Measures and models of real triodes, for the simulation of guitar amplifiers

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This paper deals with measurements of vacuum tubes to improve models and guitar amplifier simulations. A measurement device has been created. A first experiment is dedicated to study the static regimes of real dual triodes 12AX7s. A second one is used to study the dynamic regimes mainly involved by parasitic capacitances. By using these measurements, a new static triode model is proposed. it includes : refinements of the Norman Koren's plate current model; associated parameters estimation procedure; grid current modeling, dependent on both plate-to-cathode and grid-to-cathode voltages around triodes. The parameters of the static model are estimated for each real triode measured, as well as the "so-called" Miller capacitance. It is available as a SPICE model.

1 Introduction

Studying vacuum tube guitar amplifiers particularly for simulation purposes has been an important research subject these last years, with several papers published in the scientific literature (see e.g. [1, 2, 5, 6, 7]). The main goal of these studies is to allow musicians to replace the original devices with numerical equivalents, without sacrificing their sonic personality.

In this article, the triode, one of the most important electronic component of vacuum tube guitar amplifiers, is studied and measured, in order to improve the realism of amplifier simulations. For this issue, a set up has been developed, and the results of the experiments have been exploited to deduce accurate models of triodes.

This paper is organized as follows. In section 2, our experimental setup is presented. An introduction is done about the 12AX7s, used in the experiments, and the measurement set up we have developed. In section 3, the procedure to measure the static behaviour of the 12AX7s is displayed. Then, a new triode model is proposed, thanks to observations done on measures, and its parameters are estimated for each real triode. In section 4, another measurement procedure is described, to capture the dynamic behaviour of triodes, and to measure one of its parasitic capacitance. And finally, in section 5, the results and their use for guitar amplifiers simulation are discussed.

2 Presentation of the experiment setup

2.1 Triodes and 12AX7s

Triodes are vacuum tubes, with three pins : the grid (G), the cathode (K) and the plate (P) (see figure 1). Indirectly heated cathode causes electrons to be attracted to the positively charged plate, and creates a current, controlled by a negative voltage applied to the grid, repelling back towards the cathode an amount of electrons. However, when grid to cathode voltage becomes positive, some current is leaking into the grid. Consequently, the triode is considered equivalent to two current sources I_g and I_p (see figure 1), dependent on the voltages V_{gk} (grid-cathode) and V_{pk} (plate-cathode). These currents are always positive, around a few mA for 12AX7s.

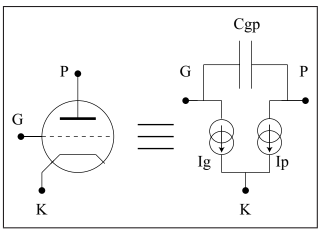


Figure 1: Triode electronic model

Triodes are also known to have small parasitic capacitances (see table 1). In the common cathode triode (CCT) amplifier, the most common triode circuits in guitar amplifiers, only one parasitic capacitance has a significant impact on the triode frequency response, which is the grid-to-plate capacitance C_{gp}. As seen in [6], its practical influence is multiplied by the circuit gain. It is due to the Miller effect. For this reason, the effective capacitance is often called Miller capacitance. Consequently, the capacitance C_{gp} is often the only one considered in triode electronic models (see figure 1).

C_{gp}	C_{gk}	C_{pk}
1.7 pF	1.6 pF	0.33 pF

Table 1: Parasitic capacitance values in 12AX7s

In this paper, the dual triode 12AX7/ECC83 is studied and measured. It is widely used in audio preamplifiers because of its high gain factor μ around 100. Two sets of real 12AX7s to measure have been chosen, from different firms: warn 12AX7s W1-W9, and new 12AX7s N1-N9 bought in music stores. Each triode in dual 12AX7s is denoted X_a and X_b.

2.2 Measurement setup

To measure each triode, several elements are needed. First, to perform the measurements successfully, the A/D board DSpace DS2004 available at Ircam has been used [9]. It is a high speed A/D board which can measure and send signals very precisely, continuous or with frequencies very low to several GHz, and execute real-time applications for data acquisition using MATLAB and Simulink. The interface takes as inputs and outputs voltages exclusively between -10 V and +10 V.

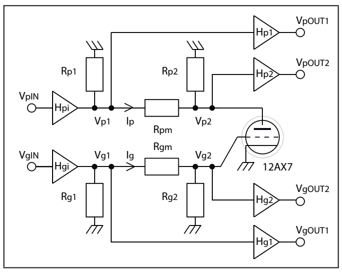


Figure 2: Schematic of the triode measuring device

Second, a device has been developed by Bruno Ferren in Orosys firm, to measure behaviour of each triode in dual 12AX7s, modeled as seen in figure 1. Figure 2 shows a simplified schematic of the triode measuring device. Its inputs fix potentials V_{gIN} and V_{pIN}, whereas its outputs give potentials V_{gOUT1}, V_{pOUT1}, V_{gOUT2} and V_{pOUT2}. The voltages values are mapped from the DSpace working range (between

-10 and +10 V) to the working range of triodes (between 0 and 400 V for the triode plate, and between -20 and +5 V for the triode grid). This is another function of the measurement device. With this data, and knowing the resistances in the circuit (see table 2), it is possible to obtain the current values I_g and I_p, using Ohm's law, cathode being connected to the ground ($V_k = 0$ V).

R_{gm}	$R_{g1} = R_{g2}$	R_{pm}	$R_{p1} = R_{p2}$
468 Ω	115 kΩ	10.05 kΩ	480 kΩ

Table 2: Resistance values in the measuring device

And third, software allowing us to send and receive data is developed with MATLAB, Simulink and the real-time workshop toolbox.

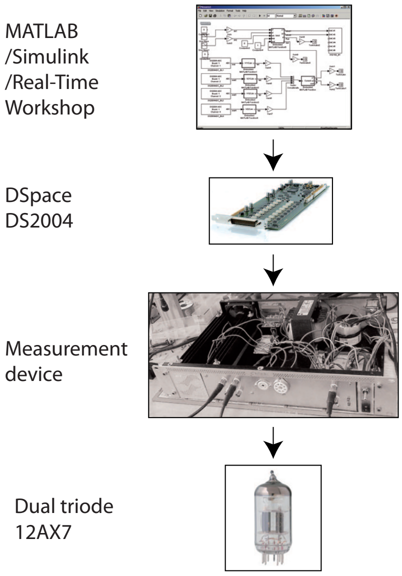


Figure 3: Schematic of the measurement setup

Another mapping is done using a separate voltage measurement device, to calculate the equivalence between the DSpace input/output voltages, and the data to record (see equation (4).

$$V_{p1} = 47.483 V_{pOUT1} + 0.2227 \quad (1)$$
$$V_{p2} = 47.319 V_{pOUT2} + 0.1791 \quad (2)$$
$$V_{g1} = 7.7619 V_{gOUT1} + 0.0133 \quad (3)$$
$$V_{g2} = 7.7836 V_{gOUT2} + 0.006 \quad (4)$$

3 Static behaviour of real 12AX7s

The static behaviour of real 12AX7s is studied in two steps. First, real 12AX7s are measured, with the measurement procedure of the section below. Second, a new triode model is proposed, and its parameters are estimated from each triode measurement.

3.1 Measurement of static triodes behaviour

3.1.1 Static measurement procedure and verifications

An experiment protocol is defined in a program developed with MATLAB and Simulink, with a particular attention on the conditions for the measurement. It needs to be done in a quasi-static context, in a way which the measurement of currents can be obtained for the whole range of input voltages around the triode, in a short time T_m. To succeed, low frequency sinus signals will fix the voltages V_{gIN} and V_{pIN}, with frequencies f_1 and f_2, to obtain the complete map of the static currents.

The frequencies f_1 and f_2 must be chosen low enough so signals are not affected by parasitic capacitances (see section 4), and high enough so they do not affect the temperature (see equation (5)), when the triode dissipates a lot of power. Indeed, the emission of electrons in 12AX7s is performed by heating a filament, connected to the cathode, and the electron flux is dependent on temperature. After a few preliminary measurements, it has been seen that 200 seconds need to separate the ignition of the measurement device, and the beginning of the following measurement procedures of real 12AX7s. This time constant gives also the low frequency limit for f_1 and f_2.

$$f_{\text{temp}} \le f_1 \le f_{\text{parasitic}} \quad (5)$$

with $f_{\text{temp}} \approx 1/200$ Hz and $f_{\text{parasitic}} \approx 10$ kHz

As a result, $f_1 = 40$ Hz has been chosen, which satisfies both constraints about temperature and frequency response of the triode. In the experiments, a duration $T_m = 60$ seconds is chosen, and the sampling frequency is $f_s = 44100$ Hz. The frequency f_2 is calculated so T_m is a period for both sinus signals :

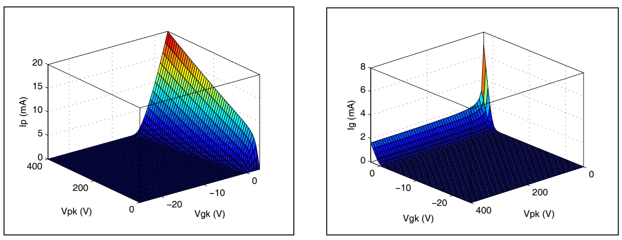
$$f2 = f1 + 1/T_m \quad (6)$$

In section 2, we have seen how the values of potentials V_{g1}, V_{g2}, V_{p1} and V_{p2} can be measured. Knowing these potentials, and the values of the resistances in the circuit (see table 2), the plate and the grid current values are :

$$I_p = (V_{p1} - V_{p2})/R_{pm} - V_{p2}/R_{p2} \quad (7)$$
$$I_g = (V_{g1} - V_{g2})/R_{gm} - V_{g2}/R_{g2} \quad (8)$$

3.1.2 Results for one triode

Finally, measurements are obtained as quadruplets $(I_p, I_g, V_{g2}, V_{p2})_i$, for plate voltages between 0 and 400 V, and grid voltages between -20 and 5 V. Then, measured quadruplets are used in MATLAB to generate a surface $I_p = f(V_{gk}, V_{pk})$ (see figure 4a) and a surface $I_g = f(V_{gk}, V_{pk})$ (see figure 4b).



(a) (b)

Figure 4: Characterization surfaces for the plate (a) and grid (b) currents of triode (O1a)

These measurements show that plate current is monotonic, and increases with V_{gk} and V_{pk}. It is null when plate voltage is null and when grid voltage is below -15 V. Grid current is seen negligible when V_{gk} is below -5 V, but it is as important as plate current for positive values of V_{gk}. Moreover, grid current is dependent with both voltages V_{gk} and V_{pk}, whereas its dependence with V_{pk} is usually neglected, as in [6].

3.2 New models and estimation of parameters

In [6], the triode model is the Norman Koren's one for the plate current :

$$I_p = \begin{cases} 2E_1^{E_x}/K_g & \text{if} \quad E_1 \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

$$E_1 = \frac{V_{pk}}{K_p} \ln\left[1 + \exp\left(K_p\left(\frac{1}{\mu} + \frac{V_{gk} + V_{ct}}{\sqrt{K_{vb} + V_{pk}^2}}\right)\right)\right]$$

Table 3 shows typical parameters values of Norman Koren's model for 12AX7s.

	μ	E_x	K_g	K_p	K_{vb}	V_{ct}
12AX7	100	1.4	1060	600	300	0.5

Table 3: Typical parameters values of Norman Koren's triode model

Moreover, a grid current model has been proposed in [6]. In this model, R_{gk} is the slope of the grid current behaviour for high grid-to-cathode voltages, and K_n parameters the transition between this area, and the area where the grid current is null. The constant a, b, and c are dependent on K_n, V_γ and R_{gk}.

$$I_g = \begin{cases} 0 & \text{if } V_{gk} < V_\gamma - K_n \\ aV_{gk}^2 + bV_{gk} + c & \text{if } V_\gamma - K_n \leq V_{gk} \leq V_\gamma + K_n \\ \frac{V_{gk} - V_\gamma}{R_{gk}} & \text{otherwise} \end{cases} \tag{10}$$

By using our measurements, it is possible to obtain the parameters values for our real triodes. Indeed, Norman Koren suggests several methods to estimate these parameters from measurements, better than a trial-and-error method [1]. Another one has been proposed in [6]. They use often a number n of measurements, optimization methods from MATLAB, or direct calculation of the parameters with measurements chosen at key locations.

However, by using our measurements, these models have shown some limits. The estimated model does not match perfectly with measurements curves, more particularly for positive grid voltages. Moreover, our grid model is not able to model the dependence of I_g with V_{pk}, displayed in the measurements. That suggests that the previous model - with Norman Koren's plate current model and our grid current model - is not able to mimic exactly the static real triodes behaviour. Consequently, we have decided to propose a new triode model :

$$I_p = \begin{cases} 2E_2^{E_x}/K_g & \text{if} \quad E_2 \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

$$E_2 = \frac{V_{pk}}{K_p} \ln\left(1 + \exp\left(K_p\left(\frac{1}{\mu} + \frac{V_{gk} + V_{ct}}{f(V_{pk})}\right)\right)\right)$$

$$f(V_{pk}) = \sqrt{K_{vb} + V_{pk}^2 + K_{vb2}V_{pk}}$$

$$I_g = \ln(1 + \exp(a(V_{gk} + V_\phi)))^\gamma \times (1/(bV_{pk} + 1) + 1/c)$$

The new plate current model is an improvement of Norman Koren's model with the introduction of an additional parameter K_{vb2}. It allows the correction of the original Norman Koren's model for low plate-to-cathode voltages and positive grid-to-cathode voltages.

Moreover, the new grid current model is dependent on both plate-to-cathode and grid-to-cathode voltages, unlike the previous grid current model introduced in [6]. It has been developed to mimic the measurement of grid currents from real triodes.

The figures 5 and 6 show the accuracy the new model and the old model, compared with the measurements.

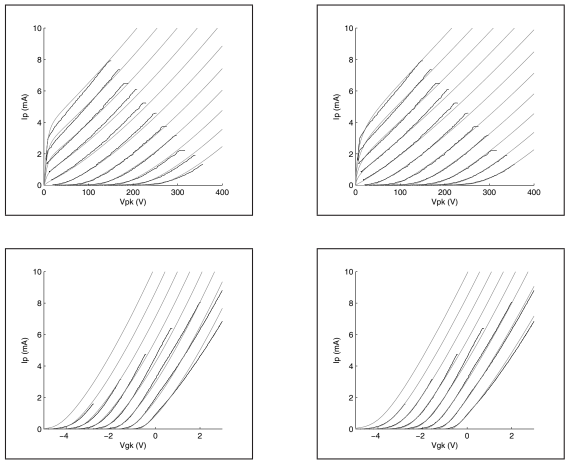


Figure 5: Measurements and estimated model for the old and improved Norman Koren's plate current models

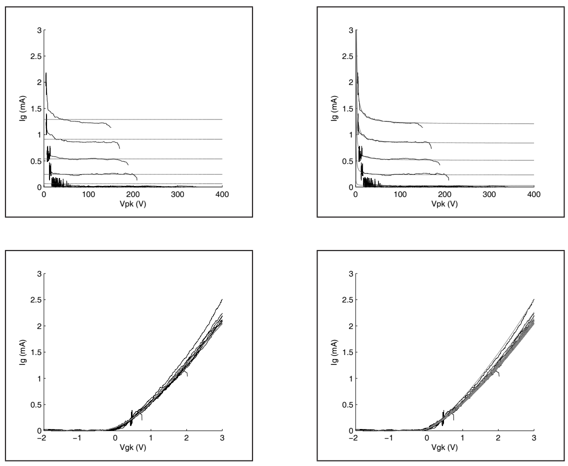


Figure 6: Measurements and estimated model for the old and new grid current models

4 Parasitic capacitances of real 12AX7s

4.1 Measurement of dynamic triodes behaviour

Static behaviour of triodes is well known in textbooks, but less information is available about their dynamics. One advice about this question is given in the datasheets, with measured values of several parasitic capacitances (see table 1). To improve our triode model, we propose in this section a method to obtain the value of the parasitic capacitance C_{gp}, in a real 12AX7. Indeed, a simple capacitance meter is not useful to get their very low values. Instead, the frequency response of the measurement device to a logarithmic sinusoidal sweep is measured, leading to a deduction of the capacitance value.

4.1.1 SPICE model of the measurement device

At first, the SPICE model of the measurement device is studied (see figure 2) with a generic triode model and three parasitic capacitances. Because the triode is in a CCT amplifier in the measurement device, the capacitance C_{gp} is expected to cause a lowpass-like filtering in the audible bandwidth, thanks to the Miller effect (see [6]). Let $f^*_{\text{-3 dB}}$ be the cutoff frequency of the low-pass-like filtering caused by the Miller effect for small input signals. This frequency depends on circuit parameters, but also on circuit voltages at equilibrium around the triode, functions of the input voltages.

With voltages $V_{gIN} = -2$ V and $V_{pIN} = 300$ V at equilibrium, SPICE simulations display a cutoff frequency $f_{\text{-3 dB}}$ of 5.22 MHz. This frequency is too high to give the information needed, because the measurement device has been developed to work with audible frequencies. Consequently, the values of components R_{pm} and R_{gm} have been changed in SPICE (see table 4) to reduce the value of $f_{\text{-3 dB}}$. It becomes 15.49 kHz with these new values, which is more acceptable. The real measurement device has been modified accordingly, to perform the triode dynamics measurements.

R_{gm}	$R_{g1} = R_{g2}$	R_{pm}	$R_{p1} = R_{p2}$
300 kΩ	115 kΩ	300 kΩ	480 kΩ

Table 4: New resistance values in the measuring device

4.1.2 Measurement of frequency response to small input signals

Now, the value of $f_{\text{-3 dB}}$ is going to be measured on real triodes, by using the Hammerstein series [4, 3]. They are simplifications of the Volterra series, limited to their diagonal elements, called kernels and denoted h_i. Hammerstein series are a great tool to estimate weakly nonlinear systems behaviour, and to measure very accurately their impulse response, which are the kernels h_1. They are represented the following way :

$$y[n] = h_1[n]x[n] + h_2[n]x^2[n] + h_3[n]x^3[n] + ... \qquad (12)$$

To identify the kernels, a logarithmic sweep sine is sent into the system, and a deconvolution operation is applied to the output. Because of the properties of the logarithmic sweep sine, the impulse response of the system is obtained, but also each kernel of the Hammerstein series.

In our case, the voltage V_{p1} is fixed to 300 V, and a logarithmic sweep sine is sent into V_{g1}, with an amplitude of 1 V, and a variable offset which defines an equilibrium point U^*. Then, the deconvolution is applied to V_{p2}, to get the impulse response, and a frequency response of the measurement device to small signals.

The frequency response obtained for the triode W1a is seen in figure 7. For low frequencies, the amplitude is a constant A_g. The cutoff frequency $f_{\text{-3 dB}}$ may be obtained at the frequency where the amplitude is $A_g/\sqrt{2}$.

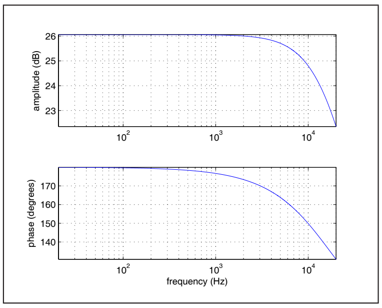


Figure 7: Frequency response measured on V_{p2} for the triode W1a

For each triode, $f_{\text{-3 dB}}$ is measured, but also the equilibrium values of V_{p2} and V_{g2}. This information will be important later. The table 5 shows these values for the triode W1a.

$f_{\text{-3 dB}}$	V^*_{g2}	V^*_{p2}
9.756 kHz	-1.214 V	138.6 V

Table 5: Measured parameters for the dynamics of the triode W1a

4.2 Estimation of the parasitic capacitances

In section 4.1, the frequency response of the measurement device for small input signals, similar to a lowpass-like filtering, has been obtained for each triode. Associated "dynamic" parameters have been deduced : the cutoff frequency $f_{\text{-3dB}}$, the voltages at equilibrium V^*_{g2} and V^*_{p2}. They are used to calculate the value of the parasitic capacitance C_{gp}.

The estimation procedure is the following. First, the measurement device is modeled, by knowing the circuit and the exact values of each components. Secondly, an expression of the cutoff-like frequency $f_{\text{-3dB}}$ is deduced, as a function of the parasitic capacitance C_{gp}, the circuit components, and local derivatives of the triode static model. Third, the voltages at equilibrium V^*_{g2} and V^*_{p2} are used to calculate the model derivatives and finally, the parasitic capacitance value can be obtained.

A model of the measurement device circuit, with the parasitic capacitance C_{gp} (see figure 8), is proposed with extended state-space representations (ESSRs).

To simulate electronic circuits, a set of differential algebraic equations is derived from their formulation, using Kirchhoff laws and component models, and converted into discrete-time equations with discretization methods, to be solved in real-time. However, a specific formatting of original equations needs to be done, to allow the systematic discretization. The ESSR is such a formulation of circuit equa-

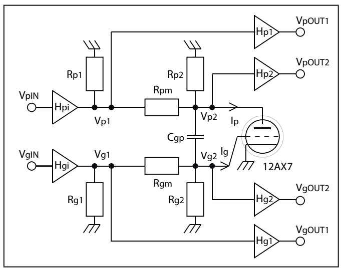


Figure 8: Schematic of the 12AX7's measuring device

tions, which gives a general modeling of nonlinear systems in the time-continuous domain, without any dependence on a particular discretization method, which can be applied directly. More information about ESSRs is available in [6].

To enlight the qualitative low-pass effect due to C_{gp}, the local linear dynamics of this circuit are studied around its equilibrium point, for inputs $U = U^* + u(t)$ (U is a constant, $u(t)$ is small) with $U^* = [300 - 2]^T$. The circuit behaviour can be expressed by a transfer function $H(s) = \frac{b_0+b_1 s}{a_0+a_1 s}$ in the Laplace domain. After a few calculations, it is possible to obtain the expression of its zero z and its pole p (strictly negative) :

$$\begin{aligned} z &= g_m/C_{gp} \qquad (13)\\ p &= -((R_{g2}+R_{gm})(R_{p2}+R_{pm}+R_{p2}R_{pm}/r_p)) \\ &\quad /(C_{gp}((R_{g2}+R_{gm})R_{p2}R_{pm}+R_{g2}R_{gm}(R_{p2}+R_{pm}) \\ &\quad +(g_m+1/r_p)R_{g2}R_{gm}R_{p2}R_{pm})) \end{aligned}$$

The pole gives an analytical expression of the cutoff-like frequency :

$$f_{\text{-3dB}} = abs(p)/2\pi \qquad (14)$$

With this formula, it is possible to obtain C_{gp} knowing $f_{\text{-3dB}}$ (from measurements), g_m and r_p. These last parameters are calculated using the static model of the real triode measured, depending on V^*_{g2} and V^*_{p2} measured previously. For example, with the triode W1a, and the parameters values seen in table 5, $g_m = 0.9605$ mA/V, $r_p = 92.247$ kΩ, and the value of the capacitance C_{gp} is around 1.6 pF.

5 Discussion of the results

The improvement of the static triode model is obvious for the grid current, introducing the dependence on plate-to-cathode voltage, and perfectly matching the curves from measurements. For the plate current, the new model is better but still imperfect for voltages involving high dissipating power. However, it is satisfactory for simulation purposes.

Moreover, by using the measurements of dynamic parameters, and the estimated static models obtained previously, the parasitic capacitances C_{gp} of each triode have been estimated and are around 1.8 pF. A SPICE model with static and dynamic parameters estimation of each real triode is available on `http://www.orosys.com/cohen/acs2012.htm`.

However, new estimation procedures must be proposed, with refined dynamic measurement devices, to estimate more accurately the parasitic capacitance C_{gp}, but also the parasitic capacitances C_{pk} and C_{gk}. Indeed, these capacitances become important when triode circuits which are not CCT amplifiers are modeled.

6 Conclusion

By using measurements of several 12AX7s, we have been able to develop accurate models of triodes, and estimate their parameters for each triode in our disposal. They have been used for simulation purposes yet, in real-time and SPICE applications. Future work will show the influence of tubes in real and simulated guitar amplifiers, using the real 12AX7s from our sets, and corresponding estimated models. Then, a specific study will evaluate the realism of guitar amplifiers simulations.

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