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On the comparison of absorbing regions methods

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Numerical simulation methods are very useful in Non Destructive Testing because they save time, lower cost and allow for the investigation of diverse experimental configurations. However, these methods consume relatively long CPU time and system memory. Different solutions exist to minimize these limitations. Absorbing region methods are among them when it's possible. These kinds of regions are also made to minimize or eliminate the spurious reflections at the boundaries of the simulated structure for more efficient signal processing. There are different ways to design appropriate absorbing regions.
Different approaches will be investigated and compared to show the advantage and limits of each one. Some examples will be presented.

1 Introduction

Nowadays, numerical modeling is widely adapted in ultrasonic guided wave NDT to help improve in-situ experimental results, contribute in reducing false alarms, and consequently making better decisions. Different numerical methods have already been developed and can be used. Finite Element (FE) modeling is among them, and it is now available in many commercial FE packages. This numerical method permits to:

- understand the wave's behavior even in complex waveguides,
- simulate the interaction between waves and realistic defects (with arbitrary fashions and sizes).
- simulate the generation of a pure mode and supply accurate data [1],
- …

However, this method needs a relatively large amount of memory and a powerful processor, and the CPU time is therefore long. Some solutions exist to reduce the CPU time. When possible, Absorbing region (AR) can be used.

From another side, FE numerical simulation can be executed in time domain as well as in frequency domain. And the AR application is nearly required for frequency domain simulations. Different from simulations in time domain, frequency domain simulations give results of stationary distribution of the acoustic field. For this reason, it is impossible to have snapshots of the acoustic field at different time points during the propagation process, especially before the reflections from boundaries occur. The total field hence appears as a summation of the incident wave and multiple reflections from all the boundaries before the full attenuation of the waves. This makes the simulation results complicated for analysis to the point of being nonsense. Besides, for better comparisons with experimental results, which separate the incident waves from reflections by different propagating times, it is necessary to remove all the reflections by plate boundaries. The most convenient solution is to add AR all around the edges of the plate to suppress any unwanted reflection. This aids in providing meaningful results in the frequency domain, avoiding the time-consuming drawback of calculation in the time domain, mainly by taking advantages of the material damping.

These methods are employed in many other fields such as seismology, telecommunication, etc [2-4] since it is related to the wave propagation cases. There are different ways to design appropriate absorbing regions. Essentially, there are typically two kinds of AR, one is called Perfectly Matched Layer (PML) [5-7], and the other is Viscoelastic Absorbing Region (VAR) [8, 9].

In this paper, different approaches to design the AR will be investigated and compared to show the advantages and limitations of each one. Basing on the PML conception, a novel VAR model will be proposed. To show the efficiency of this approach, a 3D numerical example will be presented.

2 Viscoelastic absorbing region : some approaches

The use of absorbing region is not really recent. Lysmer and Kuhlemeyer [9], who are probably the first researchers who worked on this topic, published a paper in 1969 in which they propose an artificial boundary for the purpose of wave propagation in an infinite domain. They suggested minimize the reflected wave energy by introducing damping at the plane of the finite boundary and choosing appropriate damping constants.

The visco-elasticity of materials is usually described by complex stiffness moduli, where the real and imaginary parts are related to the elastic and damping properties, respectively. The imaginary parts, which will subsequently appear in the damping term of displacement solutions, seem to be directly linked to attenuations of the wave amplitudes, so VAR is set by continuously increasing the imaginary parts of the complex moduli, in order to increasingly absorb the waves without causing reflections because of big changes in material properties.

The criteria for achieving the required VAR are:

- sufficient damping such that the effect of the boundary is negligible,
- damping is gradual enough so that there is no reflection caused by a sudden change in condition,
- cross-section is equal to that of the propagation domain (PD),
- the AR length is as short as possible, to reduce the CPU time,
- acoustic impedance is quazi-equal to that of the PD near the interface separating the AR and the PD,
- AR length is proportional to the wave's amplitude.

To satisfy these criteria, different approaches have been developed. For brevity, we give three AR models designed in the last ten years. Liu and Quek Jerry [10] use a gradually damped artificial boundary composed of 10 viscoelastic layers. The density of the PD is kept the same for all the 10 regions. However, Young's modulus E was modified to viscoelastic one E_k containing an imaginary part, as following:

$$E_k = E + i\alpha_k E,\ k = \{1, 2, ..., 10\} \tag{1}$$

where k refers to k^{th} region. This equation was inserted in a equation of motion which was solved by the usual Gaussian elimination method.

Castaings et al [1] improved this model by adding only one region to the PD instead of ten. Moreover, since the equation of motion is not always easily accessible in commercialized software, the imaginary parts of the viscoelastic material moduli, which are required in model setting, were defined by the following form:

$$(C_{IJ})_{absorber} = C'_{IJ} + iC''_{IJ}f(x) \quad (2)$$

where C'_{IJ} and C''_{IJ} are the elastic and damping matrix (given in Einstein notation) of the PD respectively, i, the complex number ($i^2 = -1$), f, a function, chosen to increase the damping in the AR. This function depends on the variable x, which denotes the distance from a point in VAR to the interface between VAR and PD (see Fig. 1 with $x=x_1$). This figure gives a 2D plate model with the PD and the AR having lengths L and L_a respectively. The AR is located in the area outside the PD which can be defined by:

$$\begin{cases} L \leq x_1 \leq L + L_a, & \text{on the right hand side} \\ -L_a \leq x_1 \leq 0, & \text{on the left hand side} \end{cases}.$$

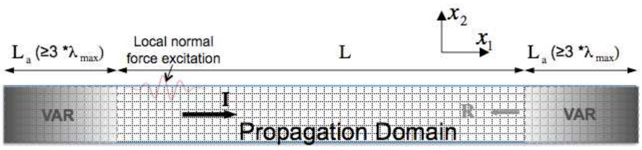

Fig. 1: Settings of 2D VAR

For a purely elastic PD, this formula can be simplified to:

$$(C_{IJ})_{absorber} = C'_{IJ}[1 + if(x)], \quad (3)$$

where C'_{IJ} is evidently real.

The function f taken in ref. [1] is exponential, and given by:

$$f(x) = e^{(x-L)/A},\ L \leq x \leq L + L_a, \quad (4)$$

where A is a constant for the optimization use.

The exponential function (discretized in the 1st approach and continuous in the 2nd one) ensures that the rate of increase in damping is low in the beginning and becomes higher as x increases. An exponential function with a well optimized parameter A is a good choice since it prevents a sudden change in the damping that will itself cause a reflection to the propagating wave. However, since the $\lim_{x\to L}(f(x)) = 1$, some spurious reflection will occur when waves cross from the first medium (PD) to the second one (AR).

Hosten et al [5] reformulated this model by using a function which is equal to zero at the interface PD / AR and increase exponentially. A function proportional to x^n, $n \in \aleph^*$ can be a solution. In addition, this function must not increase very quickly to avoid reflected waves (even if the AR is short), nor very slowly because the AR will be consequently long. In Fig. 2, we illustrate this idea by plotting x^n, $n = \{1, 2, 3, 5\}$. From this plot and with these two criteria in mind, we can deduce that x^3 is the optimal function.

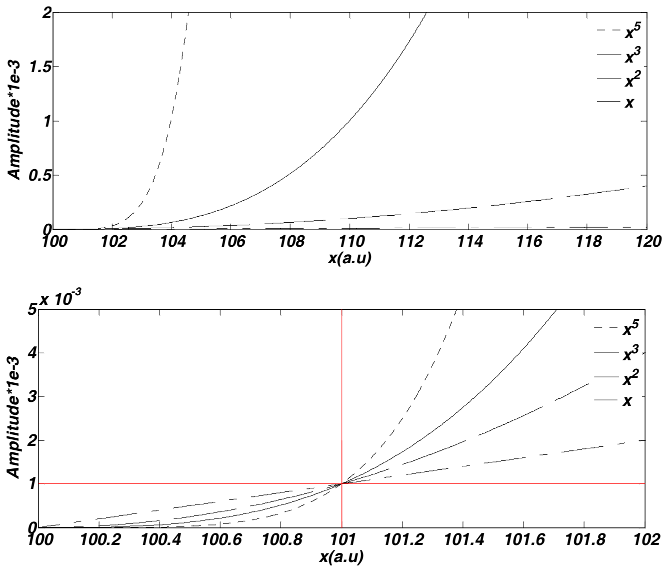

Fig. 2: Plot of $f \propto x^n$, $n = \{1, 2, 3, 5\}$ in the interval [100, 120] (top) and a zoom on the first two units (bottom).

The chosen function in ref. [5] is given by:

$$f(x) = A\frac{(x-L)^3}{L_a},\ L \leq x \leq L + L_a, \quad (5)$$

To attain the aim of attenuating the amplitudes of wave reflections to at most 0.1% of that of incident waves, the length of the VAR can ultimately be reduced to 3 times the maximum wavelength of all propagating modes in the plate after optimization.

Obviously, though the number of mesh elements within absorbing regions is reasonably small in 2D models, this is likely to be very different in 3D models because absorbing regions would then have to be defined as voluminous bands or rings surrounding the domain of propagation. For this reason, the use of absorbing regions may become a serious drawback in 3D simulations, if not properly defined or optimized.

3 Improved Viscoelastic Absorbing Region

As we can see in the previous section, the design of an AR is based on the viscoelastic constant. The question which can be asked is: why can the density not be included in the design of such an AR? This is applied in the conception of the perfectly matched layer (PML). To make the PML clear, we dedicate the following sub-section for a didactic example.

3.1 Perfectly Matched Layer

Similar to VAR, PML is also set all around the PD, and as indicated by its name, the main principle to set a PML is to always match the acoustic impedance in the AR with that of the PD with the overall aim of absorbing waves. To reveal the nature of the performance of the PML, a simple example is cited in ref. [7].

For linear acoustics in a one-dimensional case in fluid with mean density ρ_0 and speed of sound c_0, the linear wave equation for acoustical pressure p is:

$$\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0 \qquad (6)$$

where *x* is the position in the direction of propagation.
For a given medium, the acoustic impedance is given by:

$$Z_a = \rho_0 c_0 \ . \qquad (7)$$

For the simplest case, when a sound wave is incident perpendicularly from medium 1 into medium 2, the reflection coefficient R, which is related to the acoustical impedances of the adjacent media, is defined as:

$$R = \frac{Z_{a2} - Z_{a1}}{Z_{a2} + Z_{a1}} \qquad (8)$$

with $Z_{a1} = \rho_1 c_1$ et $Z_{a2} = \rho_2 c_2$.

As we can see in the equation, in the case where $Z_{a1} = Z_{a2}$, R=0, which means non-reflection.

With this in mind, it is possible to set the density ρ^{AR} and wave velocity c_{AR} in the AR to make the wave from the fluid fully attenuated by:

$$\rho_{AR} = \rho_0(1 - i\sigma_0),\ c_{AR} = c_0/(1 - i\sigma_0) \ . \qquad (9)$$

Rewriting the motion equation for a harmonic wave with angular frequency ω gives:

$$\frac{\omega^2}{c_0^2}(1 - i\sigma_0)^2 p - \frac{\partial^2 p}{\partial x^2} = 0. \qquad (10)$$

In the function reported here, σ_0 (which can also be a function of *x* [7]), is a constant related to the damping ratio. The corresponding solution is:

$$p(x,t) = p_0 e^{i(\omega t - kx)} e^{-\sigma x}, \qquad (11)$$

where $k=\omega/c_0$ is the wavenumber, $\sigma=\sigma_0 k$ is the damping coefficient, and the term $e^{-\sigma x}$ leads to the attenuation of the wave during propagation.
In this way, the wave is entirely absorbed and no wave is reflected to the PD. However, the application of a PML is not as easy as that of a VAR in the current FE software.

3.2 Novel Viscoelastic Absorbing Region

Through the two previous sections, it is easy to note that VAR and PML have basic principles in common: one is “absorbing”, the other one is avoiding reflection, either by continuously changing the material properties or keeping the acoustical impedance matched. If we have a deeper insight, we can find that in the aspects of absorbing, the nature of performance by changing material properties in VAR is quite similar to that of PML, as for isotropic material, the phase velocities of longitudinal wave and transverse wave are defined respectively, by:

$$c_p = \sqrt{\frac{C_{11}}{\rho}},\ c_s = \sqrt{\frac{C_{66}}{\rho}}, \qquad (12)$$

where C_{11} and C_{66} are the correspondent stiffness component.

Based on the definition of the PML, it seems that something can be done to improve the performance of the VAR.

In the two-dimensional case of a Lamb wave, the propagation is along the x_1 direction, and it will perpendicularly impinge onto the AR. In this case, if the material properties C_{IJ} in the VAR are defined as (3), to reduce the reflection during propagation by improving the match between acoustical impedances of the two regions, the density ρ_{AR} of the VAR can be correspondingly defined as:

$$\rho_{AR} = \frac{\rho_0}{\left[1 + iA\left(\frac{x}{L_a}\right)^3\right]^2} \ . \qquad (13)$$

As phase velocities of elastic waves are related to elastic constants, to further minimize the size of the AR, a new definition of the VAR that reduces the stiffness when increasing the imaginary part of C_{IJ} in the AR will lead to a higher number of wavelengths within the AR. This is introduced as:

$$\begin{cases} C_{AR} = C_{ij}\left[1 - \left(\frac{x}{L_a}\right)^3 + iA\left(\frac{x}{L_a}\right)^3\right] \\ \rho_{AR} = \frac{\rho_0}{\left[1 - \left(\frac{x}{L_a}\right)^3 + iA\left(\frac{x}{L_a}\right)^3\right]} \end{cases} . \qquad (14)$$

This definition is confirmed by comparisons between different VAR using a 2D FE model as shown in Fig. 3. In the model, both the parameters A and L_a are optimized for the most efficient performance of VAR, attenuating the incident waves by 60 dB while keeping the AR as short as possible.

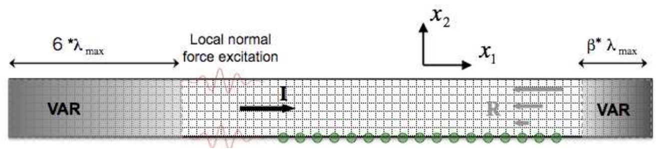

Fig. 3: 2D model for testing the efficiency of the new VAR.

In the 2D model, the left end is the classical VAR defined by (5) with a length of $6\lambda_{MAX}$, which is more than sufficient to remove all the reflections from this side, while the right side is the newly defined VAR with a length of $\beta^*\lambda_{MAX}$, with β being an optimization parameter. An incident wave of a single mode such as A_0, S_0, A_1, or S_1 is generated with a Gaussian-distributed normal force applied on both surfaces of the plate. The displacement at the plate surface in the region between the excitation zone and the AR, as noted with solid points in figure 3, is monitored.

Since mode conversion during reflection will give rise to some new modes not existing in the incident wave, a Fourier transform is applied to distinguish all of these modes and further attest to the efficiency of the AR. The reflections are plotted in figure 4(a). The reflection coefficients *R* are calculated and plotted in figure 4(b). This is done with the aim of checking the efficiency of the VAR, as well as optimizing the length L_a and the coefficient *A*.

After checking all four cases with the different single incident mode, it is confirmed that the length of the new VAR can be reduced from $3\lambda_{MAX}$ to almost $1.5\lambda_{MAX}$ with the requirement of R<0.001 being satisfied.

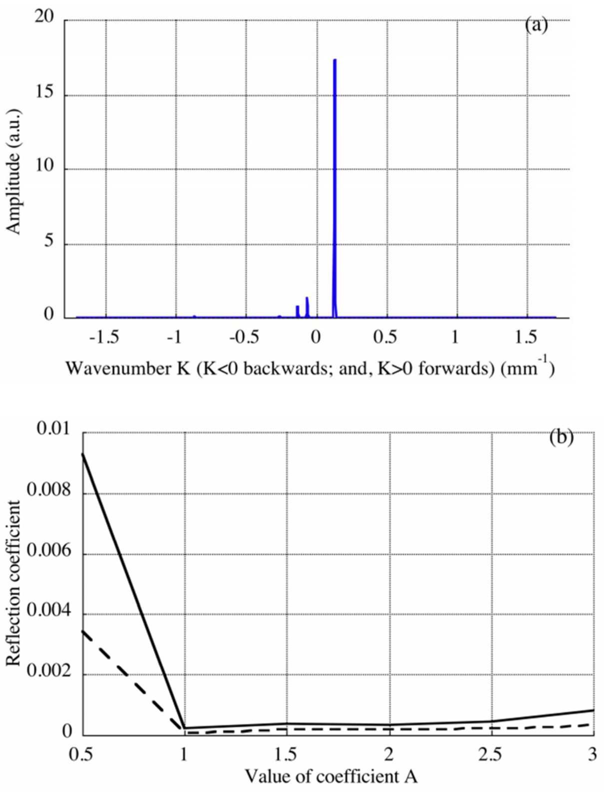

Fig. 4: Optimization of AR with 5mm thick aluminium plate using monitored *u* displacement of S_1 mode incident at frequency 0.6MHz, (a) Fourier transform result of *u* displacement, A=3, β=0.5; (b) calculated reflection coefficients for optimum coefficient A with β=1.5, (—) and (—) for the two biggest reflected modes, respectively.

4 3D Viscoelastic Absorbing Region

In this section, we present an example of ultrasonic guided waves propagation in 3D plate to test the performance of the new defined VAR. The VAR with reduced size is used all around the PD of the plate as shown in figure below. We keep the same VAR width as in the 2D study (previous section). Concerning the VAR thickness, it should be equal to that of the plate to avoid any reflection of waves.

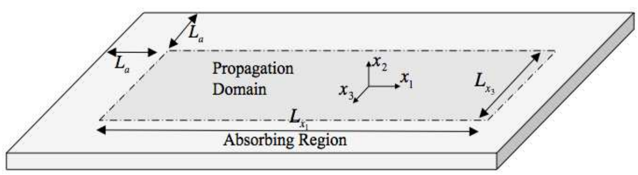

Fig. 5: The 3D setting of VAR

The efficiency of the VAR is confirmed in 3D modeling by comparing the attenuations caused by a classical VAR and the new VAR using a 4 mm-thick Aluminium plate model with coexistence of A_0, S_0 and SH_0 modes. To demonstrate clearly the processes of full attenuation of all waves, L_a is set to be 4 times $max(\lambda_{A0}, \lambda_{S0}, \lambda_{SH0})$, in the direction of propagation (x_1), at the excitation frequency. The frequency used in this numerical experience is 250 KHz, so the maximum wavelength λ_{MAX} is equal to that of S_0 (λ_{S0}) which is about 21 mm. Corresponding to S_0, A_0 and SH_0 modes respectively, the displacement components *u*, *v* and *w* monitored at the median plane of the plate (thickness/2) and in the middle of the PD width (L_{x3}/2), are plotted in figure 6. As we can remark, the amplitudes of the waves decrease significantly during the propagation in the absorbing region. By comparison with the classical AR (plotted with a black line with squares), the new AR appear more efficient: only 1.5 λ_{MAX} is needed to make the waves completely vanish in the new AR versus 3 λ_{MAX} in the classical one. In addition, we can also clearly observe continuous reductions of wavelengths, which are mainly due to the changes in material properties, and lead to the accelerated attenuations of waves.

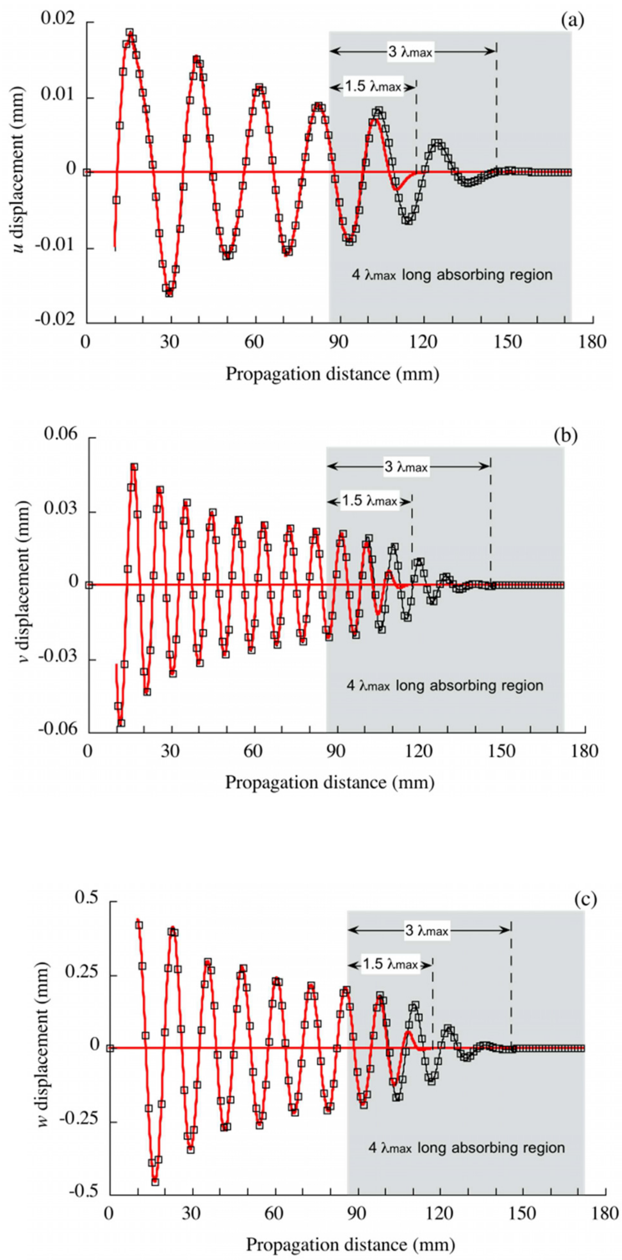

Fig. 6: Comparison between mode attenuations using (-) new AR and (-☐) classical AR: S_0 mode (a), A_0 mode (b), and SH_0 mode (c).

5 Conclusion

In this paper, a brief review of the use of absorbing region was given. The main difference between a perfectly matched layer and a viscoelastic absorbing region was outlined. Concerning the absorbing region, different mathematical expressions (discrete exponential, continuous exponential, polynomial) used to simulate the AR in different previous works were listed and compared. These expressions are all based on the definition of the viscoelastic constant (the commonly noted C_{IJ}). The most efficient among them was extended to a new formula which includes a modification on the density of the AR to make it a complex number. The mathematical expression for density is selected such that the acoustic impedance of the AR is equal to that of the propagation domain. This new absorbing region was tested in a case of 3D numerical simulation of ultrasonic guided wave propagation. Some comparisons with a classical AR were performed. In conclusion, thanks to this VAR:

- CPU time is saved since the VAR size was reduced by 50%,
- the displacement field is continuous at the PD/AR interface,
- no reflection appeared at this interface.
- post-processing of numerical simulations is simplified.

Acknowledgments

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