Phase shift and group delay in phononic crystals of pillars on a surface

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We report the study of the group delay in a 2D phononic crystal, consisting of a square lattice of nickel pillars on a lithium niobate substrate surface. We first focus on the phase shift induced by the periodic structure on the surface acoustic waves within and lying out of the sound cone. An abrupt phase-jump is observed in the vicinity of the locally resonant band gap (low frequency regime) whereas it becomes constant inside the Bragg band gap. This latter behavior has been previously observed in the case of the propagation of bulk waves in 3D phononic crystals [1].

1 Introduction

Electrical measurements of the transmitted acoustic wave’s magnitude are rapid and practical way to evaluate the attenuation inside an involved medium. Combined with the measured backscattered wave, we can roughly estimate the amount of the transmitted, reflected and dissipated energies. However, this physical quantity (i.e. magnitude) does not reveal any information about the wave propagation inside the medium, especially when the considered medium induces an intensive group delay function of frequency such as phononic crystals[2, 3] and acoustic metamaterials [4].

Phononic crystals are periodic media made of at least two different materials. They can exhibit frequency absolute band gaps in certain frequency ranges where acoustic and elastic waves are prohibited to propagate in all directions. Band gaps in phononic crystals occur because of destructive Bragg interferences. Furthermore, the prohibited wavelengths could be expected to approach double the pitch of periodicity. An attenuation of several dBs can be observed in the transmission magnitude. Theoretically, the study of the band structures shows a total absence of modes and only evanescent Bloch waves are allowed to exist inside the periodic structure. Far bellow the band gap range (long wavelength), if inclusions do not present any frequency resonance, frequency versus the wave vector is a straight line. Consequently, the phase and the group velocities are equal and the periodic structure induces no wave dispersion. When we approach the band gap, the inhomogeneous medium becomes more dispersive. For frequencies above the band gap, negative refraction, diffraction, strong and complex coupling between modes can be observed.

The case of acoustic metamaterials is a bit different. Although they are usually presented as periodic structures, the principal mechanism behind the special way of steering waves is a consequence of the local resonance of each individual scatter. This internal modal frequency resonance interacts with the continuum modes supported by the host matrix leading to the occurrence of the locally resonant band gap [5]. Therefore, the propagated waves are significantly slowed down for frequencies approaching the resonance illustrated by flat bands in the band diagram.

In a recent paper, we investigated the propagation of acoustic waves in a two dimensional periodic structure consisting of a square lattice of cylindrical pillars on a lithium niobate substrate surface. We demonstrated theoretically and experimentally that such structure exhibit the locally resonant and Bragg band gaps. The theoretical study has been performed using the efficient Finite Element Method to study both the dispersion (band diagram) and the attenuation (electrical transmission). By contrast however, dispersion has not been investigated experimentally. In this paper, we focus on the study of the phase shift and the group delay of the surface acoustic waves in the phononic crystal of pillars on a surface.

2 Pillars on a semi infinite medium

Figure 1 represents schematic of the sample used to investigate the propagation of surface acoustic waves in pillars based phononic crystals. Pillars were grown on a non pyroelectric Y+128° lithium niobate substrate using electroplating (Black niobate). The process steps are based on lithography, vapor deposition and etching (See [6] for the process details). Achieved pillars are 4.7 μm high and have a radius of 3.2 μm arranged according to a 10μm-pitch square lattice. Surface acoustic waves are generated and detected using chirped interdigital transducers (CIDTs). These transducers are broadband sources placed on both sides of the periodic structure, as depicted in Figure 1. The useful measurement bandwidth extends from 70 to 260 MHz but split into two for practical reasons. Indeed, a threshold should be taken into account between the number of the considered fingers, the dynamic response and the bandwidth of interest. With a SAW velocity in the range of 3600 m/s, the Bragg band gap is expected at frequencies around 180 MHz and the locally resonant band gap at 80MHz [6].
3   Locally resonant band gap

3.1   Attenuation :

In figure 2 we depict the transmission magnitude using the network analyzer for evaluating the attenuation level. The green and the red lines present the transmittance with and without (reference) periodic structures in between respectively. It is shown that for frequencies below and above 80 MHz, an excellent match between the two curves is observed. Therefore, the surface acoustic wave is transmitted with practically no loss in these frequency ranges. Around 80 MHz, a clear attenuation larger than 20 dB is noticed suggesting the existence of a band gap for surface guided waves around this frequency. Several different samples have been fabricated and characterized in order to vary the direction of propagation and the investigated frequency range for the demonstration of the complete band gap [6]. Here, we report only the measurements along the X-crystallographic direction in order to avoid the eventual leaky surface acoustic waves (LSAWs) excitation that will induce a complicated study of the dispersion.

![Figure 2: Measured electrical transmission using chirped interdigital transducers with (green line) and without (red line) the periodic structure in between. Case of the locally resonant band gap (around 80 MHz).](image2)

3.2   Phase shift :

According to our previous work, the optical characterizations reveal the energy storage inside the pillars at the resonance frequency and a very good wave transmission for frequencies below and above the band gap range. These wave propagation mappings are static in the way that we cannot be well-informed on the dispersion inside the inhomogeneous medium. The dispersion relation \( \omega = kc \) (\( \omega \) the angular frequency, \( k \) the wave number and \( c \) the sound velocity) could hence be deduced by the study of the phase shift. Indeed, \( k=\varphi/L \) (\( \varphi \) is the cumulative phase and \( L \) the sample width). In figure 3, we depict the cumulative phase shift induced by the periodic structure. Cumulative phase shift is as well useful to evaluate the difference of the phase and group velocities between the inhomogeneous medium and the free surface. Figure 3 shows that the phase shift induced by the periodic structure for frequencies below 70 MHz almost linear with a weak slope. This means that the energy is delayed slightly with no dispersion. Once the resonance is reached, an abrupt phase jump is observed revealing that the energy is slowed down at these frequencies. Above the resonance frequency, the inhomogeneous medium becomes non dispersive but the wave propagation as well as the energy travel with velocities lower than the propagation in the free surface. This is the consequence of a linear variation of the phase shift function of frequency.

![Figure 3: The frequency dependence of the cumulative phase shift between the free medium and the periodic structure along the X-crystallographic direction of a Y+128° lithium niobate substrate in the vicinity of the locally resonant band gap.](image3)

4   Bragg band gap

4.1   Attenuation

We afterward resume the same study at higher frequency. In figure 4, we represent the electrical transmissions with and without the presence of the periodic structure (green and red lines respectively). It is demonstrated that the wave is attenuated up to 20 dB from 140 MHz to 260 MHz with a partial transmission at 200 MHz. It has been demonstrated that till 185 MHz, the attenuation is due to Bragg scattering (perfect mirror). Above 185 MHz in contrast, the surface acoustic wave is radiated into the bulk.

![Figure 4: Measured electrical transmission using chirped interdigital transducers with (green line) and without (red line) the periodic structure in between. Case of the Bragg band gap.](image4)
line) the periodic structure in between. Case of the locally resonant band gap (around 150 MHz).

4.2 Phase shift

In figure 5, we notice that the cumulative phase shift continue to grow linearly till 140 MHz and a second jump is observed at the edge of the band gap. Inside the band gap, the phase shift becomes constant (in more than 50 MHz). Inside the radiative zone (starting from 210 MHz and rising), a sharp linear variation with negative slope is representing an abnormal behavior.

Figure 5: The frequency dependence of the cumulative phase shift between the free medium and the periodic structure along the X-crystallographic direction of a Y+128° lithium niobate substrate in the vicinity of the Bragg band gap.

5 Conclusion

In summary, we demonstrated that the presence of pillars upon a semi-infinite medium induce a variable phase shift function of frequency. It is indeed clearly demonstrated that for frequencies below the locally resonant band gap, practically no phase shift is observed. Inside the bad gap, an abrupt phase enhancement is noticed especially at the frequency resonance (80 MHz). Above this frequency, the phase shift has a linear dependency with respect to frequency. In this region, a quasi-perfect transmission of the wave is noticed. This linear variation will induce a constant group delay compared with the propagation in the free surface. Inside the frequency band gap, the phase shift remains unchanged. Inside the radiative zone (starting from 210 MHz and rising), a sharp linear variation with negative slope is representing an abnormal behavior.

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References