Simulation and application of beam-shaped subwoofer arrays

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This paper discusses the prediction and application of DDS-controlled, directional subwoofer arrays with configurable 3D sound radiation characteristics (e.g. cardioid, hyper-cardioid, dipole etc.). In order to optimise and control the dispersion of beam-shaped sub arrays, an accurate sound radiation model is indispensable. The radiation impedance of the transducers (i.e., the acoustic load) and the cabinet diffraction are determined by the size of the array and the relative position of the loudspeakers in the array. Besides these diffraction and coupling effects, other acoustic boundary conditions can have a significant effect too. The most common one is the ground plane condition. Therefore, the hybrid PSM-BEM model implemented in the DDA software, has been extended with an optional ground plane condition. Measurements show that the improved model yields very accurate results for various array set-ups with different radiation characteristics.

1 Introduction

Low frequency radiation pattern control has increasingly gained interest among sound system designers and sound engineers the last 15 years. This is because directional subwoofer systems offer some clear benefits. In a room the direct-to-reverberant ratio can be improved by aiming the LF beam to the audience, while keeping it off the walls and ceiling, resulting in better 'definition' and 'punch' of the bass. Also during outdoor events directional subwoofer arrays have proven to be useful, especially in cases where the maximum sound immission in built areas is restricted by environmental regulations.

Due to the long wave lengths involved (typically 3 to 10 m), a single subwoofer shows a near omni-directional radiation pattern. By stacking multiple subwoofers in an array the directivity (i.e., Q-factor) increases proportionally to the ratio between the size of the array and the wave length. Consequently, large array dimensions are required to achieve significant directivity at low frequencies.

By applying delays to the loudspeaker signals, the main lobe can be steered electronically. In the steering direction the loudspeaker contributions are in phase and consequently sum coherently, while in other directions they (partially) cancel each other out. As the individual subwoofers are almost omni-directional, the radiation pattern is mirror-symmetrical in the line or plane of the array, i.e., besides the desired front lobe, an almost equally strong rear lobe will arise.

On the basis of their beamforming mechanism, these arrays are often characterised as 'delay-and-sum' arrays. By nature, delay-and-sum arrays are power-efficient. By doubling the number of subwoofers in a uniformly weighted delay-and-sum array, a 6 dB on-axis gain is achieved while the total electrical power only doubles (+3 dB). They are also robust, i.e., relatively insensitive to loudspeaker positioning errors and acoustic deviations between loudspeakers as a result of production tolerances or ageing.

Another class of beamformers are the 'differential' arrays, which consist of two or more axially spaced subwoofers. Here the pressure difference between the sound waves emitted by the front and rear loudspeakers is crucial for the beamforming. Differential subwoofer arrays are 'super-directional' in the sense that a substantial spatial selectivity can be obtained with array dimensions much smaller than the wave length. Unfortunately, they tend to be less efficient, and less robust than delay-and-sum arrays.

A well-known example of a differential bass array is the cardioid subwoofer, which is useful in live concert applications due to its strong backward LF sound rejection. The theoretical principle behind the cardioid subwoofer is quite simple. Basically, only two axially spaced (<1/4 of a wave length) subwoofers are required. Assuming the two subwoofers are truly omnidirectional, only a phase reversal and an electronic delay would be required for the rear-facing loudspeaker. The practical implementation however is less straightforward, as small loudspeaker deviations or modelling errors could lead to major errors in the rearward sound cancellation. Often the power efficiency is compromised and the realised backward rejection is band-limited. Moreover, many systems require precise on-site tuning of the system.

The present work focusses at the acoustic modelling and optimisation of DDS-controlled [1] differential subwoofer arrays. These 'beam-shaped' subwoofer arrays offer some very interesting properties. For instance, with a vertical differential subwoofer array, i.e., a line array with loudspeakers in the front as well as at the back, it is possible to steer and shape the front lobe in the vertical plane and simultaneously reject sound to the rear or side. In this way the benefits of a delay-and-sum array and a differential array are combined.

This work is an extension of the study reported in 2005 [2]. In that study a computationally efficient, hybrid PSM-BEM method was introduced to accurately model subwoofers in an array facing full-space radiation conditions. Now, the PSM-BEM method is extended with a half-space radiation condition which is expected to be more valid for ground-stacked subwoofer arrays. Using this novel modelling approach, several differential subwoofer configurations with various radiation characteristics (cardioid, hyper-cardioid and dipole.) have been simulated and, subsequently, verified by measurements.

2 Acoustic modelling of subwoofer arrays

2.1 Problem description

Probably, one of the most widely applied models in acoustic simulation software is the Point Source Model (PSM) [3]. In the PSM each loudspeaker in the array is modelled as a directional point source, positioned in free space. The model assumes that the sound field (magnitude and phase of the acoustic pressure in all directions) of a loudspeaker is unaffected by the presence of other cabinets in the array.

This free field assumption leads to accurate predictions at mid and high frequencies. However, at low frequencies the sound field of a subwoofer is strongly affected by the radiation impedance (i.e., the acoustic load) and the cabinet diffraction. Both the acoustic load and the diffraction are defined by the size and shape of the array and the position of the loudspeaker in the array..

In practice, subwoofer arrays are often stacked on the floor or stage. Assuming the ground plane is acoustically hard and large compared to the acoustic wave length, it can be modelled as an infinite baffle. At first sight it might be expected that the radiation pattern of the individual array elements doesn't change and that the effect of the ground plane can be simply modelled by adding a mirror-image of the source. However, due to the contact interface between the array and the ground plane, the sound diffraction around the array is affected too, because the 'path' under the array is now blocked.

From the above it's evident that the directional response of a subwoofer is not only affected by the array geometry, but also by the radiation condition (full or half-space). So, in order to accurately model and optimise a (differential) subwoofer array, the actual Acoustic Boundary Conditions (ABC) should be taken into account.

Anechoic measurement of the radiation characteristics of a subwoofer under realistic radiation conditions is practically impossible due to the large array dimensions and unlimited number of variations in array set-up. Therefore, a computationally efficient, hybrid PSM-BEM approach was developed. The principles behind the PSM-BEM approach are repeated in the next section.

2.2 The PSM-BEM model

Using the acoustic Boundary Element Method (BEM) [4], it is possible to accurately model diffraction and coupling effects for low and low/mid frequencies. The BEM is based on the Helmholtz integral Equation (HIE). On the basis of the discrete distribution of the normal component of the particle velocity at the boundaries of a radiating object, the sound radiation can be calculated in all directions outside the object. Unfortunately, direct implementation of the BEM into the DDA modelling software would lead to dramatically increased computation times.

The idea behind the PSM-BEM approach is the following: Just like in the PSM, the array is modelled as a set of directional point sources. But, in contrast to the PSM, the spectral and directional behaviour of each point source is no longer given by the measured free field response of the loudspeaker, but is replaced by BEM-calculated directivity data of the loudspeaker facing the actual acoustic boundary conditions. A prerequisite is that the volume velocity of a moving loudspeaker cone is unaffected by the acoustic load. As the particle velocity right in front of the cone and the ports is almost completely dictated by the driver, this is a valid assumption.

In order to model ground-stacked arrays, the half-space formulation of the HIE is implemented in the propriety BEM algorithms. The PSM-BEM model now handles flown subwoofer arrays, radiating into full-space, as well as ground-stacked arrays, which are facing half-space radiation conditions.

3 Calculation of directivity data

3.1 Procedure

Using the BEM, a library with far field radiation data of each AXYS subwoofer model has been pre-calculated for the most common acoustic boundary conditions. Each ABC is defined by three parameters: The number of cabinets in the array, the position of the active subwoofer in the array and the radiation condition (either full or half-space). The BEM calculation procedure that was followed will be shortly explained now.

First, the normal particle velocity is measured just in front of the cone and the ports of the subwoofer using a pressure gradient microphone. As the particle velocity is assumed to be independent of the ABC, the measurements only need to be done for a single cabinet. Next, the boundaries of the array are partitioned into a large number of small boundary elements. The measured particle velocity data from the single cabinet is applied to one of the cabinets in the array. The velocities at the rigid parts of the active cabinet as well as at the boundaries of the inactive, neighbouring cabinets are set to zero.

In the next step the sound pressure at each boundary element is calculated by solving the discrete Helmholtz Integral Equation. Depending of the radiation condition, either the full or the half space version of the HIE is used.

Knowing both the measured velocity and the calculated pressure distribution at the boundary elements, the complex far field directivity balloons can be calculated with the help of the exterior HIE. This is done by defining a spherical receiver grid (sampled every 5 degrees) centred around the active subwoofer. For both radiation conditions the full-space version of the external HIE is used, i.e., only the direct sound is processed into the directivity balloons. In case of half-space radiation, the reflection from the ground plane is modelled by adding a second, mirror-symmetrical point source.

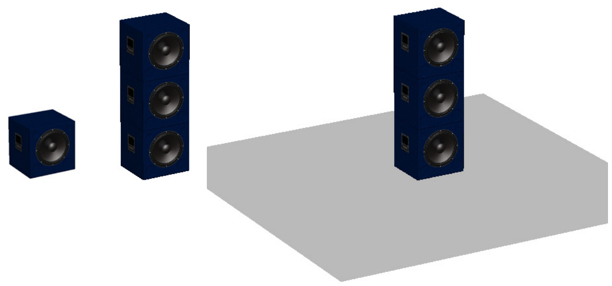
The calculation procedure described above has been repeated for each radiation condition.

3.2 Example

As an example, the BEM-calculation of the directivity data will be studied in more detail for an 18” subwoofer (AXYS B-07 model) facing three different radiation conditions.

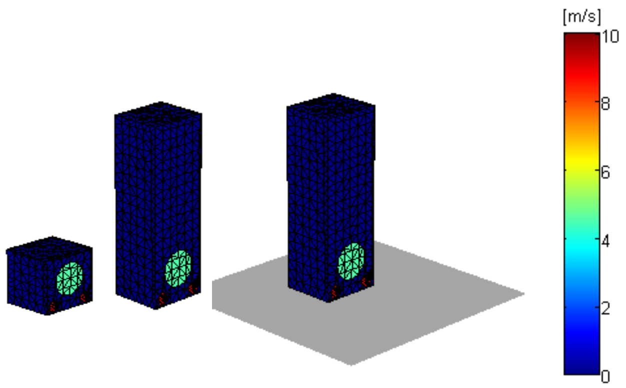
First, the subwoofer is positioned in free-space as a single unit. This will be called the 'free-field' condition. Secondly, the subwoofer is supplemented with two additional cabinets, forming an array of three units ('3U1 full-space' condition). Note that only the first subwoofer (i.e. the lowest unit) is active. Merely, the presence of the upper two cabinets is studied here. Thirdly, the same 3-element array is placed on a ground plane ('3U1 half-space' condition). The three radiation conditions are visualised in Figure 1a-c.

The measured particle velocity at 80 Hz is shown in Figure 2a-c for an arbitrary input level. As pointed out in section 2.2, the particle velocity in front of the cone and ports of the active subwoofer is independent of the radiation condition.



a) b) c)

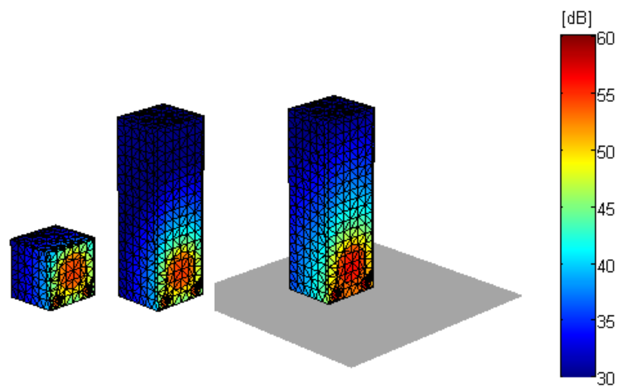
Figure 1: B-07 subwoofer facing different acoustic boundary conditions: a) As a single unit, 'free-field' condition. b) First unit in a 3-unit array, '3U1' full-space condition. c) As b) but ground-stacked, '3U1' half-space condition.



a) b) c)

Figure 2: Measured normal particle velocity @80Hz for a B-07 subwoofer facing different acoustic boundary conditions: a) 'free-field' condition. b) '3U1 full-space' condition. c) '3U1 half-space' condition

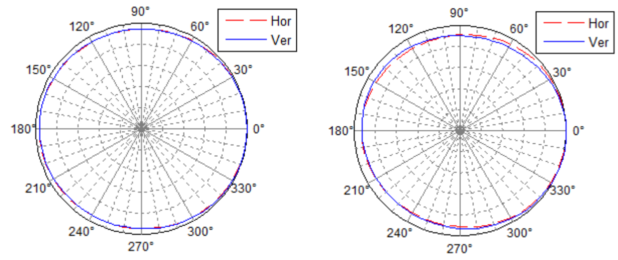
From the measured normal particle velocity data the sound pressure at the array boundaries is calculated. The results are shown in Figure 3a-c for the three radiation conditions. As expected, the SPL distribution at the array boundaries differs between the three radiation conditions.



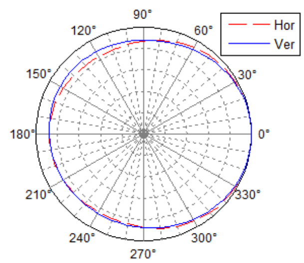
a) b) c)

Figure 3: BEM-calculated SPL @80Hz for a B-07 subwoofer facing different acoustic boundary conditions: a) 'free-field' condition. b) '3U1 full-space' condition. c) '3U1 half-space' condition

Using the measured normal particle velocity and the calculated pressure data, the far field complex 3D directivity balloon for each 1/3-octave band (32-200Hz) is calculated for the active subwoofer. The horizontal and vertical magnitude polar diagrams are shown in Figure 4a-c.



a) b)



c)

Figure 4: BEM-calculated polar diagrams @80Hz for a B-07 subwoofer facing different acoustic boundary conditions (scale: 6 dB/div): a) 'free-field' condition. b) '3U1 full-space' condition. c) '3U1 half-space' condition

By comparing Figure 4a and b it can be verified that vertical polar diagram of the '3U1 full-space' condition has become slightly asymmetrical, which is obviously a result of the asymmetrical position of the subwoofer in the array. Also, the front-to-back ratio is higher compared to the 'free-field' condition. This can be explained by the larger front baffle of the array compared to the single unit.

Also note that the vertical directivity pattern for the '3U1 half-space' condition clearly differs from the '3U1 full-space' condition. Most striking is the higher front-to-back ratio for the half-space condition compared to the full-space condition, which will be explained in further detail below. Note that, as pointed out in section 3.1, the contribution of the reflected sound is not included in the balloons for the half-space condition, but will be modelled by an additional, mirror-symmetrical point source.

In order to make an easy comparison, the front-to-back ratio for the three acoustic boundary conditions is calculated as a function of frequency. The results are shown in Figure 5. As expected, the F/B ratio increases with frequency for all conditions. Note that the F/B ratio for the '3U1 half-space' condition is higher than for the '3U1 full-space' condition. This can be explained by the changed diffraction of the sound waves around the array, as discussed in section 2.1.

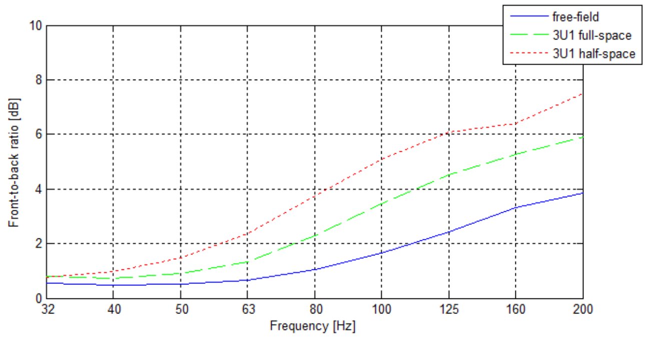


Figure 5:BEM-calculated front-to-back ratio of a single subwoofer facing different three different acoustic boundary conditions

Besides changes in the directional behaviour, the BEM calculations show that the size of the array and presence of a ground plane also affect the sensitivity. To illustrate this, the unfiltered on-axis sensitivity (i.e., response @1m for a 2.83V input at the loudspeaker clamps) for the three conditions is shown in Figure 6.

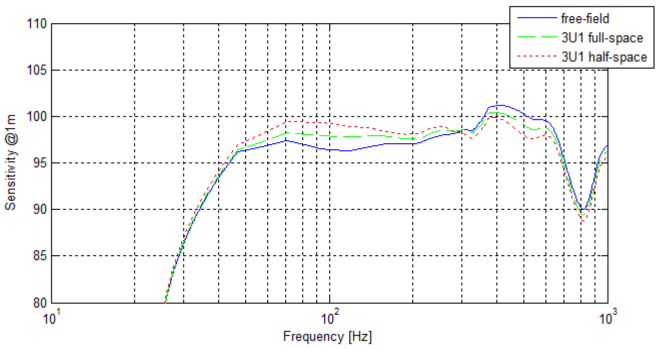


Figure 6: BEM-calculated sensitivity of a single subwoofer facing different three different acoustic boundary conditions

As expected from the larger baffle size, the sensitivity for the '3U1 full-space' condition exceeds the 'free-field' sensitivity at low frequencies. The '3U1 half-space' configuration, in its turn, is slightly more sensitive than the '3U1 full-space' configuration. Note again, that the contribution of the reflected energy has not been taken into account here as it will be modelled by an additional point source.

4 Validation of the PSM-BEM model

In order to verify the validity and accuracy of the extended PSM-BEM model, outdoor measurements have been done on ground-stacked, differential subwoofer arrays.

By the DDA software various radiation characteristics were DDS-optimised and modelled using half-space PSM-BEM subwoofer data. The DDS algorithm automatically balances the demands of matching the desired radiation pattern on the one side and maximising the total sensitivity of the array (i.e., the maximum output level) on the other side. In this way not only an accurate, but also a robust solution is found.

In the next sections the prediction and measurement results will be presented for one of the tested array set-ups.

4.1 Measurement set-up

The tested set-up consisted of two ground-stacked, self-powered, DSP-controlled B-121 subwoofers. The upper 21" sub was aimed to the front while the lower sub was facing to the rear, as shown in Figure 7. The dimensions of the cabinets are 620x620x676 mm (HxWxD).

Using DDA, the radiation pattern of this array was optimised with various radiation patterns, of which the cardioid and the dipole setting will be presented here. The on-board DSP can be controlled via the RS-485 network. Each settings was uploaded into a memory pre-set, using the WinControl software.

During the measurements the subwoofer array was placed on the ground with a hard concrete surface. Impulse responses were taken at the ground plane along a horizontal semi-circle with a radius of 7m, starting in front of the array towards the back with 10 degree steps. The reflections from nearby buildings were relatively weak and sufficiently spaced in time from the direct sound, making it possible to use a time window of 75 ms.

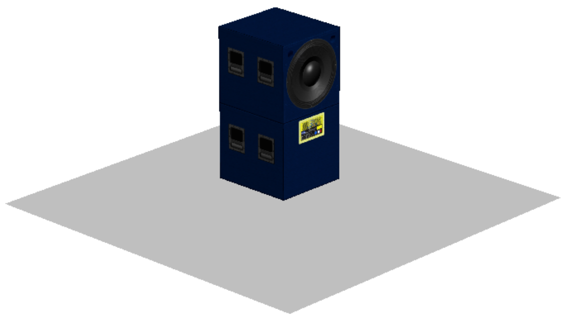


Figure 7: Differential subwoofer set-up of two B-121 units, placed on an acoustically hard ground plane

4.2 Cardioid setting

First, the differential array has been optimised for cardioid radiation, as shown in Figure 8.

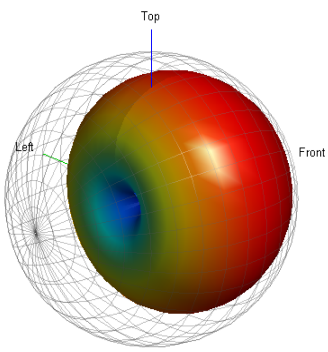


Figure 8: Desired cardioid radiation pattern

The predicted and measured horizontal polar data for the cardioid setting are shown in Figure 9. It can be verified that the match between the modelled and measured data is very good. At lower frequencies (40-80 Hz) the maximum backward rejection is found at 180 degrees. For higher frequencies the radiation dip moves slightly sideways. A backward rejection between 14 and 24 dB can be realised with this set-up.

By comparing the on-axis sensitivity of this cardioid differential set-up with a 'reference' summing array, consisting of two ground-stacked, front-facing subwoofers, only a 2 dB reduction was found. This indicates that the efficiency and the robustness of the cardioid array are very good.

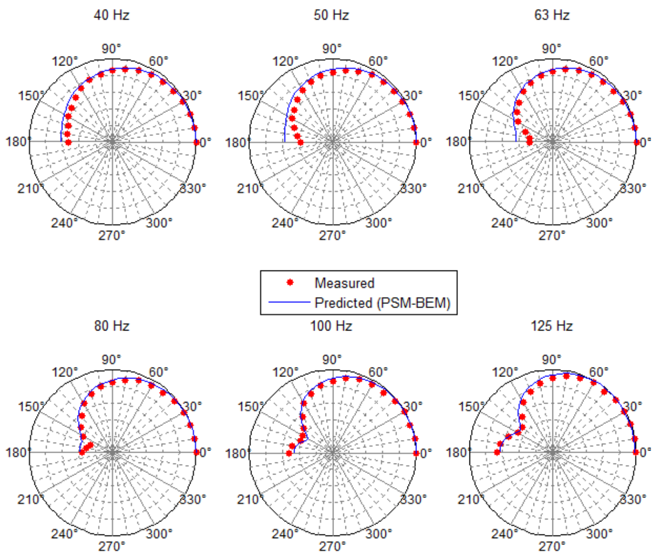


Figure 9: Measured and predicted horizontal polar data for a ground-stacked cardioid subwoofer array of two cabinets.

4.3 Dipole setting

The dipole directivity balloon is shown in Figure 10. The prediction and measurement results for the dipole setting are shown in Figure 11.

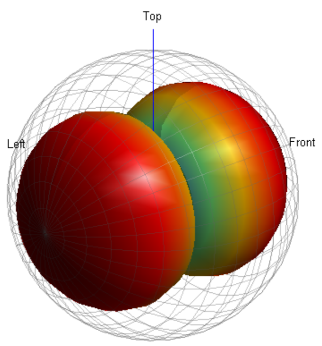


Figure 10: Desired dipole radiation pattern

Also in this case, the match between the modelled and measured data is very good. For all frequencies a strong side-ward reduction is found. The sensitivity of the dipole settings is slightly lower than for the cardioid setting. Now, the sensitivity is almost 4 dB less than for the reference array.

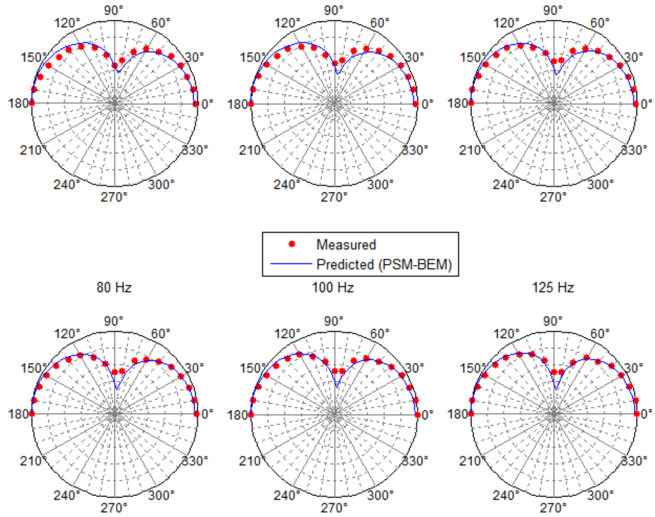


Figure 11: Measured and predicted horizontal polar data for a ground-stacked dipole subwoofer array of two cabinets.

5 Summary and Conclusion

In order to improve the acoustic modelling and optimisation of DDS-controlled, differential subwoofer arrays, the previously developed hybrid PSM-BEM model has been extended. In addition to the existing full-space radiation condition, which is more valid for flown subwoofer arrays, a half-space radiation condition has been introduced for ground-stacked arrays. Now, both the effect of the array geometry and the presence of a boundary plane, such as a hard reflective ground plane can be modelled in the DDA software.

The validity of the extended PSM-BEM model has been tested by comparing simulations and measurements on ground-stacked, differential subwoofer arrays. Various directivity characteristics have been optimised, of which the cardioid and the dipole setting were presented in this paper.

The directivity measurements confirm that the DDA predictions are very accurate. It has also been verified that these differential subwoofer arrays are very power-efficient and robust, i.e., insensitive to modelling errors.

References

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