the device. For example, for the above cone-loudspeaker, we use the mechanical velocity $v_k$ of the diaphragm at the voice-coil former in axial direction. Additionally the diaphragm may feature a velocity distribution $\upsilon$, which is normalised in such a way that a uniform velocity-distribution would yield $\upsilon = 1$. The local mechanical velocity at a certain position r on the diaphragm is

$$v_k(r) = v_k \cdot \upsilon_k(r) \tag{1}$$

The flow-parameter of the acoustical part of the lumped element network is the volume-velocity, V, which we define according to the principal motion of the diaphragm as

$$V_k = v_k \cdot S_k' \tag{2}$$

with

$$S_k' = \int_{S_k} \upsilon(r_0)\, dS_0 \tag{3}$$

$S_k$ is the surface area of all boundary elements, which belong to the diaphragm with index k. $S_k'$ is the effective and projected area of the diaphragm in direction of the principal rigid body motion taking into account a possible distribution of the normalised velocity $\upsilon$.

Note, that here there is only the rigid-body-coupling and not a structural-vibrational coupling. Thus any velocity-distribution is regarded static, i.e. not altered by the acoustic loading, and may arise from analytics or from measurement.

The vibration of all diaphragms causes an acoustic pressure-field, which is given by the Helmholtz Integral

$$c(r_s) \cdot p(r_s) = \tag{4}$$
$$j\rho c \cdot k \cdot \sum_k v_k \cdot \int_{S_k} G(r_s, r_0) \cdot \upsilon_{nk}(r_0)\, dS_0 -$$
$$\int_S G_n(r_s, r_0) \cdot p(r_0)\, dS_0$$

$\upsilon_n(r_0)$      Projection of the normalised velocity $\upsilon(r_0)$ in direction of the normal-vector of the element.

$v_k$      Mechanical lumped velocity of diaphragm with index k, otherwise Neuman boundary conditions assumed here.

$\rho c$      Product of air density and speed of sound.

$k = \omega/c$      Wave-number, with angular frequency $\omega$.

$r_s$      Position of any receiving point.

S      Total area of all boundary elements ($S \geq S_k$).

$p(r_s)$      Sound pressure at receiving point $r_s$.

$c(r_s)$      Free Term, c = 1 away from the boundaries otherwise needs to be calculated [1].

$G(r_s, r_0)$ and $G_n(r_s, r_0)$ are Green-functions.

# 3    Coupling

Because each diaphragm radiates and receives, these areas will influence each other. The coupling is two-fold. First, via the acoustic domain, where a sound-pressure is produced by one diaphragm and received by another, which in turn induces motion at the receiver. Second, via the lumped element network, where motions are linked by components, which velocities eventually cause radiation. All parameters should be in balance according to physical rules and boundary conditions.

Thinking in terms of lumped elements we seek to derive a component, which stands for the coupling. This component is called acoustic radiation-impedance, which relates sound-pressure, p and volume velocity, V:

$$z_a = \frac{p}{V} \tag{5}$$

Assuming we know $z_a$ and the potentials of the lumped element network, which in the acoustic domain are pressures, the idea is, that we would be able to calculate velocities via V = $p/z_a$. Further, by having a measure for the velocity, we can also calculate field-values with the help of the Helmholtz Integral, equation (4).

## 3.1    Field Radiation Impedance

Using the idea of an acoustic radiation-impedance and by assuming linearity, the formula for the total sound pressure at a field-point could be expanded

$$p(r_s) = \sum_k z_{ak}(r_s) \cdot V_k \tag{6}$$

In order to be able to use such a formula we would need to have available independent functions for $z_{ak}(r_s)$. This can be achieved by solving the Boundary Element Matrix several times, each time with another diaphragm driving the acoustic field exclusively. Then single terms of equation (6) are available.

Starting with a modular version of equation (4):

$$c(r_s) \cdot p_t(r_s) = \tag{7}$$
$$j\rho c \cdot k \cdot v_t \cdot \int_{S_t} G(r_s, r_0) \cdot \upsilon_{nt}(r_0)\, dS_0 -$$
$$\int_S G_n(r_s, r_0) \cdot p_t(r_0)\, dS_0$$

with $p_t(r_0)$ surface-pressures of the solution of the Helmholtz Integral for driving the boundary element system with only diaphragm with index t active. Note, that S of the second integral covers the total boundary-area of the domain.

The field radiation impedance $z_{at}$ from the diaphragm t to any field point at position $r_s$ is:

$$z_{at}(r_s) = \frac{p_t(r_s)}{V_t} \tag{8}$$

or by using equation (2)

$$z_{at}(r_s) = \frac{p_t(r_s)}{v_t \cdot S_t'} \tag{9}$$

with $p_t$ sound pressure calculated by equation (7) with $v_t = 1$ set at solving- and observation stage.

## 3.2    Lumped Radiation Impedance

So far, $z_{at}(r_s)$ is a field-value valid for a single point $r_s$. In order to be able to construct an acoustic lumped element network we would need a lumped version of the radiation impedance, equation (5). A reasonable ansatz for forming a mean- or lumped value is to derive the impedance from the acoustic power, which is received by one diaphragm, whereas another (or itself) is driven. The acoustic power from source t at diaphragm s is (the bar indicates conjugate complex):

$$P_{st} = \frac{1}{2} \cdot \int_{S_s} p_t(r_0) \cdot \bar{v}(r_0)\, dS_0 \tag{10}$$

or, similar to equation (9) and defining $z_{ast}$ lumped, yields

$$P_{st} = \frac{1}{2} \cdot z_{ast} \cdot S'_s \cdot v_s^2 \cdot \int_{S_s} \upsilon^2(r_0)\, dS_0 \tag{11}$$

By combining equations (1), (2), (10) and (11) and by assuming $v_t = v_s = 1$ yields the acoustic impedance from t to s

$$z_{ast} = \frac{1}{S'_s} \cdot \frac{\int_{S_s} p_t(r_0) \cdot \overline{\upsilon}(r_0)\, dS_0}{\int_{S_s} \upsilon^2(r_0)\, dS_0} \tag{12}$$

with $z_{ast}$ acoustic lumped radiation-impedance. If s = t $z_{ast}$ is called self-radiation impedance otherwise mutual radiation impedance.
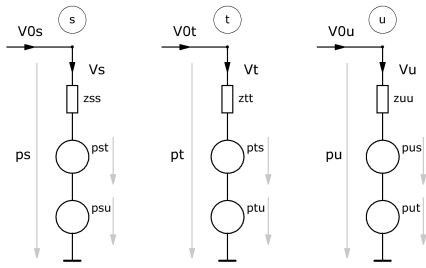
Figure 3

For each diaphragm we need to provide a branch with lumped radiation impedances. Only the self-term forms a classic network-element. The mutual radiation-impedance terms rather form flow-controlled potential sources, as in the example-circuit above, where three diaphragms interact. For this example Kirchhoffs' laws for flows and potentials at nodes s, t and u yield

$$\begin{bmatrix} p_s \\ p_t \\ p_u \end{bmatrix} = [Z_a] \cdot \begin{bmatrix} V_s \\ V_t \\ V_u \end{bmatrix} \tag{13}$$

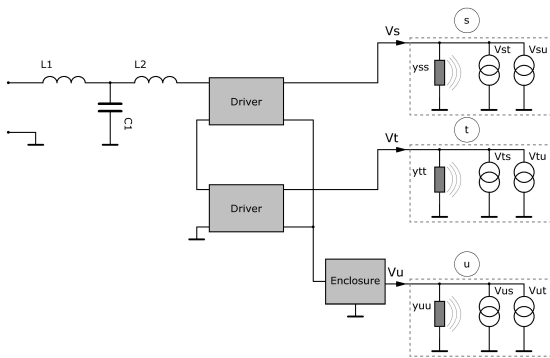with components of $[Z_a]$ given by equation (12).

Figure 4

For setting up the Nodal Formulation for lumped element analysis it is of advantage to transform the impedance matrix $[Z_a]$ into an acoustic admittance matrix $[Y_a]$:

$$[Y_a] = [Z_a]^{-1} \tag{14}$$

The diagonal-elements of $[Y_a]$ are the self radiation admittances and can be assembled into the Nodal Matrix as usual.

The off-diagonal elements of $[Y_a]$ are potential-controlled flow-sources as symbolised in the above circuit. These elements would be simply added to the Nodal Matrix at nodes indicated by the indices (s, t and u in the above example).

# 4 Radiation

After solving the lumped-element network for potentials, i.e. sound pressure values, $p_s$, $p_t$ and $p_u$ at node s, t and u would be known. We can then calculate the volume-velocities $V_s$, $V_t$ and $V_u$ with the help of $[Y_a]$:

$$\begin{bmatrix} V_s \\ V_t \\ V_u \end{bmatrix} = [Y_a] \cdot \begin{bmatrix} p_s \\ p_t \\ p_u \end{bmatrix} \tag{15}$$

Finally, with the help of equation (6) we are able to calculate the sound pressure at any field point, which would make necessary to apply the Helmholtz Integral for this point, equation (7). Note, that albeit we make use of the super-position-principle we only need to calculate the Helmholtz Integrals once for each position.

# 5 Verification

The above coupling algorithm has been implemented, with a lumped and boundary element solver [10]. The following example is an excerpt from a laboratory-project designed for the verification of the algorithm versus measurement.

The design of this experiment emphasises on electro-mechano-acoustic rigid-body coupling in presence of a complicated acoustic field. Care is taken to minimise issues outside of the modelling capacity of the solver, such as structural vibration.

Figure 5: Cabinet used in experiment

As can be seen in Figure 5, the system features three electro-dynamic loudspeaker-drivers and two acoustic domains, one interior and one exterior. In order to emphasise mutual-coupling all three drivers share a little cavity before exiting into open space.
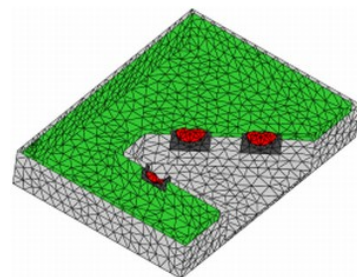
Figure 6: Meshing of cabinet

The boundary element method is used to model the interior and exterior field of the cavity. The sketch in Figure 6 shows the finite elements from the Delaunay mesh-generator. Top-bottom symmetry is exploited.
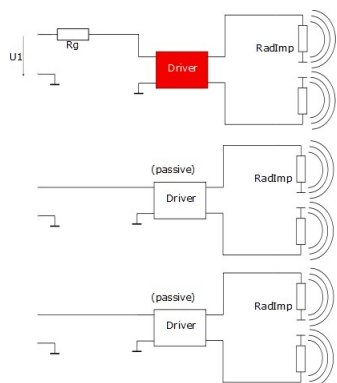


Figure 7: Lumped element network

The voltage-source, a generator resistance, three drivers and six radiation-impedance components form the lumped element network. Note, that elements for mutual radiation are hidden in Figure 7.

In this experiment, only a single driver is connected to the voltage-source (driver on the extreme right). The other two are passive and connected only via mutual radiation.

The following observations are spectral curves and overlay with the associated measurements.

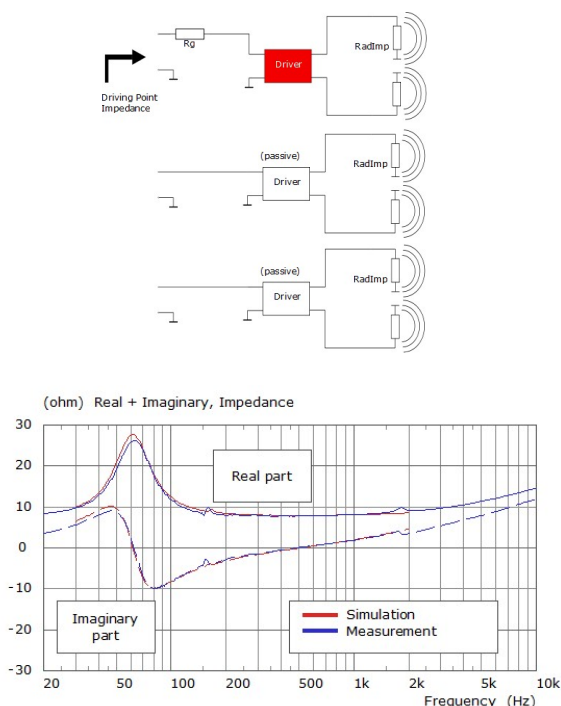## 5.1 Electrical Driving Point Impedance





Figure 8: Driving point impedance

For verification it is wise to use as many independent observation points as possible. A good start is the spectrum of the electrical driving point impedance. In Figure 8 the red curve is the simulation and the blue the spectrum of the measurement. The upper curve is the real part of the impedance-spectrum and the lower curve shows the imaginary part. This can be regarded as a good fit. The little

kinks in the measurement curve are due to structural-resonances of the diaphragm and its supporting devices.

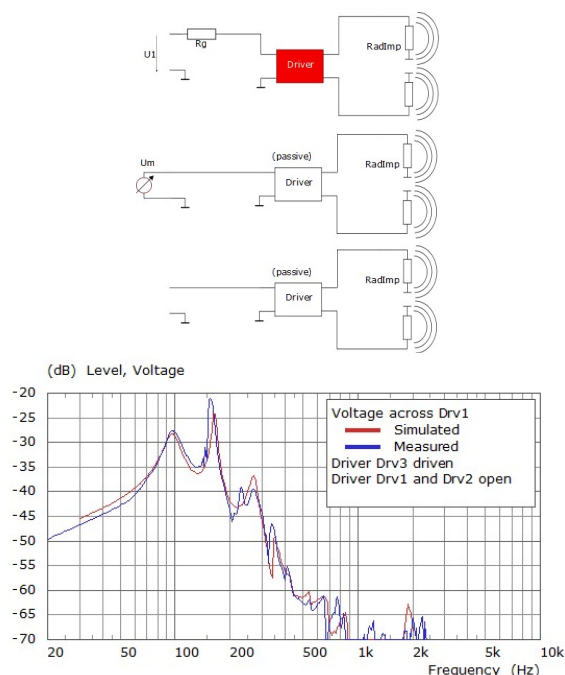## 5.2 Voltage at Driver Terminals

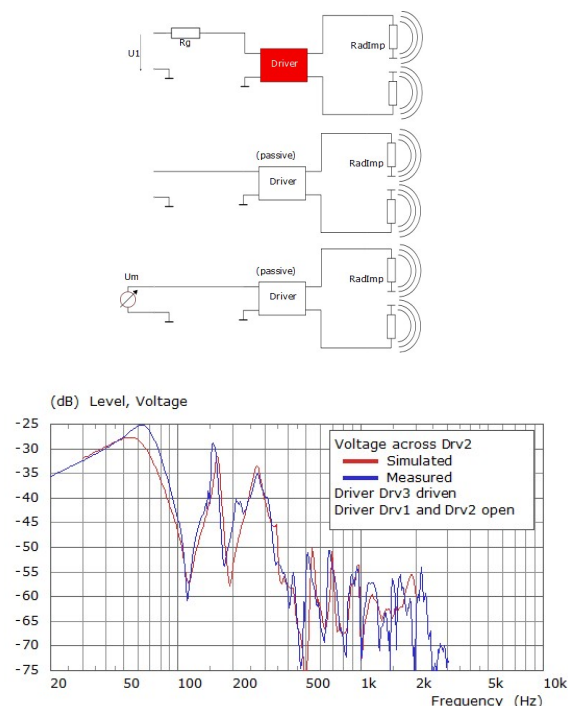



Figure 9: Voltage response at Drv1





Figure 10: Voltage response at Drv2

Figure 9 and 10 are observations of the open-loop voltage at the terminals of the passive speaker-drivers. Hence, these drivers act as a microphone. The simulation curves follow closely the measurement. Above 1.5 kHz the simulation-quality could be enhanced by increasing the mesh-density, however on cost of calculation time.
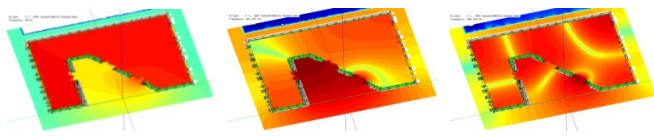
## 5.3 Field Pressure



Figure 11: SPL-fields at 50 Hz, 261 Hz and 560 Hz

The field-plots of Figure 11 display the SPL contours for a range of 100 dB on 2 dB steps. The 50 Hz plot reveals the active driver. In the 261 Hz plot we can see the strong resonance of the funnel. And the 560 Hz plot features standing waves in the outer domain against an infinite wall, which is also part of the model.
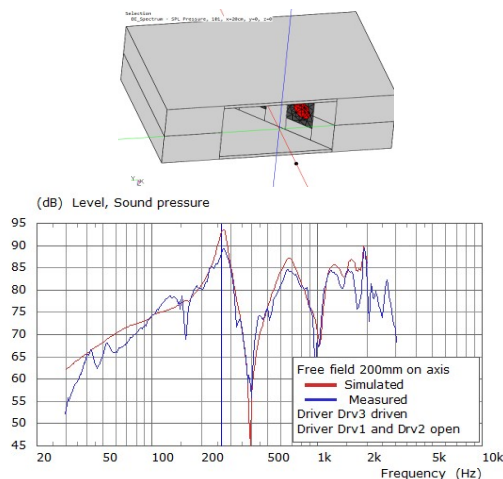


Figure 12: SPL spectrum

Finally the sound pressure spectrum at a field-point at 200 mm in front of the funnel is compared to measurement as displayed in Figure 12. The deviations at low frequencies are due to reflections, which are caused by the measurement chamber.
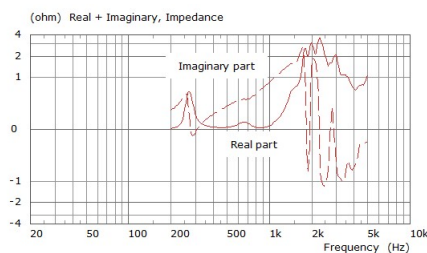
## 5.4 Radiation Impedance



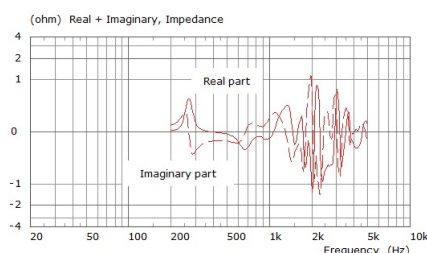Figure 13: Self Radiation Impedance Drv1 front



Figure 14: Mutual Radiation Impedance Drv1 - Drv3 front

It is interesting to observe the radiation impedance. The plot of Figure 13 displays the self-radiation-impedance of driver Drv1 (the driver on the very left). The peak at 270 Hz is due to a dominant funnel-mode. Towards 2 kHz the impedance curve approximates a typical impedance curve of a rigid diaphragm, however, emphasised by the duct resonances and various cabinet modes. Figure 14 shows the interaction between Drv1 and Drv3.

## 6 Conclusions

This paper shows the application of super-position and radiation-impedance-components in order to provide coupling of a lumped-element network and the boundary-element method for acoustics. The advantage is the possibility to apply the lumped network at observation stage of the boundary element method. The draw-back is that application of super-position makes it necessary to solve the system-matrix of the boundary element domain for each driving exclusively. Further development goes into using laser-measurements for the velocity-distribution of a diaphragm.

## Acknowledgements

## References

[1] Ciskowski R., Brebbia C.: *Boundary element methods in acoustics*; Elsevier Southampton 1991

[2] Brebbia C., Telles J., Wrobel L.: *Boundary Element Techniques*; Springer-Verlag, 1984

[3] Vlach J., Singhal K.: *Computer methods for circuit analysis and design*; Van Nostrand 1983

[4] Panzer J.: *AkAbak - An Electroacoustic Network Simulator*; IOA Reproduced Sound 12, Oct 1996 (*)

[5] Bauer B.: *Equivalent circuit analysis of mechano-acoustic structures*; JAES Vol 24 Oct 1976

[6] Lampton M.: *Transmission matrices in electroacoustics*; ACUSTICA Vol 39 1978

[7] Murphy D.: *Modelling compression drivers using T matrices and finite element analysis*; AES Convention paper 6580 Oct 2005

[8] Makarski M.: *Determining two-port parameters of horn drivers using only electrical measurements*; AES Convention paper 6098 May 2004

[9] Cremer L., Heckl M.: *Körperschall* or *Structure borne sound*; Springer 1996

[10] *ABEC - Acoustic Boundary Element Calculator*; Software application (*)

(*) available at www.randteam.de