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Modeling of the interaction of an acoustic wave with immersed targets for telemetry of complex structures

B. Lu[a,b], M. Darmon[a], C. Potel[b], L. Fradkin[c] and S. Chatillon[a]

[a]CEA, LIST, CEA Saclay, Point Courrier 120, 91191 Gif-Sur-Yvette Cedex, France, Metropolitan
[b]Laboratoire d'acoustique de l'université du Maine, Bât. IAM - UFR Sciences Avenue Olivier Messiaen 72085 Le Mans Cedex 9
[c]Sound Mathematics Ltd., 11 Mulberry Close, CB4 2AS Cambridge, UK
sylvain.chatillon@cea.fr

This study is part of the development of simulation tools for ultrasonic telemetry. Telemetry is a technique chosen for monitoring sodium-cooled fast reactors, which consists in locating various reactor structures using an ultrasonic inspection performed by immersion. In order to model the interaction between the acoustic wave and the immersed structures, classical scattering models have been firstly evaluated for rigid structures, including the geometrical theory of diffraction (GTD) and the Kirchhoff approximation (KA). These two approaches appear to be complementary. Combining them so as to retain only their advantages, we have developed the so-called refined KA based on the physical theory of diffraction (PTD). Applying all these models to the problem of high-frequency acoustic wave scattering from immersed rigid half planes enables to show their deficiencies, advantages and applicability domains. A theoretical comparison of these models is carried out to define the more adequate one for the development of a software simulation tool for ultrasonic telemetry of rigid structures. It is shown that the refined KA provides an improvement of the prediction in the near field of a rigid scatterer.

Introduction

Monitoring and inspection of nuclear reactor are stringent requirements from operator and safety authorities. The sodium-cooled fast reactor (SFR) is one of the perspectives chosen for the 4th generation reactor. The characteristics exhibited by sodium, such as its opacity, have led the designers to devise specific monitoring and inspection techniques. Consequently ultrasonic techniques are seen as suitable candidates. Two approaches are being followed: the core monitoring where transducers are directly immersed in sodium near the reactor's core and the outside inspection with transducers located along the wall of the main vessel (outside sodium medium).

Ultrasonic telemetry is one of the core monitoring techniques that allows checking the position of the various objects contained inside the main vessel and the possible detection of defects inside these objects. The distance between the transducer and the immersed targets can be determined by measuring the time of flight of backscattered acoustic waves generated by the transducer installed inside. While in-service the flow of sodium creates turbulence that leads to temperature inhomogeneities, which convert into ultrasonic velocity inhomogeneities. A wave propagation model has been developed in a previous work to calculate the ultrasonic field radiated in an inhomogeneous medium [1, 2]. Different scattering phenomena can also be produced during the interaction between the acoustic beam radiated by the probe and the immersed targets: specular reflection, tip diffraction from boundaries and edges of the different parts and corner effect. Thus various parameters will influence this technique behavior. In order to optimize the parameters of the dedicated probes to conceive and to predict the probes performances, a simulation tool is necessary to assist the design of each element of the ultrasonic telemetry.

Firstly the scattered acoustic field can be modeled using the high-frequency asymptotics known as the Geometrical Acoustics (GA) and Geometrical Theory of Diffraction (GTD) [3], all based on ray theory. The former describes incident and reflected waves and the latter, wave diffracted by obstacle edges (the so-called edge waves). The regions that support different kinds of waves are classified as either geometrical regions (illuminated region and shadow region) or transition zones that are the boundaries between an illuminated region and a shadow region. The sum of GA and GTD gives a perfectly adequate description of geometrical regions but fails inside transition ones. A more sophisticated uniform GTD is required to complete the description.

In some NDE applications, another approach, the so-called Kirchhoff approximation (KA) [4], is widely used in high-frequency scattering problems, particularly when dealing with obstacles of a complicated shape. The fundamental principle of this method is the use of Green's function representing in a given spatial region Σ the solution to the Helmholtz equation. The KA provides a correct description of the reflected wave and the fields inside the transition region. The integral formulation of the KA solution enables description of the field in more intricate regions, such as focusing areas, shadow boundaries of edge waves, where the known GTD procedures are no longer applicable. This approximation leads to qualitatively correct description of edge waves, but can provide errors in the amplitude prediction. To eliminate the deficiencies of KA and GTD and combine their advantages, a model called here the refinement of KA is proposed to modify KA by employing GTD diffraction coefficients: this approach is based on the Physical Theory of Diffraction (PTD) [5].

The targets to be inspected by sodium telemetry are steel structures immersed in liquid sodium and are consequently characterized by nearly rigid boundary conditions. In this paper, we will therefore focus on a comparison of scattering models applied to the scattering of an acoustic wave by a rigid half plane.

1 Geometrical Theory of Diffraction

1.1 Non uniform asymptotic solution for scattered waves

Consider a simple example: a plane wave scattered by a two-dimensional half plane as shown in Figure 1. The presence of this half plane in a plane incident wave field (with θ_0: incident angle) gives rise to a shadow region of incident wave, a reflected wave and the related shadow region. Thus two light-shadow boundaries can be identified which are function of the incident angle: $\theta_1 = \pi - \theta_0$, $\theta_2 = \pi + \theta_0$. The incident and reflected wave are given by Geometrical Acoustics and these solutions are discontinued on light-shadow boundaries jumping to zero in the shadow region. These jumps to zero of the field will be eliminated by adding the diffracted fields. Therefore the scattered field can be written as

$$U^{\mathrm{Scat}} = U^{\mathrm{GA}} + U^{\mathrm{GTD}}. \qquad (1)$$

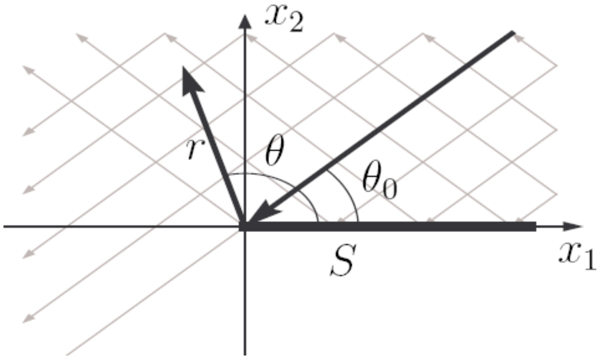

Figure 1: scattering of a plane acoustic wave by a half plane

The geometrical acoustic fields can be written as

$$U^{\mathrm{GA}} = U^{\mathrm{Inc}} + U^{\mathrm{Ref}} = e^{-\mathrm{i}kr\cos(\theta-\theta_0)} \pm e^{-\mathrm{i}kr\cos(\theta+\theta_0)}, \quad (2)$$

the plus sign is taken for the Neumann boundary condition (hard case) and the minus sign for the Dirichlet boundary condition (soft case). We'll treat in the following only the Neumann's case (perfect rigid target).

In the problem under consideration the diffraction field is a cylindrical wave generated by the obstacle edge. According to the Geometrical Theory of Diffraction [3], the "main-order" term with respect to large kr in the diffraction field has the form

$$U^{\mathrm{GTD}} \sim \frac{e^{\mathrm{i}kr}}{\sqrt{kr}} D^{\mathrm{GTD}}(\theta,\theta_0), \quad (3)$$

where θ_0 is the incidence angle of the plane wave, r and θ are the polar coordinates of the observation point, k is the wavenumber and $D^{\mathrm{GTD}}(\theta,\theta_0)$ is the diffraction coefficient [3] given by

$$D^{\mathrm{GTD}}(\theta,\theta_0) = -\frac{e^{\mathrm{i}\pi/4}}{2\sqrt{2\pi}}\left(\sec\frac{\theta-\theta_0}{2} + \sec\frac{\theta+\theta_0}{2}\right), (4)$$

where the angles θ and θ_0 are measured with respected to the illuminated surface of the half plane (S^+) as shown in Figure 1. From eqn.(4) it follows that the diffraction coefficient grows without bound if the observation point is at a light-shadow boundary ($\theta = \pi - \theta_0$ or $\theta = \pi + \theta_0$) as shown in Figure 2.

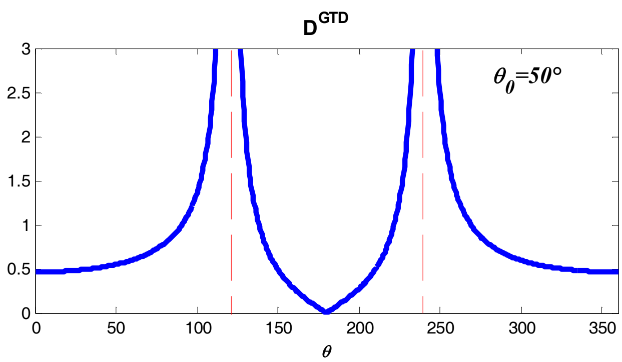

Figure 2: diffraction coefficient for Neumann boundary condition and incidence angle φ0=50°

Clearly this diffraction coefficient is inapplicable in the vicinity of light-shadow boundaries where its poles are located. Hence GTD fails on the light-shadow boundaries.

1.2 Uniform asymptotic theory (UAT) for scattered waves

Several uniform theories derived from GTD exist. One of the two most studied methods, the so-called uniform asymptotic theory of diffraction (UAT), involves the application of Fresnel integral near the light-shadow boundaries [6, 7] in order to smooth the abrupt field shift through the boundaries. Let us take the example of halfplane and there are two light-shadow boundaries: $\theta = \pi + \theta_0$ for the incident wave and $\theta = \pi - \theta_0$ for the reflected wave. The uniform solution U^{UAT} of eqn.(1) is written as

$$U^{\mathrm{UAT}} = U^{\mathrm{GA}} F\{\sqrt{k(s_e - s_{i,r})}\}, \quad (5)$$

where F is the Fresnel integral. For a plane incident wave its argument can be given as follows:

$$\sqrt{k(s_e - s_i)} = \sqrt{2kr}\cos\frac{\theta-\theta_0}{2}, \quad \sqrt{k(s_e - s_r)} = \sqrt{2kr}\cos\frac{\theta+\theta_0}{2}, \quad (6)$$

where s_i, s_e and s_r are the eikonal of respectively incident wave, the edge wave and the reflected wave. Another method proposes a modification of the diffraction coefficient and consists in suppressing the coefficient poles by multiplying it with a transition function having zeros at the poles. This procedure known as the uniform geometrical theory of diffraction (UTD) was described in [8].

2 Kirchhoff approximation

The geometrical theory of diffraction provides short-wavelength asymptotic solutions accurate to the given orders of k for some typical problems to model. Unfortunately, many scattering problems of practical interest have neither rigorous solutions to extract short-wavelength asymptotics nor appropriate asymptotics. Under these circumstances, one has to resort to approximate methods. A widely used method employed for large flaw size compared to the wavelength is the Kirchhoff approximation (KA) [4]. For any geometry, however complicated, the solution of KA is formulated as an integral of the field over the illuminated side of the reflector.

Consider how the KA method formulates a solution to the scattering field from a half plane (see Figure 1). Let us introduce the associated Cartesian coordinate system $\mathbf{x}$ (x_1, x_2), so that we have

$$x_1 = r\cos\theta \quad \text{and} \quad x_2 = r\sin\theta. \quad (7)$$

Let us consider this acoustic problem with the acoustic potential field $U(\mathbf{x})$ satisfying the Helmholtz equation. Fundamental of this method is the use of Green's function G to obtain, by superposition of elementary fields, an expression in the form of an integral equation for a given boundary S (x_1, x_2=0)

$$U^{\mathrm{Scat}}(\mathbf{x}) = \int_{\mathrm{S}}\left[G(\mathbf{x},\mathbf{x}')\frac{\partial U(\mathbf{x}')}{\partial \mathbf{n}} - U(\mathbf{x}')\frac{\partial G(\mathbf{x},\mathbf{x}')}{\partial \mathbf{n}}\right]ds(\mathbf{x}'). \quad (8)$$

Here, $U(\mathbf{x})$ refers to the field on the surface S, $\partial/\partial\mathbf{n}$ implies differentiation along the inward-directed normal to S, $\mathbf{x}'$ denotes one point on the surface S and the Green function $G(\mathbf{x}, \mathbf{x}')$ for the two-dimensional problem studied here takes the form :

$$G(\mathbf{x},\mathbf{x}') = (\mathrm{i}/4)H_0^{(1)}(k|\mathbf{x}-\mathbf{x}'|) \quad (9)$$

with $H_0^{(1)}$ the Hankel function of first kind.

The Kirchhoff approximation is based on the assumption that for $\lambda = (2\pi/k) \ll L$ (L is a typical size of the reflector), i.e. for $kL \gg 1$, one can use the approximation of geometrical acoustics for the total field $U(\mathbf{x}')$ near the surface. In the shadow region, we can set

$$U(x') = \frac{\partial U(x')}{\partial \mathbf{n}} = 0. \quad (10)$$

When evaluating $U(\mathbf{x})$ and $\partial U(\mathbf{x})/\partial\mathbf{n}$ in the illuminated region, the total field is equal to the sum of incident and specularly reflected fields for the Neumann condition or to a difference of these fields for the Dirichlet condition. Therefore, on S:

$$U(\mathbf{x}') = 2U^{\mathrm{Inc}}(\mathbf{x}') \text{ and } \frac{\partial U(\mathbf{x}')}{\partial \mathbf{n}} = 0 \quad \text{for the Neumann condition;} \quad (11)$$

$$U(\mathbf{x}')=0 \text{ and } \frac{\partial U(\mathbf{x}')}{\partial \mathbf{n}} = 2\frac{\partial U^{\mathrm{Inc}}(\mathbf{x}')}{\partial \mathbf{n}} \quad \text{for the Dirichlet condition.} \quad (12)$$

Thus, in the KA, the scattered field from a perfectly rigid surface (Neumann condition) can be written as

$$U^{\mathrm{KA}}(x) = -2\int_0^{+\infty} U^{\mathrm{Inc}}(\mathbf{x}')\frac{\partial G(\mathbf{x},\mathbf{x}')}{\partial \mathbf{n}} ds(\mathbf{x}'), \quad (13)$$

where ds is the surface element along the illuminated surface of the half plane.

3 Refinement of the Kirchhoff approximation

The Kirchhoff approximation has some limitations; the most important one is the incorrect prediction of the diffraction wave amplitudes. To overcome this limitation we are going to correct the Kirchhoff approximation by employing GTD diffraction coefficient. As we can see the GTD diffraction coefficient can be computed in an efficient manner using algorithm given by eqn.(4), this refinement of the Kirchhoff approximation should be quite fast.

In order to correct the Kirchhoff approximation, we should identify different parts inside the Kirchhoff integral (eqn.(13) for a Neumann boundary condition for example). Using the stationary phase method, we find that the integral (13) involves two critical points, a stationary point corresponding to the geometrical field U^{GA} and the lower limit contribution where $x'=0$ corresponding to the diffraction field. This diffraction field contribution has the same form as the U^{GTD} with a different diffraction coefficient:

$$U^{\mathrm{KA(Diff)}} = \frac{e^{\mathrm{i}kr}}{\sqrt{kr}} D^{\mathrm{KA}}(\theta,\theta_0). \quad (14)$$

Using **Erreur ! Source du renvoi introuvable.** to find the asymptotic contribution of integration domain boundary, the Kirchhoff diffraction coefficient, for a Neumann boundary condition (plus sign) and a Dirichlet boundary condition (minus sign), turns to be

$$D^{\mathrm{KA}}(\theta,\theta_0) = -\frac{e^{i\pi/4}}{2\sqrt{2\pi}}\left(\tan\frac{\theta-\theta_0}{2} \pm \tan\frac{\theta+\theta_0}{2}\right). \quad (15)$$

This means that outside the penumbrae areas, the non-uniform asymptotics of the Kirchhoff integral are

$$U^{\mathrm{KA}}_{\text{non-uniform}}(\mathbf{x}) = U^{\mathrm{GA}}(\mathbf{x}) + U^{\mathrm{KA(Diff)}}(\mathbf{x}). \quad (16)$$

Thus the KA integral has been decomposed in two parts. The refinement of the Kirchhoff approximation is to correct the diffraction field amplitudes by employing the GTD

$$\begin{aligned} U^{\mathrm{RKA}}(\mathbf{x}) &= U^{\mathrm{KA}}(\mathbf{x}) + U^{\mathrm{GTD}}(\mathbf{x}) - U^{\mathrm{KA(Diff)}}(\mathbf{x}) \\ &= U^{\mathrm{KA}}(\mathbf{x}) + \frac{e^{\mathrm{i}kr}}{\sqrt{kr}}\left(D^{\mathrm{GTD}}(\mathbf{x}) - D^{\mathrm{KA}}(\mathbf{x})\right). \end{aligned} \quad (17)$$

Finally the refinement of the Kirchhoff (RKA) consists in correcting, thanks to GTD, the KA contribution corresponding to the field scattered by the edge. This correction leads to add a corrective term to the KA field which is the difference of wave amplitudes diffracted by the edge given by GTD and KA. The diffraction coefficients for KA diffraction contribution and GTD have the same singularities at $\theta = \pi + \theta_0$ and $\theta = \pi - \theta_0$ (see Figure 3); when we make the difference of the two coefficients, their singularities cancel each other.

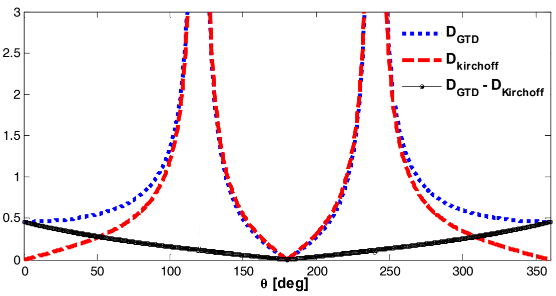

Figure 3: Diffraction coefficient for GTD and KA for θ_0 =50°.

4 Models comparisons and discussion

The scattering of a plane wave by a half plane is a canonical problem and it has an exact solution which allows us to compare with the GTD non-uniform eqn.(1) and uniform solutions eqn.(5). Those results are represented by their radiation pattern (containing the maximum power) and shown in Figure 4 and Figure 5. The incidence angle θ_0 is taken at 50°; the observation points are located around the edge for two distances from the edge $r = \lambda$ and $r = 5\lambda$ where λ is the wave length. Depending on the incidence θ_0 the light-shadow boundaries are θ_1= 130° and θ_2=230° on which we find the singularities of the non-uniform GTD solution (black dash-dot curve). However the uniform solutions (green dash curve) coincide quite well with the exact solution (red solid curve).

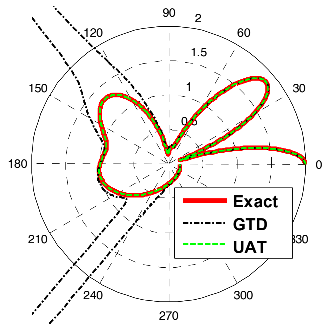

Figure 4: Radiation pattern comparison of the scattered field ($\theta_0 = 50°$, $r = \lambda$) by a perfectly rigid half plane given by exact solution, GTD and UAT.

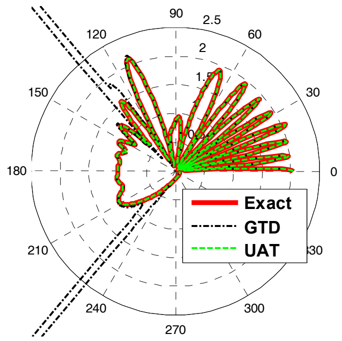

Figure 5: Radiation pattern comparison of the scattered field ($\theta_0 = 50°$, $r = 5\lambda$) by a perfectly rigid half plane given by exact solution, GTD and UAT.

Applying Kirchhoff approximation to the perfectly rigid half plane in the same configuration as in Figure 4 and Figure 5 we obtain the results in Figure 6 and Figure 7. The Kirchhoff approximation provides a qualitatively correct description of the scattered field. Errors can be found near the boundaries and in the shadow region where the edge diffraction wave dominates. When the observation is done far from the boundaries ($r = 5\lambda$) and it is usually the case in non destructive evaluation, the KA field coincide quite well with the exact solution (Figure 7).

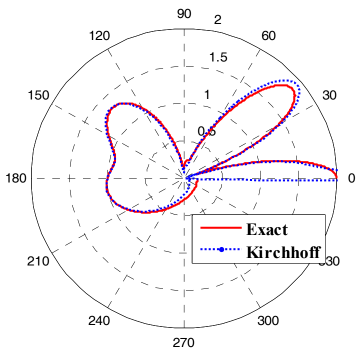

Figure 6: Radiation pattern comparison of scattered field ($\theta_0 = 50°$, $r = \lambda$) from a rigid half plane given by exact solution and Kirchhoff approximation.

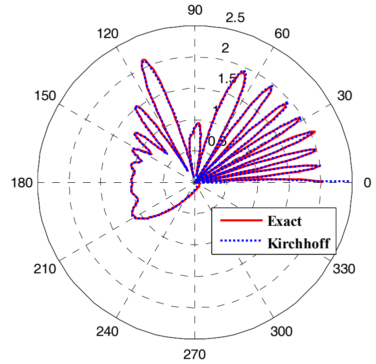

Figure 7: Radiation pattern comparison of scattered field ($\theta_0 = 50°$, $r = 5\lambda$) from a rigid half plane given by exact solution and Kirchhoff approximation.

In a two dimensional space, the distribution of KA's normalized errors around the half plane can be represented in Figure 8. The errors located near the edge are caused by the approximations we made in eqn. (11) on the boundary conditions. The errors away from the edge are indeed due to the incorrect prediction of the diffracted wave amplitudes.

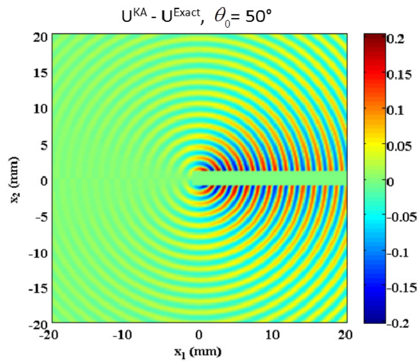

Figure 8: Distribution of KA errors to the exact solution with $\theta_0 = 50°$.

To correct the errors produced by the Kirchhoff approximation, we apply the refinement of the Kirchhoff approximation by employing GTD diffraction coefficients. The configurations of Figure 6 and Figure 7 have been recalculated using the refined Kirchhoff approximation. The results are given in Figure 9 and Figure 10. The results after the refinement coincide quite well with the exact solutions. The errors are greatly reduced in Figure 11 (from 20% max to 1.5% max).

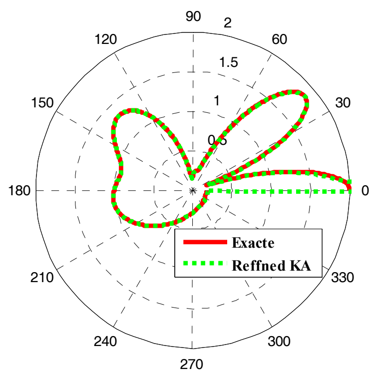

Figure 9: Radiation pattern comparison of scattered field ($\theta_0 = 50°$, $r = \lambda$) from a rigid half plane given by exact solution and refined KA

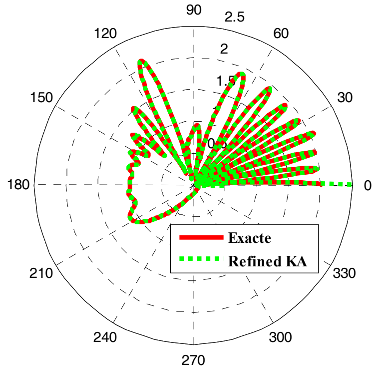

Figure 10: Radiation pattern comparison of scattered field from a rigid half plane given by exact solution and refined KA

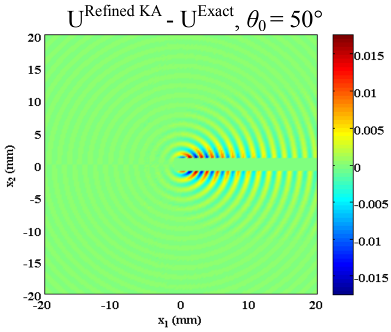

Figure 11: Distribution of refined KA's errors to the exact solution with $\theta_0 = 50°$.

5 Conclusion

Two classic wave scattering models: Geometrical Theory of Diffraction (GTD) and the Kirchhoff approximation (KA) have been studied for a rigid half plane model. The results have been compared with exact solutions. The asymptotic GTD formalism can be computed in an efficient manner and gives a perfectly adequate description of geometrical regions but is inapplicable inside transition regions. A more sophisticated uniform GTD is required to complete the description. The Green's-function-based KA formalism results are uniform with respect to the location of the observation point. KA correctly describes the geometrical field but leads to qualitatively correct description of diffraction wave but with incorrect amplitudes. In order to eliminate the deficiencies of GTD and KA and combine their advantages, the refinement of KA has been developed which consists in correcting, thanks to GTD, the KA contribution corresponding to the field scattered by the edge. This correction leads to add a corrective term to the KA field which is the difference of wave amplitudes diffracted by the edge given by GTD and KA. The refined KA gives accurate results compared to the exact solutions and with its simple formalism it can deal with obstacles of a complicated shape. The improvement provided by the refined KA and demonstrated here for a half plane is also obtained for a rigid wedge as shown in [10, 11].

Further work [10, 12] not described in this paper has been carried out to take into account the real boundary condition of the inspected structures in sodium telemetry. To deal with the scattering from a finite impedance target more representative of a reactor structure, the initial (non refined) KA model has then been extended. The obtained model, the so-called "general" KA model, has been compared to a reference model and provides a satisfactory solution for the application to telemetry. Finally, a complete simulation tool for telemetry is built by coupling this general KA diffraction model with a stochastic model developed for wave propagation in inhomogeneous media as sodium.

Acknowledgements

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