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Issues concerning using mode conversion of guided waves to size defects in plates

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Lamb waves have been used for damage detection in plates by measuring what amounts to transmission and mode conversion coefficients. These coefficients are indicative of the size of the defect, and relationships can be measured via numerical simulation. When size constraints do not allow for the modes to separate in time, the Fourier transform is often employed to resolve the strength of various modes in a single wave packet. However, due to the finite and discrete nature of the simulated signals, various issues arise concerning the detectability of low amplitude converted modes. The impacts of using various window functions in the spatial domain are investigated and compared. A technique is introduced to help detect smaller amplitude waves without necessitating dispersion curves to predict the abscissa and ordinate, i.e. frequency and wavenumber, locations based on a baseline subtraction technique employed in the two-dimensional frequency space.

1 Introduction

Lamb waves in plates have been used in Non-Destructive Testing (NDT) for over the past 50 years [1]. There are many advantages to using these guided waves instead of traditional C-scans to investigate the integrity of a plate-like structure, with time of testing being one of the most attractive. Unfortunately, use of guided waves is also complicated by several factors, most notably their dispersive nature, i.e. that a given mode's velocities are a function of the frequency in a plate of a given thickness. However, despite these dispersive relationships, Lamb waves can be successfully implemented in structural health monitoring.

Techniques for damage detection include identifying mode conversion [2, 3, 4, 5]. Mode conversion occurs at interfaces or changes in the geometry or structural properties of the waveguide, which are indicative of defects. Modes traveling in a wave guide are necessarily coupled in two directions (in the direction of propagation and at least one direction parallel to the wavefront, even for isotropic materials). Although an infinite amount of Lamb modes are theoretically possible, the two fundamental modes are often employed in damage detection. Often this is because a signal with fewer possible modes are easier to interpret. Furthermore, the two fundamental modes can propagate at lower frequencies, which are often easier to excite, and can be recorded with a lower sampling rate. When operating below the cutoff frequency for the first higher order mode, mode conversion is greatly simplified by excluding the possibility of conversion to all other higher modes as well.

Alleyne and Cawley identified that one of the fundamental problems in using mode conversion as a method of damage detection is identifying the relative amplitudes of the different modes present in the signal [2]. Correspondingly, they implemented a two-dimensional Fourier transform (2DFFT) to measure the content of the signal in both the temporal and spatial frequency domains [6]. This method involves recording the waveform, usually on the surface of the plate, at regular closely-spaced intervals. It is therefore implementable both in a real-world experimental set-up, granted that such spacing is achievable, as well as a numerical simulation via finite element analysis. In certain situations the two fundamental modes may actually separate in time. However, many circumstances can render this impossible. Geometrical constraints may not allow the waves to fully separate in time before interaction with another reflective surface, or the frequency range that is readily accessible could possible involve very similar group velocities between the two fundamental modes, depending on material properties. In these cases, the 2DFFT is an obvious solution.

Terrien et al. [3] quantified the amount of mode conversion for a given frequency-thickness product in the form of two ratios. The first ratio was quintessentially a transmission ratio and is calculated by dividing the amplitude of the wave after passing through a damage site with the amplitude of the same pure mode prior to interaction with the damage. The second ratio involved the same pure incident mode prior to damage interaction, but this time uses the amplitude of the resultant converted mode after passing through the damage zone. These ratios (henceforth referred to as transmission and conversion coefficients respectively) use only the motion of the surface in the plate, in the direction perpendicular to the surface of the plate. This motion is chosen since it is a reasonable measurable quantity for transducers and other equipment used for measuring normal displacement such as laser vibrometers. It has been noted [3] that these two coefficients have the potential to indicate the presence of straight-edged notches on the order of magnitude of 1/40th the size of the incident wavelength.

In the current study, we measure, via FEM, the transmission and conversion coefficients and find them to be in close agreement with those reported in the literature. Unfortunately, hurdles are encountered when trying to determine the coefficients for smaller damage sizes. Namely, the 2DFFT is subject to a certain amount of spectral leakage from strong modes that can mask the appearance of any mode conversion. Window functions other than the presupposed rectangular window may help limit spectral leakage at the cost of lowering resolution. This can prove to be problematic when material properties dictate very close dispersion curves. Closely spaced modes may actually end up overlapping if resolution is poor enough, making the suppression of spectral leakage a moot point. The authors therefore also implement a technique to help estimate the amount of mode conversion when analytical dispersion curves may not be available to predict the abscissa and ordinate in the 2DFFT. This technique is based on a simple baseline subtraction, often implemented in NDT for a single temporal waveform, but applied here to the two-dimensional frequency domain.

This paper is organized as follows: the method for determing the transmission and conversion coefficients via a numerical FEM experiment is detailed, and benchmarked by comparing it to the literature. Then, the lower limits of damage detection by way of extraction of mode amplitudes from the 2DFFT is investigated. Different window functions are employed in an attempt to continue to push the limit to its smallest damage detection potential. Ultimately, a baseline subtraction technique is presented and employed when window functions become impractical.

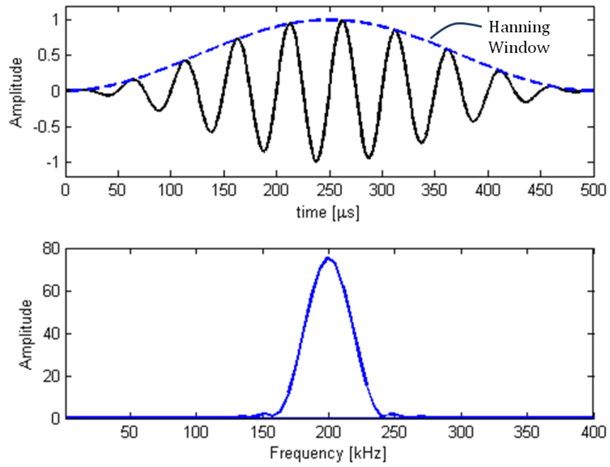

Figure 1: Top: Excitation signal used. A ten cycle, hanning-windowed signal centered at 200kHz. Bottom: Spectrum of the excitation signal. The use of a Hanning window in time excites a smooth, broad frequency band with minimal spectral leakage.

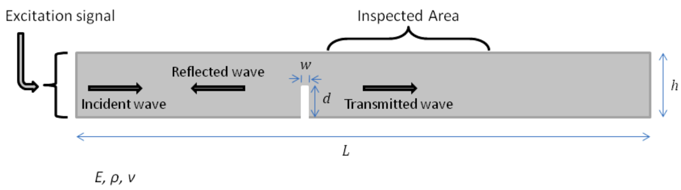

Figure 2: Schematic of the geometry for the numerical model. Waves are excited from the left end of the plate and interact with the notch located far enough away from the edges to avoid unwanted reflections. The inspection area is located directly after the notch, where the z-displacement of the surface is recorded at regular intervals.

2 Numerical Setup

The parameters and geometries for the first set of numerical simulations are chosen specifically to compare with the literature [2, 3]. To this end, cracks originating on the surface of various depths are modeled as straight edged notches with a fixed width. It has been found that the width of the notch has little bearing on the transmission or conversion coefficients as long as it is much smaller than the incident wavelength. Therefore, only results for notch depth are presented here, being the more interesting of the two variations of notch dimensions. The material used for the bench-marking simulations is steel ($\rho = 8000 kg/m^3$, $c_{Long} = 5960$ m/s, $c_{Shear} = 3250$ m/s). The frequency-thickness product needs to be 1.35 MHz-mm, to compare directly with the results published by Terrien et al. Therefore, with a center frequency for the excitation at 200 kHz, the thickness of the structure must be 6.75 mm. The excitation itself is a ten-cycle, Hanning windowed signal, as shown in figure 1. This temporal excitation shape is chosen to correctly limit the possible modes to only the two fundamental modes, confined to a frequency range with a fairly low amount of dispersion. The mesh is chosen to have ten elements per wavelength, which experience has shown is sufficient for accurate calculation. The time step for the explicit solver is sufficiently small enough so that a wave can not travel between two nodes during one time step. The inspection area is located directly after the crack location, as seen in figure 2.

Similar to a B-scan, the resultant waveform is collected at regular spaced intervals. Realistically, a range of data is available as output of an FEM numerical simulation, including displacement in any given direction at any given position (both interior and exterior) of the plate. However, to most closely mimic data accessible to a real-world procedure, surface displacement perpendicular to the surface of the plate is used.

As discussed previously, two ratios are implemented to measure notch depth, i.e. a transmission and conversion coefficient. However, since these two coefficients are based only on the amplitude of motion in one of the two coupled directions, it is worth while to define them here to avoid confusion.

$$T = \frac{U_{z,t}}{U_{z,i}} \tag{1}$$

and

$$C = \frac{U_{z,c}}{U_{z,i}} \tag{2}$$

where all motion is measured on the surface of the plate and perpendicular to the surface. $U_{z,i}$ denotes the pure incident mode, $U_{z,t}$ is of the same mode type as the incident mode, but refers to its amplitude after passing through the damage zone, and $U_{z,c}$ refers to the converted mode after passing through the damage zone. Figure 3 illuminates the case where S0 is the incident mode, and therefore the amplitude of the S0 mode after the crack is used to calculate T whereas the amplitude of the A0 mode is used to calculate C. The amplitudes, as explained previously, are extracted via the 2DFFT. A range of notch depths were simulated, from 10% penetration through the plate thickness, to just over 80%. The results are shown in figure 4 for the case where the S0 mode is incident. These values are extremely close to the ones reported in the literature by Terrien et al., successfully benchmarking the method outlined here of calculating the two coefficients. The only descrepency is for the lowest value of the conversion coefficient, which is reported here as being '0'. The following section is devoted to this discrepency.

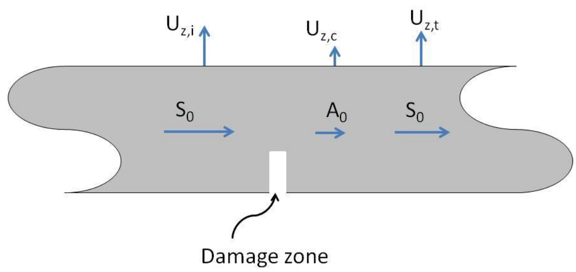

Figure 3: Case for S0 incidence. All displacement quantities are measured on the surface of the plate, and perpendicular to it.

3 Extending the lower limit of damage detection via window functions

As was seen in figure 4, a notch that reaches only about 10% of the through-thickness is undetectable according to

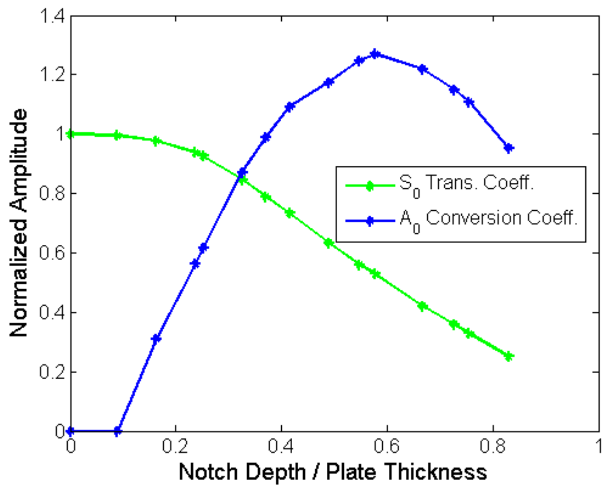

Figure 4: Numerical results for the Transmission (S0) and Conversion (A0) coefficients. Both amplitudes were extracted from the 2-D FFT at the center frequency of 200kHz, and normalized with respect to the incident S0 mode.

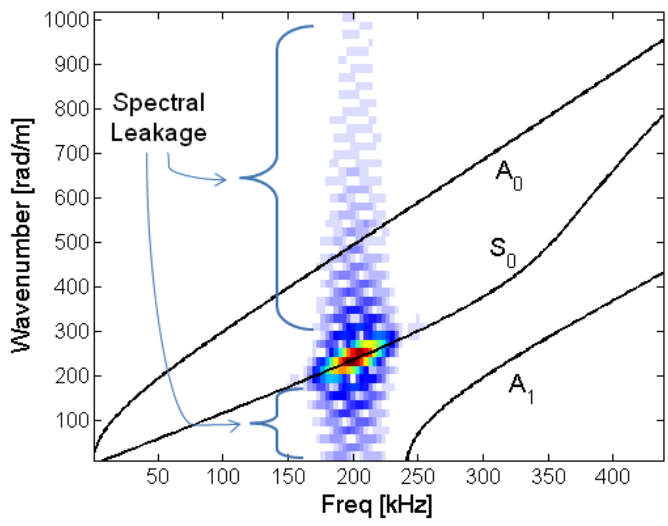

Figure 5: 2-D FFT for a healthy plate of steel when only the S0 mode is propagating. The excitation signal is centered at 200 kHz. The Hanning window used in the temporal domain for the excitation results in limited observable leakage along the frequency axis, whereas the rectangular window associated with truncating the signal results in observable leakage along the wavenumber axis.

the process explained in the previous section. This was mainly due to the spectral leakage that occurs in the 2DFFT. Spectral leakage is a common phenomenon caused by taking the Fourier transform of a windowed signal [7]. For a rectangular window, it manifests itself as side lobes which appear at smaller amplitudes on either side of the true frequency content in a signal since the Fourier transform of a rectangular window is a sinc function. Therefore, the resultant Fourier transform of a rectangularly windowed signal will be the convolution of a certain sinc function with the Fourier transform obtained in the case when no windowing is used. Spectral leakage occurs whenever a signal is altered from its inifinite length and thus arises with other windowing functions as well with varying shapes and levels of severity. It is important to note that such leakage appears from the truncation of a signal and therefore occurs even when a continuous Fourier transform is employed. For the 2-D FFT, where two independent variables are present, spectral leakage is possible in two dimensions, namely with respect to temporal and spatial frequency. For the case of a rectangular window, special care must be taken that side lobes are not confused for true modes, and interference is problematic for the detection of low amplitude converted modes. Figure 5 shows the 2DFFT for a plate of steel, when only the S0 mode is propagating, and demonstrates the appearance of spectral leakage.

One method of dealing with spectral leakage is to multiply the received signal by an intelligently chosen window function in the spatial domain. Since the signal is inevitably finite in time, any post-processing done on a signal already involves some form of window function. The automatic case is the rectangular window, which simply assumes that the function is zero outside of the recorded value, and multiplied by unity (unchanged) elsewhere. Other commonly used window functions are Gaussian, Hamming, and Hanning, just to name a few. Although the use of these functions greatly decreases spectral leakage, they tend to also decrease resolution. To illustrate, the series of regularly spaced waveforms recorded along the surface of the plate is multiplied by a rectangular window, as seen in figure 6. It can be noted that along the time axis, the function changes smoothly since the excitation signal is itself multiplied by a Hanning window. However, along the spatial axis the image changes abruptly at the boundaries. Figure 7 illustrates the use of a Hanning window. Now the waveforms change smoothly in both dimensions, greatly reducing spectral leakage.

Taking the results of fig 7 and transforming it into the two-dimensional frequency space gives figure 8. When comparing figure 8 with figure 5, it can be noted that the spectral leakage is greatly reduced by employing a gently sloping window function. However, the price for such leakage suppression is reduced resolution which manifests itself as the actual size of the mode in the 2DFFT. In figure 5 the mode was confined to a fairly tight area in the two-dimensional plane, when excluding the effects of spectral leakage. This area has greatly increased in figure 8. For example, consider the mode at 200 kHz. For the rectangular window case, the mode covers from approximately 200 to 250 rad/m. For the hanning window case, the wavenumber range is closer to 150 to 300 rad/m. Other smooth window functions are not presented here, but give similar results.

In the figures presented here, the theoretical dispersion curves have been superimposed to show that the results from the numerical simulation agree with the expected values. Furthermore, the dispersion curves for the two fundamental modes are spaced fairly wide apart. This limits the negative effect of decreased resolution in using a smooth window function, since any converted mode is well separated from the wide transmitted mode. Consider the case where the dispersion curves are unknown, i.e. when the material properties are unknown. Or, when the dispersion curves are known but excitation occurs in a region where the two modes have similar wavenumbers and are therefore not well separated in the two-dimensional frequency space. In such conditions another method can be employed to increase the detectability of small amplitude converted modes.

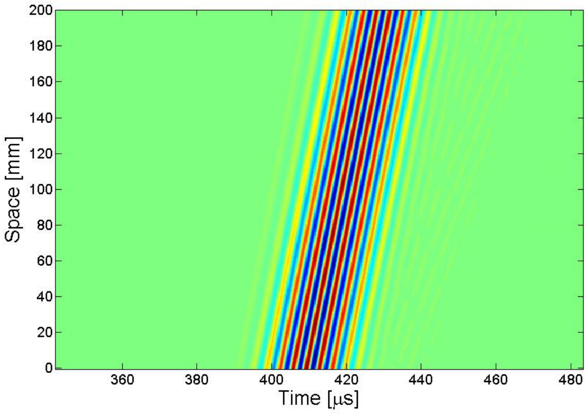

Figure 6: An S0 mode, where the excitation signal was Hanning-windowed in time, is recorded at a series of equally spaced locations along a plate surface. The data here is presented raw, which is equivalent to multiplying each spatial waveform (with a fixed unit of time) by a rectangular window in the spatial domain.

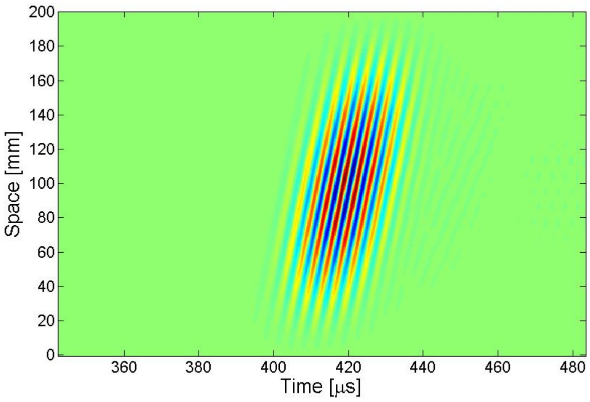

Figure 7: An S0 mode, where the excitation signal was Hanning-windowed in time, is recorded at a series of equally spaced locations along a plate surface. To provide a smoother function along the spatial dimension, the waveforms are multiplied by a Hanning window in space as well.

4 Baseline subtraction

Baseline subtraction is a concept commonly applied to waveforms in the temporal domain to illuminate small amplitude wave packets that are not overtly obvious in some initial 'healthy' signal. The same concept can be applied to the two-dimensional frequency space. Instead of small wave packets, the baseline subtraction method can highlight smaller amplitude modes that may be hidden due to poor resolution or spectral leakage.

Therefore, a baseline subtraction technique was applied to the two-dimensional frequency space according to the formula

$$\Gamma(f, \lambda) = \left\| 2DFFT\left(u_D(x,t)\right) \right| - \left| 2DFFT\left(u_B(x,t)\right) \right\| \quad (3)$$

where u_D is the signal with a defect present, and u_B is the baseline signal. Essentially equation 3 amounts to the absolute value of the difference of a damaged and healthy 2DFFT. The absolute value is taken because the incident mode loses some amplitude due to transmission loss through the defect,

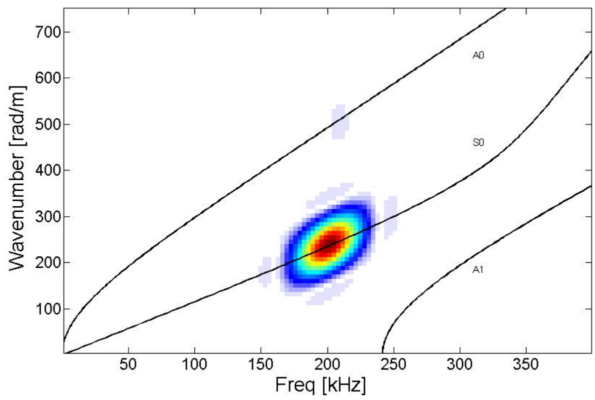

Figure 8: Two dimensional Fourier transform of a pure S0 mode propagating in a steel plate. The series of equally spaced waveforms were multiplied by a Hanning window function during post-processing as in figure 7

and therefore it would cause the region around the S0 mode at 200 kHz to be negative. To illustrate the use of the baseline subtraction, it is applied to the numerical model described above. Recall that mode conversion from a notch with a depth equal to 10% of the the width of a steel plate was previously undetectable (figure 4). Via equation 3, the resulting 2DFFT is depicted in figure 9. Now the converted A0 mode is obvious, and can be extracted to help determine the presense of the notch. The additional point resulting from employing the baseline subtraction method in the two-dimensional frequency space is displayed in figure 10. It can be noted that this notch depth is much smaller than the actual wavelength of either of the two fundamental Lamb modes.

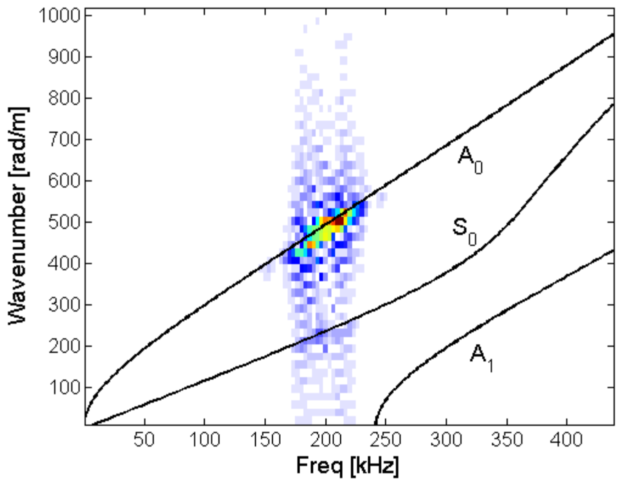

Figure 9: Results of the baseline subtraction technique for the case when the crack is 0.6 mm deep. Since the transmission coefficient was near '1', the S0 mode has almost entirely been deleted from the 2-D frequency space, along with the spectral leakage that accompanies it. The A0 mode is now highly visible.

5 Conclusion

Mode conversion, as has been previously demonstrated, is an effective tool for damage detection and sizing. How-

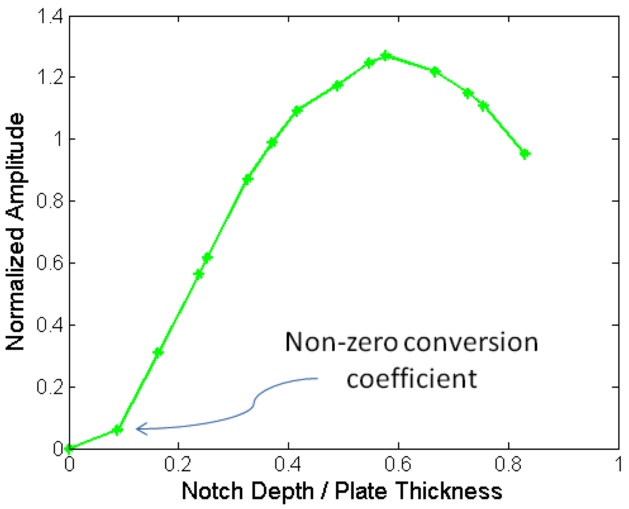

Figure 10: New point from baseline subtraction technique added to figure 4 . Amplitudes were extracted from the 2-D FFT at the center frequency of 200kHz, and normalized with respect to the incident S0 mode.

ever, situations can arise that limit the ability to detect mode conversion. Geometrical constraints may not allow for modes to fully separate in time, necessitating the use of a 2DFFT to extract amplitudes of all the modes present in a signal. Within the 2DFFT, issues such as spectral leakage or overlap of modes due to resolution restrictions may hamper the detection of small amplitude converted modes. Simple post-processing methods such as applying a window function in the spatial domain can help limit the manifestation of spectral leakage with the cost of lowering resolution. The authors have implemented the use of a baseline subtraction applied to the two-dimensional frequency domain to make low amplitude modes more obvious. This technique has been shown to be effective in detecting notches simulating cracks with a depth equal to as low as 10% of the through-thickness of the plate. The incident wavelength is much longer than the notch depth.

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