Identification of circumferential acoustic waves propagating around the tube by multiresolution analysis and time-frequency representation

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This paper describes a technique based on Multiresolution Analysis (MRA) of the wavelet transform. This technique is applied for decomposition of the original acoustic signal backscattered by a thin tube. The Multiresolution technique was used as a tool to filter the wave modes contained in the original signal. The time-frequency representation using the Smoothed Pseudo-Wigner-Ville (SPWV) distribution is applied on the decomposed acoustic signal. The results obtained show that this technique of decomposition can identify not only single circumferential wave mode but also multimode effectively. This methodology permits to obtain the interesting results.

1 Introduction

The present paper is specially concerned with the application of the Multiresolution analysis of the wavelet transform on the acoustic signal backscattered by a thin tube, The MRA concept was initiated by Meyer [1] and Mallat [2-3], which provides a natural framework for the understanding of wavelet bases. The wavelets transform are essentially applied to extract information and as a basis for signal representation. The choice of the basis function determines the information to be extracted from the process and allows the study of different signal structures, such as non-stationary short-variation transient components. The process representation using wavelets is provided by a series expansion of dilated and translated versions of the basis function, also called the “mother” wavelet, multiplied by appropriate coefficients [2,4-5]. For processes with the wavelet transform gives an approximation to the original signal.

In this work we explore the potential of Daubechies wavelet Multiresolution analysis for analysis and information extraction from an acoustic signal [6]. The main scientific objective of the work is to examine the Daubechies wavelet representation as a matching filter for to extract characteristics of interest such as identification of the circumferential waves. The wavelet coefficients are obtained from the high-pass and low-pass filter, The two filters are iteratively applied from the finest to the coarsest scales to estimate the wavelet coefficients

2 Acoustic scattering by an elastic tube

2.1 Complex backscattering pressure by a tube

The study of the acoustic scattering by targets of simple geometrical shape is the object of many works [5,7-10]. In this paper we will use a plane harmonic wave incident on an infinite tube of radius ratio b/a with an air-filled cavity (Fig. 1). The mathematical approach of impulse response of tube is based on the Rayleigh series formations that consist in the decomposition of the backscattered pressure field into infinite sum of modal components, depending on both the mechanical proprieties and the geometry of the target.

The general geometry for the backscattering of a plane wave by a tube is illustrated in figure 1.

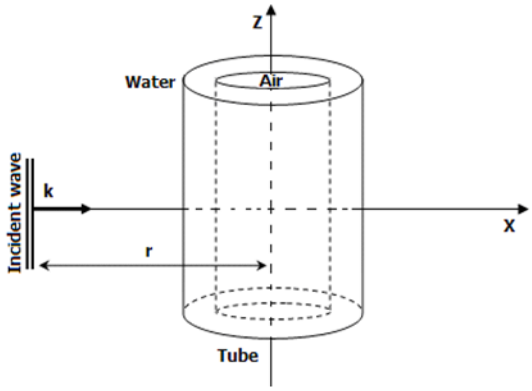

Figure 1: Geometry of the problem.

The backscattering complex pressure in far filled of the cylindrical shell immersed in water (Fig. 1) is given by the expression [7].

$$P_{scat}(\omega) = P_0 \sum_{n=0}^{\infty} \frac{D_n^{[1]}(\omega)}{D_n(\omega)} H_n^{(1)}(kr) \tag{1}$$

Where ω is angular frequency, k=ω/c is the wave number with respect to wave velocity in the external fluid and c is the velocity of sound in water. P0 is the amplitude of the plane incident wave and r is the position of the receiver. $D_n(\omega)$ and $D_n^{[1]}(\omega)$ are determinants computed from the boundary conditions of the problem (continuity of radial stress and displacements of both interfaces). $H_n^1(kr)$ is the Hankel function of the first kind.

The figure 2 illustrates the theoretical backscattering spectrum of an air-filled Aluminium tube immersed in water; this spectrum is obtained theoreticaly by the expression (1).

This spectrum is presented in function of the dimensionless frequency ka (without unit) witch defined by the expression below [7,11] :

$$ka = \frac{2\pi fa}{c} \tag{2}$$

2.2 Temporal signal backscattered by a tube

The backscattering spectrum in figure 3 shows the presence of a continuous level on which take shape abrupt transitions corresponding to resonances.

The time signal $s(t)$ of a tube is calculated by the inverse Fourier transform of the backscattering spectrum.

$$s(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}h(\omega)P_{scat}(\omega)e^{-i\omega t}d\omega \qquad (3)$$

Where $h(\omega)$ is the band-pass of the transducer.

To obtain the resonance spectrum; (i) in first time a theoretical temporal signal is calculated from the backscattered spectrum with an inverse Fourier transform, it corresponds to the time signal observed when the tube is excited with a broadband impulse (Fig. 3); on this time signal, various echoes related to the circumferential waves are observed, (ii) in second time the specular echo related to the reflection on the tube is suppressed with a Personal Computer and replaced by zeroes (Fig. 3); (iii) in third time a new Fourier transform is applied to the filtered time signal.

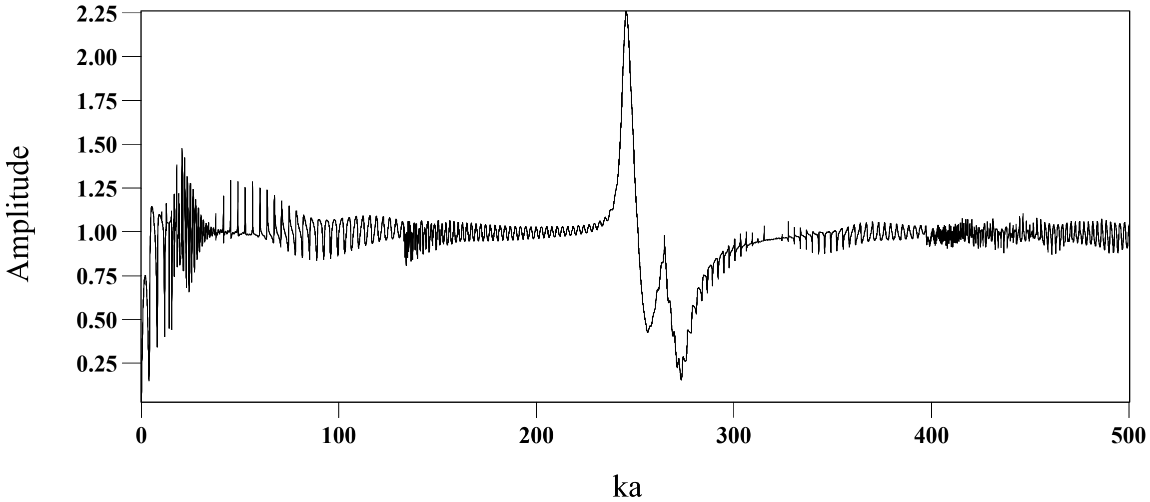

Figure 2: Backscattering spectrum of an air-filled aluminum cylindrical shell immersed in water, b/a = 0.95, a=3cm

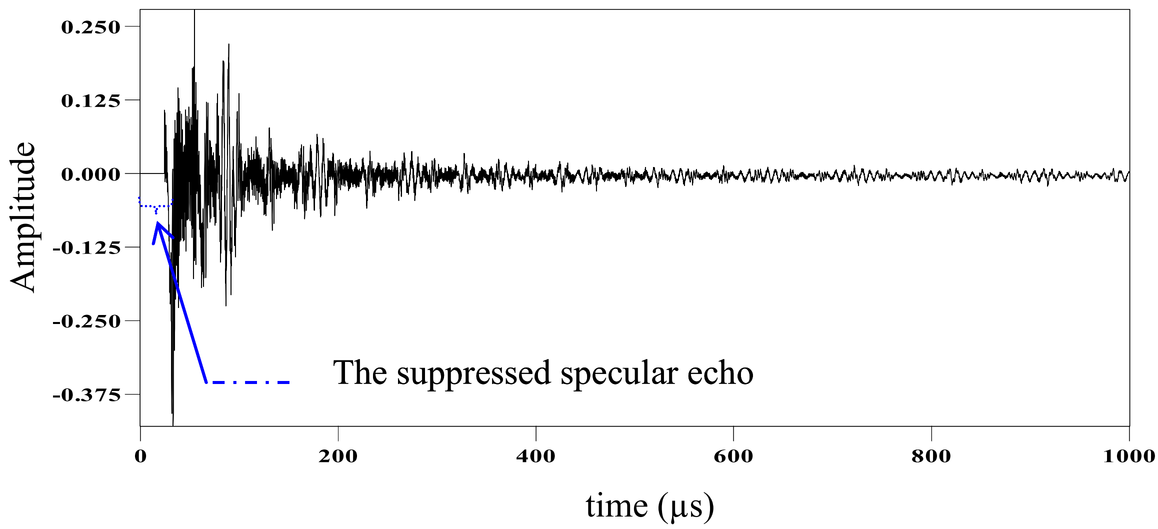

Figure. 3: Temporal signal of an aluminum tube b/a=0.95, with suppressed specular echo.

3 Multiresolution analysis and time-frequency representation

3.1 Multiresolution analysis (MRA)

The most important time-frequency transformations is the Short-Time Fourier Transform (STFT). It uses a windowing technique to analyze a small section of the signal in discrete-time [12], but the wavelet ψ , is a function used in the time scale transformations, where a comparison of the signal is done among a collection of elements where the number of oscillations remains constant when the time support (time where mother wavelet is defined) decreases. This function is dilated with a scale parameter a, and translated in time by b. These scaling and translations produce a collection of functions denoted by [2,4-6] :

$$\psi_{a,b}(t)=\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right), a,b\in\mathbb{R}, a\succ 0 \qquad 4)$$

$\psi_{a,b}(t)$ is defined as a continuous wavelet, which is derived from a mother wavelet $\psi(t)$.

The Continuous Wavelet Transform (CWT) is obtained by computing the correlation of the signal x(t) and the scaled and translated mother wavelet $\psi(t)$:

$$WT(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi\left(\frac{t-b}{a}\right) dt \qquad (5)$$

CWT, just as STFT, transforms one-dimensional information contained in a signal into two-dimensional plane. This means that there are redundant information contained in the time-frequency plane. The best sampling of the CWT is determined by the coefficients $WT(a,b)$ that allow a perfect reconstruction of the signal x(t). This perfect reconstruction is obtained by [2-3] :

$$a = 2^j \; ; b = k2^j \quad with \quad k, j \in \mathbb{Z}$$

This sampling creates a plane called dyadic and introduces the Discrete Wavelet Transform (DWT) defined by:

$$DWT(j,k) = c_{j,k} = \int_{-\infty}^{+\infty} x(t) \psi_{j,k}(t)\, dt \qquad (6)$$

Where

$$\psi_{j,k}(t) = \frac{1}{\sqrt{2^j}} \psi\left(\frac{t-2^j k}{2^j}\right)$$

The definition of the MRA is given by [2-3]: a sequence $\{V_j\}$ of closed subspaces of $L^2(R)$ is a Multiresolution Analysis if the following properties are satisfied:

- $\forall (j,k) \in \mathbb{Z}^2,\ x(t) \in V_j \Leftrightarrow x(t-2^j k) \in V_j$, any translation of a function belongs to the same space.
- $\forall\, j \in \mathbb{Z},\quad V_{j+1} \subset V_j$, the subspace embodied at level j+1 is contained in the last subspace j.
- $\forall\, j \in \mathbb{Z},\ x(t) \in V_j \quad \Leftrightarrow \quad x(\frac{t}{2}) \in V_{j+1}$, it denotes that V_{j+1} consists of all rescaled functions in V_j.
- $\lim\limits_{j \to +\infty} V_j = \bigcap_{j=-\infty}^{+\infty} V_j = \{0\}$ the intersection at all subspaces is $\{0\}$.
- $\lim\limits_{j \to +\infty} V_j = \text{Closure}\left(\bigcap_{j=-\infty}^{+\infty} V_j\right) = L^2(\mathbb{R})$ the union of all subspaces creates $L^2(\mathbb{R})$.
- There exist $\varphi \in V_0$ such that $\{\varphi(t-n),\ n \in \mathbb{Z}\}$ is a Riesz basis of V_0, which contains the approximation at level 0.

3.2 Smoothed Pseudo Wigner-Ville distribution

Wigner-Ville distribution (WV) is a time-frequency analysis method which interprets a given signal in time domain and frequency domain synchronously. It can exam how frequency content changes as a function of time, and the output of the distribution is the energy intensity of various frequency components of the signal at given points in time. Because of the existence of the nonzero interferences term in WV which are bad for the explanation for the results, in practice, it could be replaced by smoothed pseudo Wigner-Ville distribution (SPWV), in which some window functions are convolved with WV to restrain and decrease the effect of the interference terms. Supposing x is the time domain signal, then SPWV of x is given by [5,13]:

$$SPW_x(t,\nu) = \int_{-\infty}^{+\infty} \left| h(\frac{\tau}{2}) \right|^2 \int_{-\infty}^{+\infty} g(t-u)\ x(u+\frac{\tau}{2})\ x^*(u-\frac{\tau}{2})\ e^{-2i\pi\nu t}\, d\tau \qquad (7)$$

where x* is complex conjugate of x , h and g are time and frequency smooth window function respectively.

4 Results and discussion

4.1 MRA of the acoustic signal backscattered by a thin tube

The most important aspects of the Multiresolution analysis is that the detail to Discrete Wavelet Transform. It means that the decomposed acoustic signal at different scales, by the Multiresolution analysis, contains necessary information to determine the important parameters presented in the original acoustic signal.

4.1.1 Application of MRA on the original signal

The high-pass and low-pass filters of the Daubechies wavelet transform in the order 8 are applied to the current signal. The coarsest level of the decomposition analysis is the order three. The temporal series correspond to approximation and to detail coefficients are illustrated in the Fig. 7. In the anther hand, the acoustic signal is successively convolved with the two filters low and high frequencies. Each resulting temporal signal is decimated by suppression of one sample out of two. The low frequency signal is left, the high frequency signal is right.

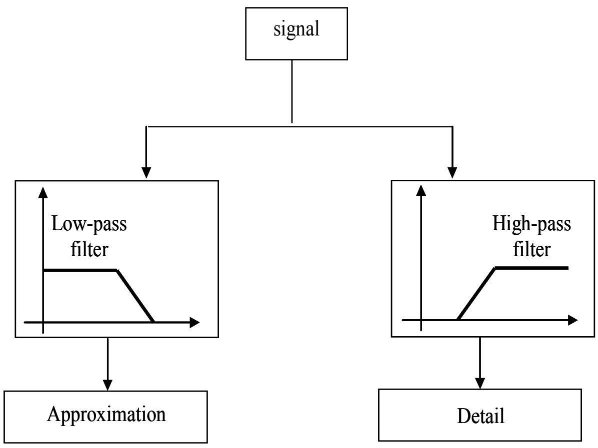

Figure. 4: Decomposition, approximation and detail of the signal.

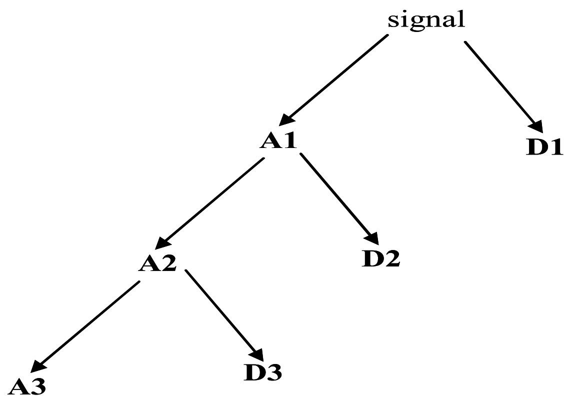

Figure. 5: wavelet decomposition of signal of 3 levels.

The approximation is the high-scale, low-frequency components of the signal. The details are the low-scale, high-frequency components. The filtering process, at its most basic levels, is given in Fig. 4.

The original signal, pass through two complementary filters and emerges as two signals: signal approximation "A" and signal detail "D" (Fig. 4).

The decomposition process can be iterated, with successive approximation being decomposed in turn, so that one signal is broken down into many lower-resolution components. This is called the wavelet decomposition (Fig. 5).

4.1.2 Time-frequency images of the composed original signal

The filter bank associated with the multiresolution analysis are shown in the Fig. 6 for the scale level j=1. The first signal corresponds to the original acoustic signal, the left temporal signal correspond to the approximation coefficients Ap1 (low frequency) and the right temporal signal series correspond to the detail coefficients (D1) (high frequency).

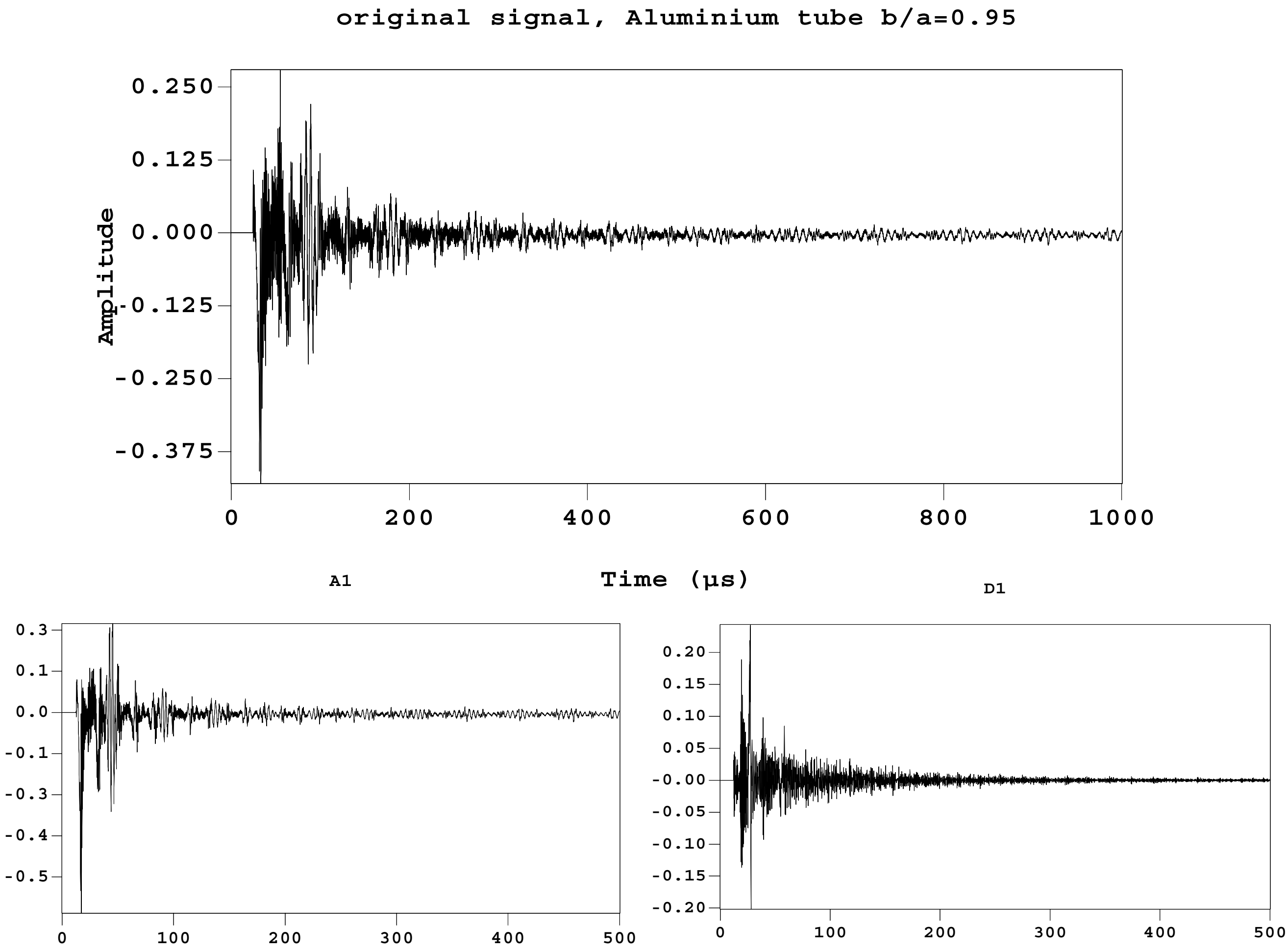

Figure. 6: Filter bank associated with the Multiresolution analysis, approximation coefficients Ap1 (left) and Detail coefficients D1(right) with the scaling function and the Daubechies wavelet the order 8

The original acoustic signal undergoes with a high-pass and a low-pass filters (filter bank), resulting the approximation Ap1 and the detail D1 corresponds to low frequency signal and the high frequency signal respectively Fig. 6A1 and Fig. 6D1 . The SPWV time-frequency method is applied on the approximation Ap1 and the detail D1 (level 1), the time-frequency images are illustrated in the figure 8. According to obtained images of the SPWV (Fig 7a and 7b), the low frequency signal corresponds to Scholte wave (A) and shell waves (S0, A1) (Fig. 7a), the reduced frequency scale ka in which the Scholte wave A appears is $0 < ka < 25$, the symmetric circumferential wave S0 is $26 < ka < 120$ and the antisymmetric circumferential wave A1 (Fig. 7b) is 132 < ka < 210. The high frequency signal corresponds to symmetric circumferential waves (S1, S2) and the antisymmetic circumferential wave A2, the reduced frequency scale ka in which the S1 and S2 waves appear is 250 < ka <380 and the range of the reduced frequencies corresponding to the wave A2 is , the symmetric circumferential wave S0 is 26 < ka < 130.

The synthetic time-frequency images ensure that the approximation Ap1 behave the Scholte (A), S0 and A1 waves and the signature temporal corresponding to the detail D1 behave the S1,S2 and A2 circumferential waves.

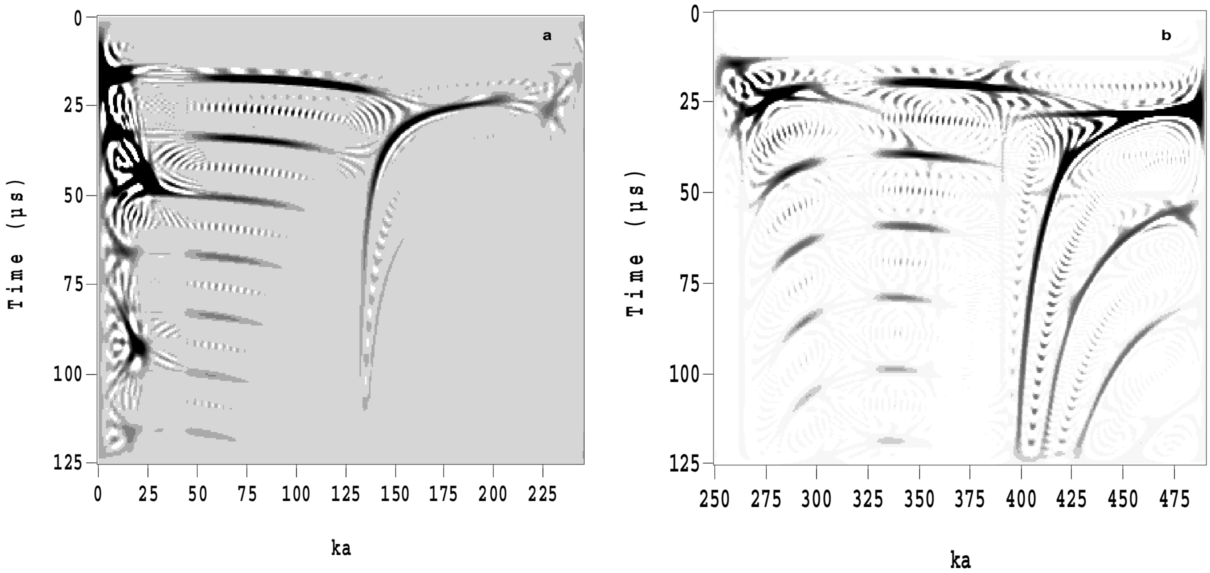

Figure. 7 : SPWV images of the approximation Ap1 (a) and the detail D1 (b)

5 Conclusion

This paper describes a methodology on Multiresolution analysis of the wavelet transform in mode identification of circumferential acoustic wave. The theoretical signal were carried out on the aluminium thin tube with a radii ratio b/a=0.95. The results have shown that the Multiresolution analysis and the time-frequency method of SPWV could be used to identify the circumferential acoustic waves in the tube not only single mode but also multimode circumferential waves effectively and the results of identification using these two methods are consistent. Finally, this methodology permits to obtain the interesting results.

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