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Modeling of cylindrical baffle mufflers for low frequency sound propagation

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Baffle mufflers are widely used in building airflow systems. These mufflers are made up of diaphragms and porous materials inserted in constant section ducts. The object of this study is to predict the acoustic transmission loss of such a silencer with the aim to improve its efficiency. We are mainly interested in the low frequency range (between 63 Hz and 500 Hz) and we shall restrict the analysis to cylindrical geometries. The first part of this paper deals with the muffler insertion into an air conditioning networks using the transfer matrix method. Indeed an analytical low frequency approach computes the global transfer matrix of one cylindrical muffler, analogous to two diaphragms in series with porous material in-between. The first transverse mode is taken into account for the matching conditions at the two discontinuities whereas a single quasi-planar wave like mode is used to model the propagation in the lined duct. Results are compared with experiments and numerical simulations computed with the finite element method. Effects of the porous material thickness, the baffle length and its geometry are discussed.

1 Introduction

Airflow networks used in the building domain are subjected to high sound pressure level in the low frequency range (typically [63 - 500] Hz) due to air conditioning installations. Mufflers are widely used to decrease the level of noise: they consist of inserting parallel porous layer in an air section of those networks. However, these mufflers show a lack of efficiency in the low frequency range.

The modeling of silencers in ducts below the first cut-off frequency may be performed by the transfer matrix method (TMM) (see [1] for instance in the case of purely reactive silencers). Using a finite element method (FEM), Peat [2] takes into account the dissipation effects due to the presence of a porous liner in the duct. In the low frequency regime the corresponding plane-wave transfer matrix for a dissipative silencer element can be then determined. However this method requires a considerable numerical effort, especially for general three-dimensional configurations. Aurégan and Debray [3] proposed an analytical approach to determine the acoustic performance of a dissipative expansion chamber at low frequency range. This approximate model allows to find out the transfer matrix of an expansion chamber partially filled with an absorbing material. In their work, an equivalent fluid model [4] is considered for the porous material.

In this paper we shall apply the same analytical approach for the prediction of a two diaphragms lined silencer. Results are also compared to experimental and numerical results.

2 Model

Let us consider a cylindrical baffle muffler (fig. 1) made of two diaphragms placed in a constant section duct and distant from each other by a length L. An acoustic foam lining is inserted in-between. The sum of its thickness δ and the aperture radii r_a is equal to the radii r_p of the waveguide. The study takes place under the first cut-off frequency of the section. Hence, only the first propagating mode is considered in the acoustic system (planar or quasi-planar waves). In order to develop the transfer matrix formalism, it is necessary to consider mean values for the acoustic pressure over a given duct cross-section [2, 3]. The transfer matrix T which links the mean pressures and the flow rates on both sides of a sudden contraction or expansion can be written

$$T = \begin{pmatrix} 1 & Z_{add} \\ 0 & 1 \end{pmatrix} \quad (1)$$

where Z_{add} is the additional impedance which takes into account the interaction between the planar mode and the other higher transverse modes at the discontinuity.

The analytical transmission loss of the silencer is then determined by TMM using rigid-framed model for the porous material [4]. The studied muffler is split into a certain number of sub-systems characterized by their own transfer matrix. The overall transfer matrix of the acoustic system is the product of the sub-systems transfer matrices. In this case, the cylindrical baffle muffler is split into three sub-systems (fig. 1):

- the first acoustic discontinuity: diaphragm with porous material downstream,
- the lined duct,
- the second acoustic discontinuity: diaphragm with porous material upstream.

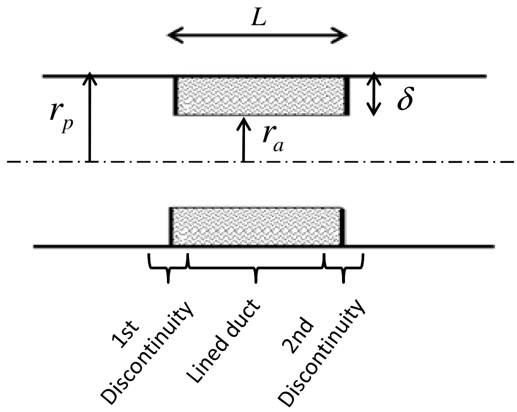

Figure 1: Schematic representation of the studied silencer split in three sub-systems.

2.1 Transfer matrix of a diaphragm

Let us consider first a single diaphragm of thickness ε without any porous material. It can be also split into three sub-systems as shown in figure 2 where planar waves propagate with the mean pressures P_0^I, P_0^{II} and the flow rates U_0^I, U_0^I respectively upstream and downstream of the discontinuity. The element 2 is a simple duct with a length ε. Its transfer matrix writes

$$T_2 = \begin{pmatrix} \cosh(jk_0\varepsilon) & Z_{c_0}\sinh(jk_0\varepsilon) \\ Z_{c_0}^{-1}\sinh(jk_0\varepsilon) & \cosh(jk_0\varepsilon) \end{pmatrix} \quad (2)$$

with k_0 the wave number of the first mode and Z_{c_0} the characteristic impedance of the section. Assuming that $Z_{c_0} \gg k_0\varepsilon$ and $k_0\varepsilon \to 0$, a first order expansion with respect to $k_0\varepsilon$ gives:

$$T_2 = \begin{pmatrix} 1 & jk_0\varepsilon Z_{c_0} \\ 0 & 1 \end{pmatrix}. \quad (3)$$

In the case where the contraction and extension are not associated with a lining in one of the large section then

$$Z_{add} = Z^0_{add}, \quad (4)$$

stands for a reactance linked to the added length of the duct radiating on both sides [5]. The transfer matrix of the contraction and the expansion (elements 1 and 3) can be then written as

$$T_{1\to2} = T_{2\to3} = \begin{pmatrix} 1 & Z^0_{add} \\ 0 & 1 \end{pmatrix}. \quad (5)$$

The overall transfer matrix T_{os} is finally the product of the three sub-systems transfer matrices. Thus, the diaphragm is represented by this matrix system

$$\begin{pmatrix} P^I_0 \\ U^I_0 \end{pmatrix} = T_{os} \begin{pmatrix} P^{II}_0 \\ U^{II}_0 \end{pmatrix} \quad (6)$$

with

$$T_{os} = T_{1\to2} T_2 T_{2\to3} = \begin{pmatrix} 1 & 2Z^0_{add} + jk_0\varepsilon Z_{c_0} \\ 0 & 1 \end{pmatrix}. \quad (7)$$

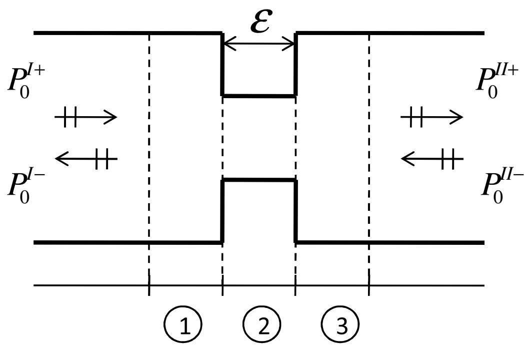

Figure 2: Single diaphragm split in three sub-systems.

2.2 Transmission loss for cylindrical baffle mufflers

To get the transmission loss of the baffle muffler shown on figure 1, the overall transfer matrix has to be evaluated. In this way, a lining is added in one of the large section of the diaphragm to be compatible with both discontinuities of the studied muffler. Hence, for the first discontinuity, the element 3 of the diaphragm is now a lined duct characterized by the additional impedance Z_{add}. The transfer matrix for both discontinuities writes

$$T_{discontinuity} = \begin{pmatrix} 1 & Z^0_{add} + jk_0\varepsilon Z_{c_0} + Z_{add} \\ 0 & 1 \end{pmatrix} \quad (8)$$

and the one from the lined section writes

$$T_{lined\ section} = \begin{pmatrix} \cosh(jk_1L) & Z_{c_1}\sinh(jk_1L) \\ Z_{c_1}^{-1}\sinh(jk_1L) & \cosh(jk_1L) \end{pmatrix} \quad (9)$$

with k_1 the wave number of the quasi-planar mode and Z_{c_1} the characteristic impedance of the lined duct. Finally, the model takes into account quasi-planar waves in the lined duct and evanescent waves at the discontinuities with a second wave number included in Z_{add}. The overall acoustic system can be written as

$$T_{os} = T_{discontinuity} T_{lined\ section} T_{discontinuity}. \quad (10)$$

Some examples of transmission loss of a purely reactive system with no absorbing material (bare configuration) and a dissipative system with porous material (lined configurations) are presented in figure 3. The porous material is a melamine foam which parameters are taken from reference [6]. The radii of the waveguide is 42 mm and the thickness of both diaphragms 5 mm. The two reference curves (thick lines) for both configurations refer to a length of 300 mm and an aperture radii of 22 mm. Note that dimensions are smaller than for an industrial muffler. The first cut-off frequency is 2400 Hz. The other curves (thin and dashed lines) represent respectively the system with half length and thickness. It can be observed that the reactive properties of the silencer disappear and the attenuation is largely increased when melamine foam is inserted. Between 0 Hz and 700 Hz the length of the silencer and the thickness of the porous material affect the transmission loss curve in a very similar way. This suggests that it may be worth designing the muffler with maximal thickness in order to make it as compact as possible.

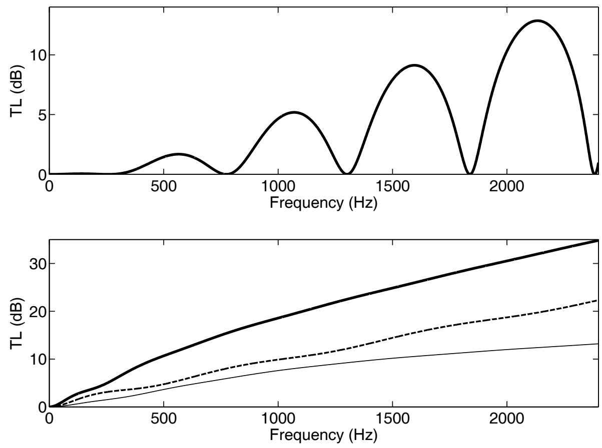

Figure 3: Analytical transmission loss of a baffle muffler for two cases: (top) bare configuration and (bottom) lined configurations. (thick line) L = 300 mm and δ = 20 mm, (thin line) L = 300 mm and δ = 10 mm, (dashed line) L = 150 mm and δ = 20 mm.

3 Validation

The studied configuration is the same than the analytical configuration: the waveguide radii is 42 mm, the thickness of both diaphragms is 5 mm and the length of the silencer is 300 mm. Two different thicknesses of 10 mm and 20 mm (fig. 5) of melamine foam have been chosen for the validation.

3.1 Experimental validation

The measurements were performed at the Roberval laboratory in the Technological University of Compiegne. The experimental test bench (fig. 4) measures the scattering matrix of the studied acoustic system (test section) using a double sources method [7]. From measurement of the scattering matrix, the transmission loss is determined for a bare and lined configurations. Figure 6 shows a good agreement between the analytical model and experimental results for the bare muffler. When adding the lining, a good tendency is also observed for the both thicknesses (fig. 7). However, discrepancies appear above 800 Hz and 1600 Hz for 20 mm and 10 mm thick, respectively. This is due to a resonance of the melamine foam skeleton that is not accounted for in the analytical model. Indeed, the determination for both thicknesses of the first $\lambda/4$ resonance frequency of the shear wave

$$f_r^{shear} \approx \frac{1}{4\delta}\sqrt{\frac{E}{2(1+\nu)\rho_1}}, \tag{11}$$

obtained from the same resonance frequency of the frame-borne wave [8], gives:

- $f_r^{shear}(20 \text{ mm}) = 1220{,}6$ Hz,
- $f_r^{shear}(10 \text{ mm}) = 2441{,}2$ Hz.

The melamine foam is characterized by its Young's modulus E, Poisson's ratio ν and frame density ρ_1. This skeleton resonance has been also observed by laser vibrometry.

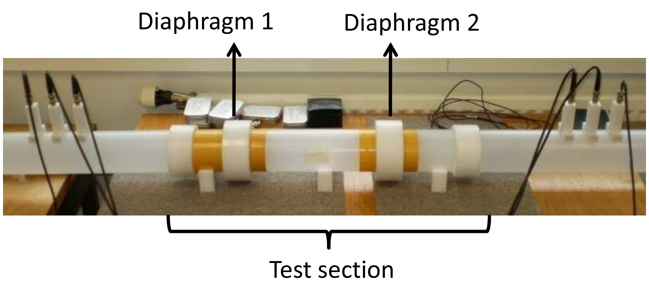

Figure 4: Test bench.

Figure 5: Test duct with melamine foam: (left) δ = 10 mm and (right) δ = 20 mm.

3.2 Effect of skeleton resonance

In order to corroborate the tendencies from the experimental results, a comparison with a finite element model (FEM) has been performed with a FEM code [9, 10], where the melamine foam is considered as a poroelastic material. Figure 8 compares the different approaches for a 20 mm thick lining. A resonance appears in the same frequency band, around 1360 Hz, followed by a decrease. The analytical model based on a rigid-framed model is a good average of the poroelastic model. When reducing the thickness (figure 9) the same tendency is observed at higher frequencies.

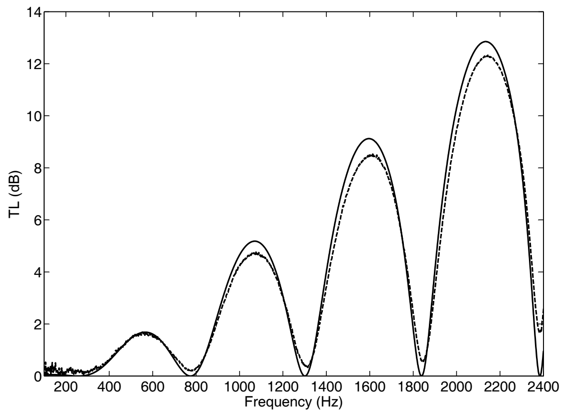

Figure 6: Experimental and analytical transmission loss for a bare cylindrical baffle muffler with δ = 20 mm. (dashed lines) Experimental, (continuous lines) Analytical.

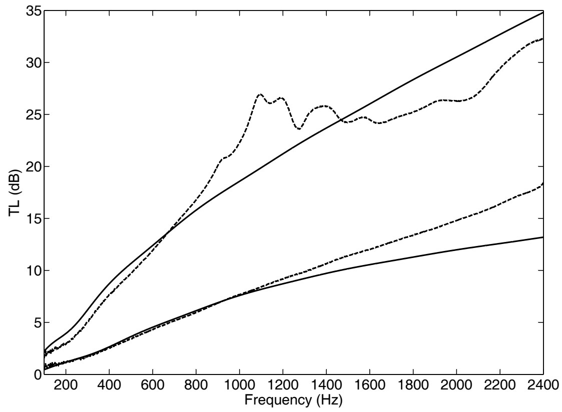

Figure 7: Experimental and analytical transmission loss for a cylindrical baffle muffler for two cases: (top curves) δ = 20 mm and (bottom curves) δ = 10 mm. (dashed lines) Experimental, (continuous lines) Analytical.

4 Conclusion

This paper examined the acoustic transmission loss of a cylindrical baffle muffler at low frequency. An analytical model based on the transfer matrix method is derived. The porous material is described with a rigid framed model. The analytical model has been compared successfully to experimental results, though the latter can not describe the skeleton resonance, that was clearly shown on the experimental results. Finally, comparison with a poroelastic FEM code shows good trends. Further work will deal with rectangular section and multiple baffles mufflers.

Acknowledgments

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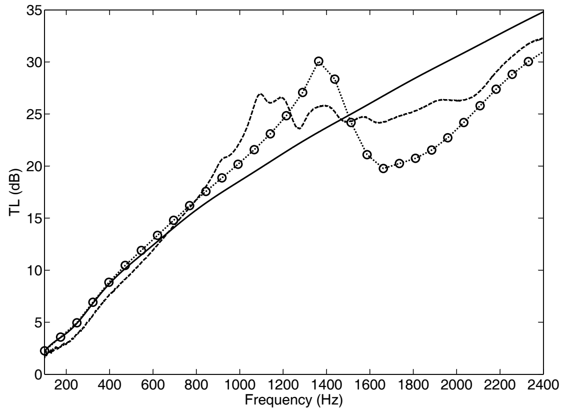

Figure 8: Comparison of transmission loss for the cylindrical baffle muffler of 20 mm thick: (continuous line) analytical, (dashed line) experimental and (circle marker) numerical.

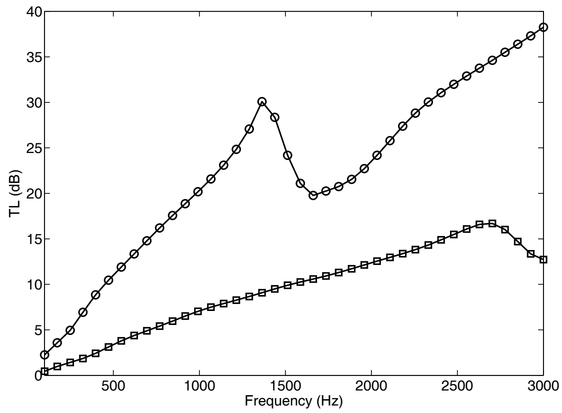

Figure 9: Simulated transmission loss by poroelastic FEM for two thicknesses: (circle marker) δ = 20 mm and (square marker) δ = 10 mm.

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