ACOUSTICS 2012

Active Absorbers

M. Rousseau[a] and J. Vanderkooy[b]

[a]B&W Group ltd., Elm Grove Lane, BN44 3SA Steyning, UK
[b]University of Waterloo, Dept of Physics and Astronomy, Waterloo, Canada N2L 3G1
mrousseau@bwgroup.com

This paper explores the use of bass loudspeakers as both acoustic sources and broadband absorbers. We develop the theory for a point active absorber immersed in the anechoic field from a point source. This will apply to normal loudspeakers used as either sources or absorbers at frequencies below about 300 Hz, where they act much like points. The result extends the theory of Nelson and Elliott for a point absorber interacting with a plane wave. An extra oscillatory interference term occurs which should largely cancel in rooms due to the varying distances between all the source images and the absorber. Responses were measured in several rooms from source and absorber loudspeakers to both a few listening microphones and microphones mounted very near the absorber diaphragms. Using pre-computed absorber signals to avoid stability issues, the case for absorption was not very clear. We analyze several aspects which might resolve theory and experiment.

1 Introduction

An active acoustic absorber must sense the sound field in a room, and generate a signal to absorb energy from that field. In 1-D such absorbers work very well in situations such as ducts, and in 3-D systems they can be effective if source and canceller are much closer than a wavelength. In actual rooms with audio equipment, such conditions seldom apply. Even the lowest modes in rooms are several wavelengths long, and reflective wall characteristics generally make the room response very ragged and unsuitable at bass frequencies. Extra absorption at such frequencies would reduce the amplitude of resonance peaks, and decrease the reverberation time of the room. This is generally considered to be beneficial for good listening conditions. Equalisation can reduce peaks and dips in the response, but it does not change the decay rate of the room resonances.

Passive absorbers such as Helmholtz resonators or membrane absorbers can be used to alleviate bass response problems, but these are large, and quite a few may be necessary to make a significant improvement. The maximum absorption cross section of a resonant absorber is $\lambda^2/4\pi$, but since the bandwidth is narrow, many would be needed to cover the bass region. It is the purpose of this paper to study active absorbers that purport to have a similar cross-section, but that work over a wide band. Our starting point for the basic theory comes from a book on active control [1]. We review this theory below and extend it to more realistic situations.

2 Simple Absorber Theory

It is helpful to understand a few basics of the acoustics of spherical sources [2]. The pressure from such sources varies as 1/r, where r is the distance from the source, while the particle velocity varies as (1+1/jkr)/r, which has a strong $1/r^2$ reactive component. The acoustic impedance, Z_a, of a spherical source of radius, r, is

$$Z_A = \rho c/4\pi r^2 \, jkr(1+jkr).$$

As the radius r shrinks to zero, the imaginary part of this impedance becomes infinite, but the real part stays finite, $\omega^2\rho/4\pi c$. The theory to follow is not dependent on the source being a point; but we shall assume that it is small relative to the wavelength. At bass frequencies, loudspeakers look like point sources located at their acoustic centre [3], so we model an active absorber as a point monopole source that is driven in such a way as to remove maximum power from a travelling plane wave. The theory in this section is mostly from [1].

The power W of the absorber is given by the rate of change of the total energy, E, ignoring dipole sources

$$W = \partial E/\partial t = p(\mathbf{r},t)\, q(\mathbf{r},t), \qquad (1)$$

where p is the pressure and q is the volume velocity applied at the absorber position.

Such a unit will produce a self-pressure, p_{self}, which will add locally to the pressure, $p_{plane\ wave}$, of the plane wave. Using frequency domain, the power output of the monopole will be given by

$$W = (1/2)\ \mathrm{Re}\{[p_{self}(\mathbf{r}) + p_{plane\ wave}(\mathbf{r})]^*\, q(\mathbf{r})\}. \qquad (2)$$

Since $p_{plane\ wave}$ is constant, the second term will be proportional to q, but p_{self} is proportional to q, being given by $p_{self} = Z_{acoustic}\, q$, where Z is the acoustic impedance of the absorber, so the first term is quadratic in q. The quadratic and linear terms result in an optimum value of q(**r**) that will minimise the power W. W is determined by the real part of the acoustic impedance of a point source [2], leading to the optimum q:

$$\begin{aligned} q_{opt} &= -\, p_{plane\ wave}(\mathbf{r})/\{2\ \mathrm{Re}(Z_{acoustic})\} \\ &= -\,(2\pi c/\omega^2\rho)\ p_{plane\ wave}(\mathbf{r}) \\ &= (2\pi c/[j\omega]^2\rho)\ p_{plane\ wave}(\mathbf{r}), \end{aligned} \qquad (3)$$

The optimum volume velocity amplitude is antiphase to the plane wave, and proportional to two time integrations of the plane wave signal. The maximum or optimum power absorbed is

$$W_{opt} = (\pi c/2\omega^2\rho)\ |p_{plane\ wave}(\mathbf{r})|^2. \qquad (4)$$

This maximum absorption can be written as

$$W_{opt} = (\lambda^2/4\pi)\ |p_{plane\ wave}(\mathbf{r})|^2/(2\rho c), \qquad (5)$$

showing that the absorbed power is equivalent to a totally absorbing cross-sectional area of $\lambda^2/4\pi$. This is a large area for low frequencies. These conditions represent the best that a point active absorber could deliver. It may not be a point, but if it is small compared to the wavelength, it will act like one.

The optimum volume velocity q results from a competition between the positive self-power of the absorber which is quadratic in q, and the absorber power which is negative and proportional to q, when chosen antiphase to the plane-wave pressure.

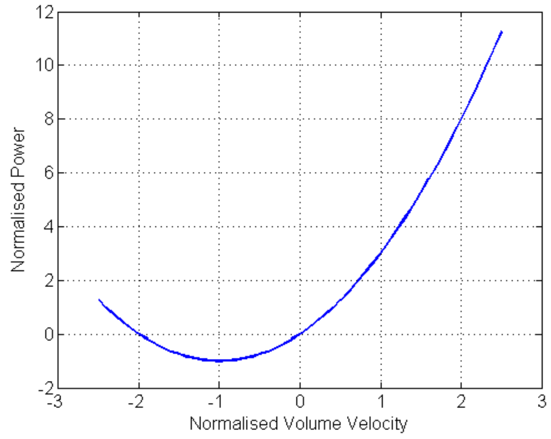

Figure 1. Normalized power output of point absorber in phase with a passing plane wave, as a function of normalized volume velocity of the absorber. Negative volume velocity indicates an antiphase relationship between plane wave and absorber.

Early experiments did not show the expected absorption, which gave us cause to re-evaluate the simple theory. Nelson and Elliott [1] do make the proviso that "*one has, of course, to bear in mind that no account is taken of the influence of the absorbing source on the output of the source from which the plane wave originates*. If there is wave curvature, then a real source must be present at the centre of curvature, and we should take into account the mutual coupling between that source and the absorber loudspeaker. In addition, for real rooms an acoustic source will have images in the walls as well as other modifications. Both the plane wave assumption and the effect of multiple sources will be dealt with in the next sections.

3 A New More Realistic Theory

We will model loudspeakers as sources of constant volume velocity since their cones are relatively heavy, and pressure at the cone has minimal effect on the cone's motion. Consider a main reference point source of volume velocity given by

$$q_0(t) = q_0 \cos(\omega t). \tag{6}$$

The power of this source can be calculated from the time average $<q(t)\ p(t)>$, where $p(t)$ is the pressure at the source. Although the self-pressure $p(t)$ is very large near the source, we only need the in-phase component, as used to derive Eq. (3). The finite self-power, W_0, of the source is easily shown to be

$$W_0 = (\omega^2\rho/4\pi c)\ q_0^2\ /2. \tag{7}$$

The main source will produce a retarded pressure, p_{0d}, a distance, d, away as given by the volume acceleration [4]

$$\begin{aligned} p_{0d}(t) &= (\rho/4\pi d)\ dq/dt \\ &= -(\rho/4\pi d)\ q_0\ \omega \sin(\omega t - kd). \end{aligned} \tag{8}$$

Now let us place a point absorber at distance, d, away of volume velocity

$$q_1(t) = q_1 \cos(\omega t + \Phi), \tag{9}$$

where Φ is a constant phase difference from the reference source. The self-power W_1 of this absorber, using an identical argument, will be

$$W_1 = (\omega^2\rho/4\pi c)\ q_1^2\ /2. \tag{10}$$

The absorber will produce a retarded pressure at the main source location of

$$p_{1d}(t) = -(\rho/4\pi d)\ q_1\ \omega \sin(\omega t + \Phi - kd).$$

The pressures of each source at the location of the other source as given above will lead to two extra cross-power terms, $<q_0(t)p_{1d}(t)>$, and $<q_1(t)p_{0d}(t)>$. Let us first look at the term relating to the change in power of the main source,

$$\begin{aligned} W_{01} &= < q_0(t)\ p_{1d}(t) > \\ &= -< q_0 \cos(\omega t)\ (\rho/4\pi d)\ q_1\ \omega \sin(\omega t + \Phi - kd) > \\ &= -\tfrac{1}{2}\ (\rho/4\pi d)\ q_0\ q_1\ \omega \sin(\Phi - kd). \end{aligned}$$

As before, quadratic terms in $\cos^2(\omega t)$ or $\sin^2(\omega t)$ will time average to ½, while cross-terms such as $\cos(\omega t)\sin(\omega t)$ will time average to zero.

For the absorber, the change is

$$\begin{aligned} W_{10} &= <q_1(t)p_{0d}(t)> \\ &= <q_1 \cos(\omega t + \Phi)\ (\rho/4\pi d)\ q_0\ \omega \sin(\omega t - kd)> \\ &= \tfrac{1}{2}\ (\rho/4\pi d)\ q_1\ q_0\ \omega \sin(\Phi + kd). \end{aligned}$$

Interestingly, these terms have the same magnitude for main and absorber sources. Note that $[(\rho/4\pi d)\ q_1\ q_0\ \omega]/[(\omega^2\rho/4\pi c)\ q_1^2] = (c/\omega d)\ q_1/q_0 = (1/kd)\ q_1/q_0$. If we normalise to the power of the main point source by itself, the total normalised power W is

$$W = 1 + (q_1/q_0)^2 + [\sin(kd - \Phi) + \sin(kd + \Phi)]/kd\ (q_1/q_0). \tag{11}$$

A surface plot of the deviation of this function from unity ΔW (= W−1) is shown in Fig.2 for $-1< q_1/q_0 <1$ and $0 < kd < 4\pi$. It is immediately obvious that the only significant negative values occur when $kd<\pi$, that is, when d is less than half a wavelength. As kd becomes very small, the absorber is essentially co-located with the main source, and when $q_1 = -q_0$, the power is completely absorbed, as expected. The two co-located sources cancel completely.

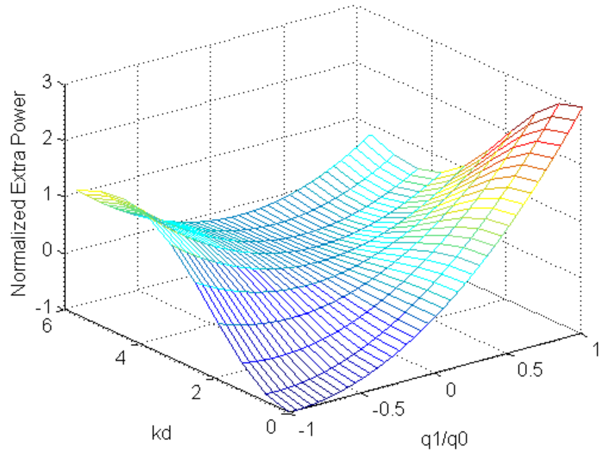

Figure 2. Showing the normalised extra power when a point absorber is introduced into the acoustic field of a main point source, as a function of kd and relative absorber strength q_1/q_0.

There is a small region where W−1 is negative for kd ~ (3/2)π and $q_1/q_0 \sim 0.22$, giving an absorption of about 4.6% of the main source power. Such a restricted region is not useful generally, so we might initially conclude that driven point absorbers are not the panacea that we hoped for.

If $\Phi = \pi/2$, the last term of Eq. (11) in square brackets is identically 0. Then the absorber always adds just its own self-power to the system. Thus sources in phase quadrature do not influence each other's power output. This is true for any source separation.

4 Comparing the Plane Wave and Point Source Models

In the plane wave model, the absorber always absorbs power if the optimum volume velocity is chosen. We always chose the phase of the absorber to be optimal. In our point-source model, we can manipulate the phase Φ to optimise absorption. The normalised change in power from just the absorber, ΔW_1, is

$$\Delta W_1 = (q_1/q_0)^2 + \sin(kd+\Phi)/kd\ (q_1/q_0).$$

We should choose Φ so that the absorber volume velocity has the same phase as the pressure from the main source, as in the earlier plane-wave theory. This means we must equate the phases of

$$q_1 \cos(\omega t+\Phi)$$

and

$$d/dt\,[q_0 \cos(\omega t-kd)] = q_0\, \omega\, [-\sin(\omega t-kd)].$$

This will happen for $\Phi = \pi/2-kd$, which is *not the same* as quadrature. With this proviso, the normalised power of Eq. (11) becomes

$$W = 1 + (q_1/q_0)^2 + [\sin(kd-\pi/2+kd) + \sin(kd+\pi/2-kd)]/kd\ (q_1/q_0)$$

$$= 1 + (q_1/q_0)^2 + [-\cos(2kd) + 1]/kd\ (q_1/q_0). \qquad (12)$$

The term −cos(2kd) in the square brackets that applies to the main source is strongly oscillatory in both frequency and distance from the absorber, and we shall argue soon that it is not significant in rooms. However the second term of unity in square brackets for the absorber, divided by kd, is always positive, so this whole term will be negative if q_1/q_0 is negative. This term resulted from making the phase of the absorber signal the same as the retarded pressure from the main source. That has caused the term for the main source to have a retardation twice as large, 2kd.

If we ignore the cos(2kd) term in Eq. (12), there will again be an optimum value for q_1/q_0

$$(q_1/q_0)_{optimum} = -1/(2kd).$$

The maximum normalised power absorbed is $1/(4k^2d^2)$. This has an inverse square dependence on frequency ($k=\omega/c$), and also on distance. The actual dependence on k and d becomes clear if we denormalise to the actual power, and express it in terms of pressure from the main source. The power of the main source from Eq. (8) is $W_0 = (\omega^2\rho/4\pi c)q_0^2/2$. Thus the denormalised optimum absorbed power becomes

$$W_{abs} = -[1/(4k^2d^2)]\,[(\omega^2\rho/4\pi c)\, q_0^2/2] = -(\pi c/2\omega^2\rho)\, p^2,$$

This is exactly the same answer, Eq. (4), as Nelson and Elliott [1] for the plane wave theory! We have thus shown that it is not necessary to have a plane wave, but we do now have an additional power term from the main source that is oscillatory in kd, and we need to assess its effect in relation to the term from the absorber.

5 The Transition to a Real Room

In a real room, there are a multitude of reflections from the source loudspeakers, and we must consider how this might affect the above. All these multiple sources have absorber terms that will always have the desired negative sign, as long as we make the absorber volume velocity antiphase to the total pressure at the absorber from the actual source and all its images. However, the −cos(2kd) terms relating to the main source will tend to cancel out because of the different distances to the main source and all the images. In addition, as frequency varies, this term oscillates and will tend to zero with frequency averaging. This somewhat justifies the underlying assumptions in Nelson and Elliott, and encourages us to try to implement driven absorbers.

The picture that emerges, then, is that if the driven absorber is fed an appropriate volume velocity signal that is in antiphase with the pressure *at the absorber from all the sources in the room*, it will always absorb more power than it generates. At the same time, the multiple sources in the room tend to have random phases with respect to the pressure at the absorber, so the extra power terms from the main source (which have the same magnitudes as the absorber terms) tend to cancel. Thus net absorption has the upper hand, but there are a few subtleties.

6 Some Enigmatic Experiments

Early experiments conducted with real loudspeakers did not clearly show net absorption; certain frequency regions were reduced while others were increased. This may be partly due to the few microphone positions used to determine the room behaviour, but nonetheless, the results were discouraging, so we set about to do some simulations using a real room model. Simulations were carried out with finite difference time domain techniques (FDTD), initially with a large enough room for which the anechoic and reverberant parts of the absorber response could be separated.

Eq.(3) gives us the prescription for the active absorber. We rewrite it in the time domain

$$q_1 = (2\pi c/\rho) \iint p(\mathbf{r},t)\, d^2t, \qquad (13)$$

displaying two time derivatives of the pressure $p(\mathbf{r},t)$ at the absorber position $\mathbf{r}$ from all the sources and reflections in the room. The two time derivatives are problematic, since very low frequencies will be emphasized without limit. We used highpass filters set at low frequencies to temper the resulting q.

For the source we used a relatively sharply bandlimited volume velocity pulse [5] for the source, and obtained both the total acoustic energy in the room versus time, and the pressure 'impulse' response at the absorber position. In order to determine the reverberant absorber response at the position of the absorber itself, for some simulations the distance to the walls was made large enough that the near self-field of the absorber impulse response could be separated from its later reverberant response. The FDTD simulations properly accounted for the volume velocity of the loudspeakers employed, and were programmed to evaluate the total acoustic energy in the room.

Typically our simulations showed a little absorption during early times of the energy decay in the room, but later on there would actually be more room reverberation with the absorber driven. This is probably due to the fact that we have not modelled the reverberation of the absorber as a real source which should also be iteratively dealt with in the simulations.

7 Discussion

We note that loudspeakers can also be used as microphones [6], with relatively simple processing of the output at their terminals. That would allow transfer functions from sources to absorbers to be measured *in situ* without any extra components. The benefit of using the loudspeaker as a microphone is that it places the "microphone" exactly at the low-frequency acoustic centre [3] of the absorber loudspeaker. Loudspeakers have efficient magnetic circuits, so their sensitivity as microphones is reasonable.

We should mention again that we have considered the absorber as producing a single anechoic pressure signal, while we have allowed the source to have images and reverberation from the room. This multiplicity of source signals is fine, since the theory just considers them all as the net source signal at the absorber. Even the averaging of all the main source cos(2kd) terms to zero seems reasonable. However, the absorber is also in the room, and it will also reverberate, which we have not taken into account. These signals must also interact with the main source and the absorber.

We should not need to take the direct anechoic signal of the absorber into account, because the self-power of the absorber has already been included in our theory. As frequency goes down there will come a point where kd is too small for our assumption that terms like cos(2kd) will average to zero. There are propagation distances involved in the absorber reverberant interactions as well, and we might think that they should average to zero, but perhaps this assumption is not warranted.

8 Conclusion

The theory presented earlier is straightforward, so we do not expect there to be any logical errors or omissions. However, there are several assumptions we have made that may need re-evaluation. We have assumed that kd is large enough that the extra terms for source images tend to cancel. This will not be true as frequencies become lower and lower. We must at some low frequency taper off the absorber signal. The lack of consistent absorption in our simulations is not presently understood, but several possibilities have been suggested.

Acknowledgments

We are grateful for discussions with Peter Fryer, Peter Craven, and Jon Moore.

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