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Modelling of reverberation enhancement systems

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Electroacoustic reverberation enhancement systems (RES) are increasingly specified by acoustic consultants to address the requests for a multi-purpose use of performance halls. However, there is still a lack of simple models to predict the effect induced by these systems on the acoustic field. Two models are introduced to establish the impulse responses of a room equipped with a regenerative reverberation enhancement system. These models are based on passive impulse responses according to the modified theory of Barron & Lee or to the diffuse stochastic fields approach introduced by Polack. The action of the system is simulated either with an energetic approach derived from Sabine's theory or by solving the frequency equation governing a multi-variable loop system (FMLSE). The acoustic criteria derived from these models are compared with those obtained with a reference method. This method is based on the numerical calculation of impulse responses by asymptotic methods (ICARE software developed at CSTB) and the resolution of the FMLSE.

1 Introduction

Wide band regenerative RES such as the MCR system [1] or the CARMEN®system [2] are designed to increase reverberation time and acoustic strength by the use of delays and gain in their signal processing units. In order to predict the effect of the MCR system on these two important room acoustic criteria Franssen [3] and DeKoning [1] proposed simple formulas based on Sabine's theory but without taking into account the eventual delays. Poletti [4] did so but the accuracy of his formula has not been checked yet. For the modification of the clarity C80 Svensson [5] proposed an analytical approach but only for a one-channel RES and without taking into account the entire feedback effect. For other criteria such as the EDT no work has been published until now.

In this paper, we present two methods to predict the effect of a diagonal regenerative RES on a room impulse response and thus on the room acoustic criteria. The changes predicted by these methods and a reference method on the reverberation time (RT), the acoustic strength (G), the clarity (C80) and the early decay time (EDT) due to the used of a regenerative RES are compared for a room which could be typical for the need of a RES.

The reference method calculates active impulse responses (with RES) from the frequency equation governing a multi-variable loop system (FMLSE) and passive impulse responses (without RES) obtained with ray-tracing algorithms. This method was already described in the literature [6] ; its principle is summarized in the first part of this paper. The two theoretical methods are presented in the following part. The first one uses a gain function derived from Sabine theory according to Poletti approach [4]. Because these method stem from classical diffuse field theory, it is mainly designed to be applied to passive impulse responses resulting from theories such as Barron & Lee's revised theory [7] which is also briefly summarized here. The second approach estimates the components in the active impulse responses due to the RES. This approach uses the FMLSE and a set of passive impulse responses synthesized according to a stochastic approach introduced by Polack [8]. The results of this comparative study are shown in the last part. Although this study focus on the CARMEN®system develloped by the CSTB, the methods and probably the results could also be extrapolated to the MCR system or other wide band regenerative RES.

2 Numerical simulation of RES

A RES is a multi-variable loop system. Thus, the transfer function of the active room can be obtained from a simple formula in the frequency domain, which is the sum of the passive transfer function and the additional components due to the RES. The impulse response is obtained by using the Fourier transform of the result.

$$H_{act} = H_{sr} + \underline{H_{lr}}\,(\underline{\underline{I_d}} - \underline{\underline{G_{ml}}}\,\underline{\underline{H_{lm}}})^{-1}\underline{\underline{G_{ml}}}\,\underline{H_{sm}} \quad (1)$$

Where H_{sr} is the passive acoustic transfer function between the source and the receiver, $\underline{H_{sm}}$ is a column vector of the acoustic transfer functions between the source and each microphone, $\underline{H_{lr}}$ is a line vector of the acoustic transfer functions between each loudspeaker and each receiver, $\underline{\underline{H_{lm}}}$ is a matrix of the acoustic transfer functions between each loudspeaker and each microphone, $\underline{\underline{I_d}}$ is the unity matrix and $\underline{\underline{G_{ml}}}$ is a matrix of the electronic transfer function between each microphone and each loudspeaker as illustrated in Figure 1 (if a microphone is connected to only one loudspeaker, the $\underline{\underline{G_{ml}}}$ matrix and the RES are diagonal).

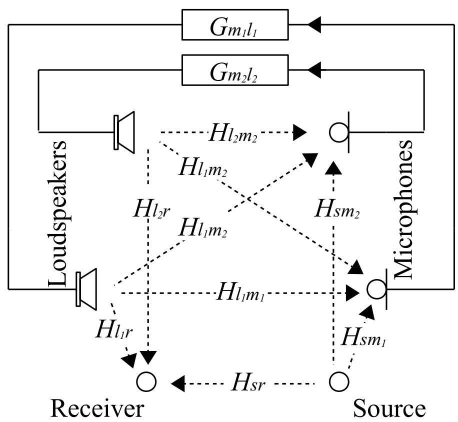

Figure 1: A schematic representation of a two channel diagonal reverberation enhancement system

The different acoustic transfer functions used in Eq. (1) can be obtained by using of a ray-tracing algorithm. For our work, we used the ICARE software developed by the CSTB [9] and based on the combination of beam tracing, source image methods and particles tracing.

The electronic transfer functions are obtained according to an iterative process in three steps to reach the desired open loop gain which is generally set around -17 dB in order to avoid instability and limit frequency coloration [10].

3 Theoretical approaches

3.1 Theory based on energetic approach

The RES's gain function

According to Sabine's assumption, the reverberant sound field in a room is governed by the differential equation [11] :

$$\frac{\mathrm{d}\omega(t)}{\mathrm{d}t} + \left(\frac{Ac}{4V} + \mu c\right)\omega(t) = \frac{1}{V}P_s(t) \quad (2)$$

Where $\omega(t)$ is the volumetric energy density, A is the total absorption area of the room, V is the room's volume, $P_s(t)$ is the source's power, c is the celerity of sound and μ is a coefficient due to air absorption.

If the source signal is a short impulse, the reverberant sound can not exist before the arrival of the direct sound. Thus Eq. (2) is valid only for time values up to t_d, where t_d is the arrival time of the direct wave front ($t = 0$ is the time when the pulse is emitted by the source). With the assumption that the reverberant sound field exists instantly after t_d, the solution of Eq. (2) is:

$$\omega(t > t_d) = \beta e^{-\left(\frac{Ac}{4V} + \mu c\right)t} \quad (3)$$

where β is a coefficient corresponding to the volumetric energy density at time t_d.

In this simple approach the RES could be seen as additional sources whose power depend on the electronic gain, the delay on each channel and the energy density [4]. If we assume a diagonal RES of N_c channels, each one with the same electronic gain and the same delay value $\bar{\tau}$, the equation (2) becomes :

$$\frac{\mathrm{d}\omega_{act}(t)}{\mathrm{d}t} + \left(\frac{Ac}{4V} + \mu c\right)\omega_{act}(t) = \frac{1}{V}P_s(t) + \left(\frac{Ac}{4V} + \mu c\right)N_c\bar{\gamma}\omega_{act}(t - \bar{\tau}) \quad (4)$$

The mean open loop gain $\bar{\gamma}$ is introduced because it is the limiting factor on the system power. It can be expressed by the product of the electronic gain, the acoustic feedback gain of a channel and a factor depending on the transducers sensitivity and directivity. According to the diffuse field theory the acoustic feedback gain is simply the level of reverberant sound produce by a stationary unity power source. Thus the mean open loop gain is:

$$\bar{\gamma} = \frac{\bar{G}\xi}{V\left(\frac{Ac}{4V} + \mu c\right)} \quad (5)$$

with $\bar{G}$ the electronic power gain of one RES′s channel and ξ the transducer factor.

Although Eq. (4) is a solvable delay differential equation, the resulting expression of the reverberated density energy is complicated because it is different for each time period [12]. For solving this differential equation Poletti [4] proposed to use the Laplace transform and to approximate the exponential term due to the delay in this mathematical space by it's first-order Taylor series expansion centered at zero [1]. Back in the time domain Eq.(4) becomes :

$$(V + N_c\bar{\gamma}\bar{\tau}(\frac{Ac}{4} + V\mu c))\frac{\mathrm{d}\omega(t)}{\mathrm{d}t} = P_s(t) - (\frac{Ac}{4} + V\mu c)(1 - N_c\bar{\gamma})\omega(t) \quad (6)$$

Considering the same source characteristics that those used to establish Eq. (3), the resolution of differential equation Eq. (6) yields to

$$\omega_{act}(t > t_d + \bar{\tau}) = \beta' e^{-\left(\frac{Ac}{4V} + \mu c\right)\left(\frac{1-N_c\bar{\gamma}}{1+\left(\frac{Ac}{4V}+\mu c\right)N_c\bar{\gamma}\bar{\tau}}\right)t} \quad (7)$$

Because the RES can only react after a short time gap corresponding to the delay value, the β' factor can be obtained using the passive energy density given by Eq. (3) at the time $t_d + \bar{\tau}$.

$$\beta' = \beta e^{-\left(\frac{Ac}{4V} + \mu c\right)\left(1-\left(\frac{1-N_c\bar{\gamma}}{1+\left(\frac{Ac}{4V}+\mu c\right)N_c\bar{\gamma}\bar{\tau}}\right)\right)\bar{\tau}} \quad (8)$$

Finally, the gain function of the system is the ratio of Eq. (6) and Eq. (3).

$$\Gamma(t) = \begin{cases} 1 & \text{for } t < t_d + \bar{\tau} \\ e^{\left(\frac{Ac}{4V} + \mu c\right)\left(1-\left(\frac{1-N_c\bar{\gamma}}{1+\left(\frac{Ac}{4V}+\mu c\right)N_c\bar{\gamma}\bar{\tau}}\right)\right)(t-\bar{\tau})} & \text{for } t \geq t_d + \bar{\tau} \end{cases} \quad (9)$$

Barron & Lee's revised theory [7]

The revised theory was introduced by Barron & Lee to take into account the decrease of the total reverberant sound energy levels in concert hall's impulse responses with the increase of the distance from the source to the receiver. Their model supposes that the reflected energy according to the original Sabine's theory is correct only for the case when the source and the receiver are at the same position. In case they are not, the part of the reflected energy density before the arrival of the direct sound is truncated from the impulse response. Thus, compared to Sabine's original theory, the total energy density of the reverberant part of the impulse response is reduced by a factor $e^{-\left(\frac{A}{4V}+\mu\right)r}$, where r is the source-receiver distance.

The impulse response obtained from this revised theory is composed by the direct sound which exist at time t_d and whose value is calculated according to the geometrical decrease of an omnidirectionnal acoustic source:

$$\omega(t = t_d) = \frac{P_s}{4\pi c r^2} \quad (10)$$

The reverberated energy density for time values up to t_d:

$$\omega(t > t_d) = \frac{P_s\delta t}{V}e^{-\frac{6\ln(10)}{RT}t} \quad (11)$$

where δt is the duration of the initial pulse. The decrease of the reverberant energy density with source-receiver distance is taken into account by the condition "$t > t_d$".

3.2 Theory based on a stochastic approach

The passive stochastic impulse response [8]

This approach is based on the diffused sound particles concept. The total reflected energy at a specific time is the summation of each particle's energy arriving at this time. The energy of one particle depends on the number of reflexions it has been subjected to and the total length it has traveled from

[1] this approximation is justifiable because in regenerative RES the delay values must not be to important in order to limit detection problems [10]

the source to the receiver. Polack suggests that in diffuse field conditions the number of sound particles arriving at the receiver position is governed by a stochastic time dependent function, as well as the number of reflexions of each particles. These two quantities can be obtained by the generation of two independent random numbers.

The reflected component of an omnidirectionnal room impulse response produced by a source whose power is equal to P_s :

$$\omega(t > t_d) = \frac{P_s}{4\pi c^3 t^2} e^{-\mu ct} \sum_{i=1}^{N(t)} (1 - \bar{\alpha})^{n_i(t)} \quad (12)$$

where $\bar{\alpha}$ is the mean absorption Sabine's coefficient, $N(t)$ is the number of incident particles on the receiver at time t during an small time interval δt (typically equal to the duration of the initial pulse) and $n_i(t)$ is the number of reflexion a particle i has been subjected to.

$N(t)$ is obtain by a random number generator following a Poisson process with a rate parameter λ_N equal to the mean number of image sources contained in two spheres of diameter equal to ct and $c(t + \delta t)$. Because the volume occupied by one image source is simply the volume of the room:

$$\lambda_N = \frac{4\pi c^3 t^2 \delta t}{V} \quad (13)$$

$n_i(t)$ is obtain by a random number generator following a Poisson process with a rate parameter λ_n equal to the total length traveled by the particle divided by the mean free path of the room :

$$\lambda_n = \frac{ctS}{4V} \quad (14)$$

where S is total surface of the room.

In order to take into account the decrease of reverberated sound with the distance, it is here proposed here to truncate the components of the impulse response in Eq. (12) arriving before the arrival of the direct sound, as it is done in Barron & Lee revised theory. Finally, in order to obtain the total impulse response, the direct sound ω_d from Eq. (10) is added at time t_d.

The combination of passive stochastic impulse responses with FMLSE

The impulse responses generated by the above described method can be injected in Eq. (1) after proceeding the Fourier transform. Thus one can calculate separately the passive and the active components of the active impulse responses. To avoid incoherences, such as the action of the RES before the arrival of the direct sound,the use of realistic distances between the source, the receivers and the transducers of the RES is needed. For this purpose a typical active shoe box concert hall from which these distances are calculated is automatically generated. The relative geometrical characteristics of this room, such as the ratio of the height and the length, are fixed according to their mean values in this kind of hall as observed by Haan [13]. The volume of the room is a parameter specified by the user. The source in this virtual room is placed on stage and receivers are located in the seat area at a distance r from the source. The transducer of the RES are homogeneously positioned on the lateral walls and the ceiling. Because in this study we focus on the CARMEN®system, the microphone and the loudspeaker of a same channel are close to each other [2].

4 Results

The above described theoretical approaches were compared to a numerical simulation for a shoe-box shaped hall. With a relative low RT at mid frequencies of 1 s and a approximative volume of 10000 m^3, it is typical of halls in which a regenerative RES could be installed. This hall also has rear and side balconies. The RES simulated is a thirty-channel CARMEN®system. Twelve of these channels are located on the ceiling, one on the back of each side walls, eight under the narrow side balconies, and three under the two deep rear balconies. Under the balconies the microphone and the loudspeaker of each channel are not placed close to each other. The microphone is on the front of the balcony and the corresponding loudspeaker is in the under balcony volume. For each RES channel, the target mean loop gain was set to -18 dB, and the delay value was set to 0.02 s. The source position is in the middle of the front stage. The receiver positions are chosen beyond five meter from the source.

The relative variations of RT, G, EDT and C80 are calculated from their mean values over the 500 Hz and 1000 Hz octave bands. These variations are averaged over 30 receiver positions. 30 main volume receiver positions and 30 under balconies receiver positions have been considered separately, because under deep balconies the behavior of the some acoustical criteria differ from what is observed in the main room. For example C80 in the main volume is known to decrease with the increase of the distance from the source, whereas the opposite is observed under deep balconies [14]. This behavior has been checked from the passive ray tracing simulated impulse responses.

The theoretical gain due to the RES obtained from Eq. (9), and the extra active components calculated from the combination of passive stochastic impulse responses and FMLSE can be applied either on theoretical passive impulse responses or on numerically simulated ones in order to evaluate the accuracy of the passive impulse responses on the predicted variations of RT, G, EDT and C80. The evolutions of this acoustic criteria obtained with numerical simulations withing their just-noticeable differences (JND) [15] have been compared to four theoretical or semi-theoretical approaches:

- Eq. (9) applied on simulated passive impulse responses from the revised theory (E_{BL})
- Eq. (9) applied on simulated passive impulse responses from the ray-tracing algorithm (E_{Ray})
- The active part of the impulse responses calculated from the theory based on the stochastic approach and the FMLSE added to passive impulse responses obtained from the same stochastic approach (S_{ST})
- The active part of the impulse responses calculated from the theory based on the stochastic approach and the FMLSE added to passive impulse responses obtained from the ray-tracing algorithm (S_{Ray})

4.1 Passive criteria

Table 1 presents the acoustical criteria deduced from the passive impulse responses generated by the ray-tracing algorithm, Barron & Lee's revised theory and the stochastic approach. The RT values are obviously the same in the three different approaches because the RT value obtained from the ray-tracing algorithm is the parameter used to deduce the mean absorption coefficient used in the two other theory to establish the impulse response. The three other parameters differ in small proportions from one approach to another, especially in the main volume. Thus this hall can be considered as diffusive in the sense that it is in good agreement with the Barron & Lee's revised theory and the Polack's stochastic approach, which are both based on a diffuse field assumption.

Table 1: Mean passive acoustical criteria

	Impulse response	RT (s)	EDT (s)	G (dB)	C80 (dB)
Main volume	Ray tracing	1.0	0.9	4.1	5.7
	Revised theory	1.0	1.0	4.1	5.1
	Stochastic	1.0	1.1	3.7	4.8
Under balconies	Ray tracing	1.0	0.7	2.8	5.7
	Revised theory	1.0	1.0	2.5	4.3
	Stochastic	1.0	1.0	2.2	4.0

4.2 Reverberation time

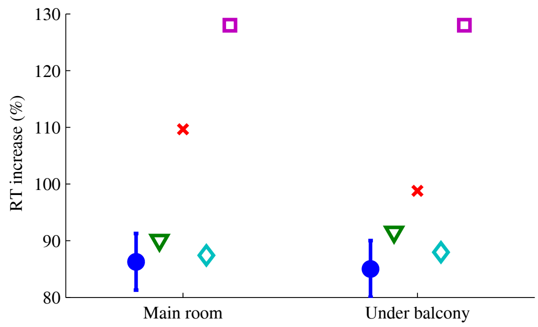

Figure 2: Numerical simulated and theoretical evolution of RT in the main room and under balconies
•: Numerical simulation ; I: JND
◊: S_{ST} ; □: E_{BL} ; ▽: S_{Ray} ; ×: E_{Ray}

As shown in Figure 2 each of the theoretical approaches based on the Sabine's energetic theory tend to excessively overestimate the effect of the RES on the RT in the main volume as well as under balconies. Both of the approaches based on stochastic impulse responses predict RT sufficiently close to the numerical simulations, within the JND. This tends to prove that for the prediction of this criterion, the theoretical approach used for simulating the action of the RES seams important, whereas taking into account the original passive impulse response or the simulated one as no effect. This is easily explained since the RT is a robust parameter withing the seat positions [11] and since the value of RT obtained with numerical simulations is one of the parameter used to establish the passive impulse responses from the two diffuse theoretical approaches.

4.3 Strength

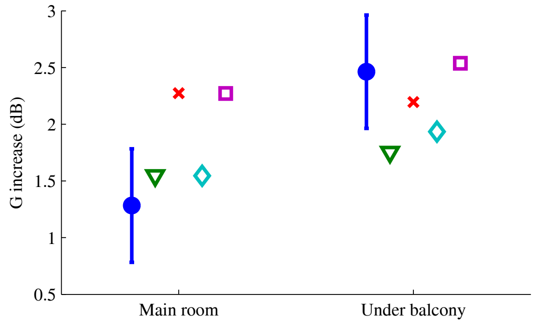

Figure 3: Numerical simulated and theoretical evolution of G in the main room and under balconies
•: Numerical simulation ; I: JND
◊: S_{ST} ; □: E_{BL} ; ▽: S_{Ray} ; ×: E_{Ray}

Regarding the acoustic strength increase in the main volume, the approaches based on stochastic impulse responses are close enough to the numerical simulations whereas the approaches based on Sabine's theory overestimate the action of the RES (see Figure 3). Once again, the passive impulse response has no effect on the accuracy. This results is not surprising since the passive strength index predicted with the numerically simulated impulse responses and the theoretical approaches are almost the same.

Under the balconies, the approaches based on Sabine's theory are more accurate than the stochastic based approaches (see Figure 3). The RES channels located under the balconies have a less important feedback effect because their microphones and their loudspeakers are not in the same acoustical volume due to coupling phenomena. So, for an equal mean loop gain, the under balcony RES channels provides more energy to the under balcony volumes than in the main room. Thus, the approaches based on Sabine's theory which usually overestimate the effect of the RES are for these particular seat positions closer to what is obtained with numerical simulations.

4.4 Clarity

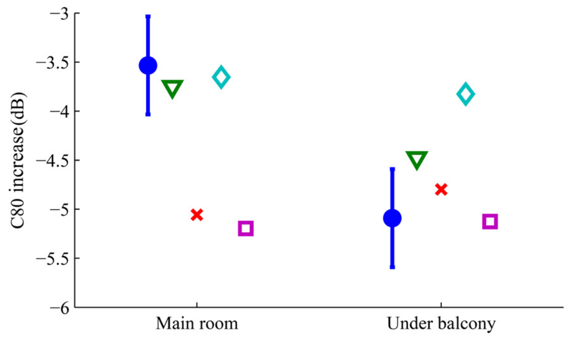

Figure 4: Numerical simulated and theoretical evolution of C80 in the main room and under balconies
•: Numerical simulation ; I: JND
◊: S_{ST} ; □: E_{BL} ; ▽: S_{Ray} ; ×: E_{Ray}

As shown on Figure 4, to predict the clarity index increase, the four theoretical approaches compared to the numerical simulations behave as for the predictions of the strength index. Therefore similar conclusion apply.

4.5 Early decay time

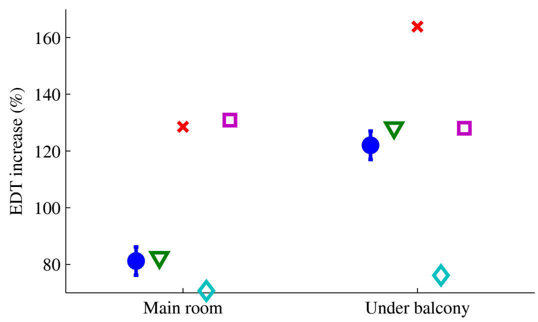

Figure 5: Numerical simulated and theoretical evolution of EDT in the main room and under balconies
•: Numerical simulation ; I: JND
◇: S_{ST} ; □: E_{BL} ; ▽: S_{Ray} ; ×: E_{Ray}

In the main volume, the EDT increase predicted by the method S_{Ray} is the only of the four theoretical approaches which is close enough to the numerical simulation within the JND. Under the balconies this method is still sufficiently accurate. It must be noticed that under the balconies E_{Ray} dramatically overestimates the effect of the RES. E_{BL} gives predicted EDT increase very close to the numerical simulations even then E_{Ray} is more approximative because it doesn't take into account the exact passive components. Thus this latter result should be otherwise confirmed.

5 Conclusion

In the main volume of the hall, the stochastic based approaches seam to be accurate enough to predict the evolution of the RT, G and C80 due to the RES compared to the numerical simulations. The approaches based on Sabine's theory overestimate the action of the RES. For these criteria the used of the exact passive impulse response or a theoretical one has no significant effect. However this result must be carefully considered since the passive hall is initially in good agreement with the different theoretical approaches. For the EDT only the stochastic based approach with the used of the exact passive impulse response is accurate enough.

Under balconies the same conclusion can be made for the EDT evolution due to the RES. For the RT, G and C80, the fact that the approaches based on Sabine's theory overestimate the action of the RES in the main hall's volume is, for the under balcony volumes a lucky coincidence. Indeed, RES channels placed under balconies provides more energy in these particular seat positions due to the coupling effect, whereas this aspect is not taken into account in the theoretical approaches.

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