



## Sound speed profiles linearisation for engineering methods

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Due to the large distances and the numerous sources to be considered, environmental noise impact is calculated using engineering methods. These methods are based on path finding approaches. Many studies have shown that the influence of meteorological effects has to be taken into account to get realistic results. This can be achieved by curving the ray paths or by using the curved ground analogy considering a linear variation of the sound speed along the height. As the real sound speed profiles are well defined by a combination of linear and logarithmic functions of the height, a suitable linearization approach has to be done. This is the scope of this paper. It is based on the comparison of two sets of results from Parabolic Equation calculations. A first set of calculations is done using realistic linear/logarithmic profiles. A second set is done using linear profiles. A comparison between the two sets shows that the equivalent linear sound speed profile depends mainly on the geometrical parameters. Moreover, the errors in sound levels due to the linearization are sizeable especially on an absorbing ground.

## 1 Introduction

The use of engineering methods for calculating noise impact from human activities, as industrial processes or transportation noise, is a necessity. This is due to the complexity of real situations: the number of sources, the large areas and complex topographies to deal with, that cannot be taken into account using costly numerical methods based on the solving of the equations of acoustics. There are several engineering methods (e.g. [1],[2]) all based on path findings between a source and a receiver.

Among the most important phenomena in outdoor sound propagation, the influence of meteorological effects has to be considered to get realistic results [3]. In engineering methods, this can be achieved by curving the ray paths [2] or by using a curved ground analogy [4]. The latter is easier, provided one can assume a linear effective sound speed profile.

This paper presents a way to get a suitable linear form of the sound speed profiles by using a set of numerical results from parabolic equation (PE) calculations. As the higher levels are determinant for noise impact, the work focuses on downward conditions only.

## 2 Linearization of realistic sound speed profiles

Realistic (effective) sound speed profiles close to the ground are well approximated by a combination of linear and logarithmic (lin-log) functions of the height [5], this can be written as:

$$c(z) = c_0 + b_{lin}z + a_{log} \log_{10} \left( \frac{z + z_0}{z_0} \right) \quad (1)$$

Where  $c_0$  [m.s<sup>-1</sup>] is the sound speed at  $z=0$ m,  $b_{lin}$  [s<sup>-1</sup>] the coefficient of the linear part,  $a_{log}$  [m.s<sup>-1</sup>] the coefficient of the logarithmic part and  $z_0$  [m], the (aerodynamic) roughness.

A suitable linearization of the sound profile could be achieved by searching, for a given value of  $z_0$ , a value  $b'_{lin}$  such as the sound level calculated with a sound speed profile defined by:

$$c(z) = c_0 + b'_{lin}z \quad (2)$$

is as close as possible to the sound level calculated with a sound speed profile given by Eq. (1) with  $b_{lin}=0$  i.e. considering a pure logarithmic sound speed profile.

The linearization is then equivalent to define a functional relation  $b'_{lin} = f(a_{log})$  by comparing sound level results obtained, on one side with a pure logarithmic profile and on the other side, with a pure linear profile.

At the end of the process, the most representative sound speed gradient value for the lin-log sound speed profile could be expressed as:

$$\frac{dc}{dz} = b_{lin} + b'_{lin} \quad (3)$$

$b'_{lin}$  can then be seen as the gradient of the linear sound speed profile equivalent to the logarithmic part of the lin-log profile. It can be used to define a constant optimal ray curvature  $R$  of the sound rays or a curved ground analogy, as in [4], by:

$$R = \frac{dc}{dz} / c_0 \quad (4)$$

This approach differs from previous works which were based on searching a “spatial averaged” sound speed gradient between the source and the receiver heights [6]. The aim of the present study is to find a sound speed gradient value that gives the best estimate of the sound level at the receiver.

## 3 Set of numerical calculations

Two series of calculations described below have been done using a PE code called PE\_FORTRAN. This code is a result of a partnership between EDF and IFSTTAR and is based on the IFSTTAR WAPE code [7].

For both series, the source to receiver horizontal distance  $x$  ranges from 50 to 2000m; calculations are done in third octave bands from 50 to 2 kHz; the ground is flat; the Delany and Bazley's one parameter impedance model is used and two types of ground are considered: a soft and a hard ground with an air flow resistivity  $\sigma$  of respectively 300 kNsm<sup>-4</sup> and 3.10<sup>4</sup> kNsm<sup>-4</sup>; the source height  $h_s$  is 2m; receiver heights  $z$  range from 1 to 100m.

The specificities of each serie are the followings.

Serie1: pure logarithmic sound speed profiles ( $a_{log}=0:0.01:2$ ;  $b_{lin}=0$ ) are considered;

Serie2: pure linear sound speed profiles ( $b'_{lin}=0:0.01:1$ ) are considered.

These 10000 CPU hour calculations were done on a massively parallel computer of EDF in one day.

## 4 Optimal value of $b'_{lin}$

The optimal value of  $b'_{lin}$  is determined for each case (one receiver height, one source-receiver distance and one type of ground) as the value which minimize the absolute deviation  $\Delta L$  between the two series of calculations for all the third octave bands. Strictly, an optimal value can be different for each frequency band but, as geometrical engineering methods use a constant ray path with the frequency, one must define a frequency independent optimal value.

The behaviour of  $\Delta L$  as a function of  $b'_{lin}$  is shown in Figure 1. Blue dots give the value of  $\Delta L$  for each frequency, and each  $b'_{lin}$ . The red line gives the average of  $\Delta L$  over the frequency ( $\langle \Delta L \rangle_{freq}$ ). The red plus curve give the sum of  $\langle \Delta L \rangle_{freq}$  and the positive average deviation ( $\langle d_{>0} \rangle$ ) of the blue dots to the red curve. The best estimate of  $b'_{lin}$  is found at the minimum of the red plus curve (see the cyan circle in Figure 1). This criterion ensures a low value and a low dispersion of  $\langle \Delta L \rangle_{freq}$ .

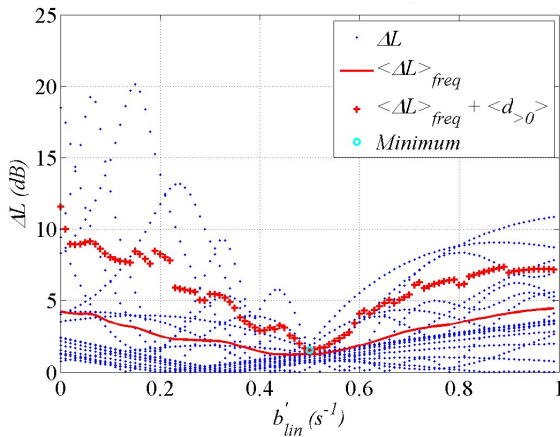


Figure 1:  $\Delta L$  as a function of  $b'_{lin}$  for  $a_{log}=1\text{m.s}^{-1}$ . ( $x=100\text{m}$ ;  $z=2\text{m}$ ;  $\sigma=3.10^4\text{ kNsm}^{-4}$ ). The optimal value of is found to be  $0.5\text{ s}^{-1}$  in that case.

## 5 Relation between $a_{log}$ and $b'_{lin}$

The process to find the optimal value of  $b'_{lin}$  for each  $a_{log}$  value is as follows, for each value of  $a_{log}$ :

- Start from the shortest source-receiver distance (i.e. 50m);
- Find an optimal value of  $b'_{lin}$ ;
- Continue to the next distance (e.g. 100m);
- Find an optimal value of  $b'_{lin}$  with a constraint on  $b'_{lin}$  to be lower or equal than the value found for the previous shorter distance and greater or equal than the value found for the previous lower value of  $a_{log}$ .

These constraints ensure:

- A physical coherent decreasing behaviour of  $b'_{lin}$  with the distance for a given  $a_{log}$  that is due to the increase of the average height of the rays with the distance between the source and the receiver;
- An increase of  $b'_{lin}$  with  $a_{log}$  at a given receiver.

The behaviour studied for several cases leads to find a relation between  $b'_{lin}$  and  $a_{log}$ . It is well approached by the following function:

$$b'_{lin} = P_1 (1 - e^{-P_2 a_{log}}) \quad (5)$$

where  $P_1$  and  $P_2$  depend on the type of ground,  $x$  and  $z$ . They are found thanks to a minimization process (fminsearch function in MATLAB®).

The result is shown in Figure 2.

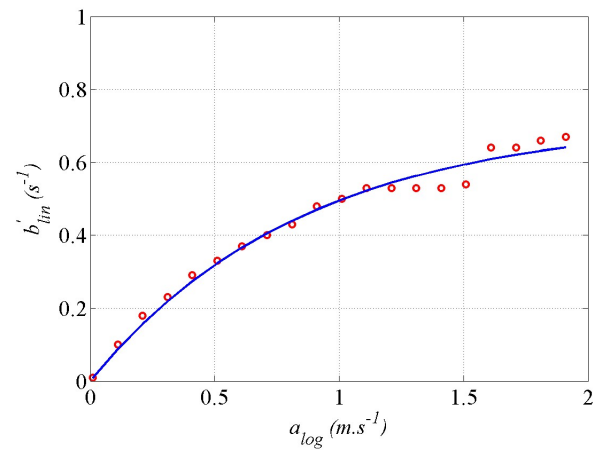


Figure 2: Relation between  $b'_{lin}$  and  $a_{log}$ .

( $x=100\text{m}$ ;  $z=2\text{m}$ ;  $\sigma=3.10^4\text{ kNsm}^{-4}$ ;  $P_1=0.7168\text{ s}^{-1}$ ;  $P_2=1.1735\text{ m}^{-1}\text{.s}$ ). The red circles are the best estimates of  $b'_{lin}$ .

The parameters  $P_1$  and  $P_2$  can be precalculated from a database of reference results and Eq. (5), (3) and (4) can be used in engineering codes to calculate the curvature of the rays.

## 6 Geometrical and ground parameters dependency

At a given horizontal distance  $x$ , for a given value of  $a_{log}$ , an increase of the receiver height induces a decrease of  $b'_{lin}$ . This is shown in Figure 3. It appears that the meteorological effects can be neglected for high receivers. This can be symmetrically applied for high sources and low receivers as for example for wind turbines. According to these results, the downwind conditions should not give a significant increase of the levels for high sources, which is physically coherent.

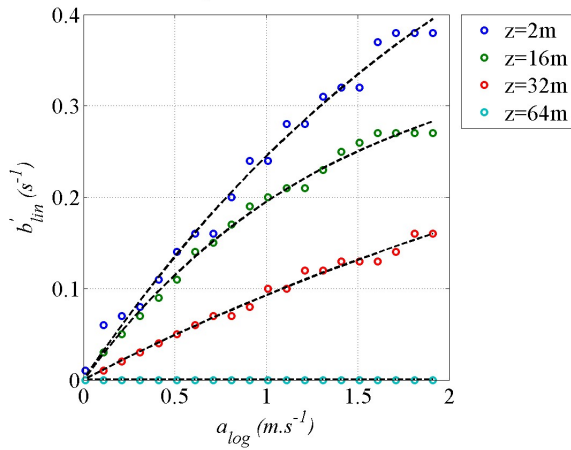


Figure 3: Relation between  $b'_{lin}$  and  $a_{log}$  for  $x=300m$ , on a hard ground, for different values of the receiver height. The circles are the best estimates for  $b'_{lin}$ . The dashed lines interpolate the results using Eq. (5).

Considering the behavior along the horizontal distance  $x$ , one can observe that the relation between  $b'_{lin}$  and  $a_{log}$  reaches a limit. An example is given in Figure 4: the results are almost the same at 500m or 1000m, for a source and a receiver at 2m above the ground.

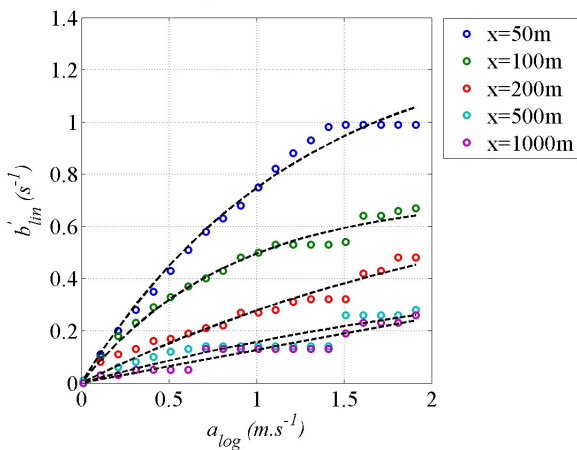


Figure 4: Relation between  $b'_{lin}$  and  $a_{log}$  for  $z=2m$ , on a hard ground, for different values of the horizontal distance  $x$ . The circles are the best estimates for  $b'_{lin}$ . The dashed lines interpolate the results using Eq. (5).

Figure 5 compares the results obtained with the same geometrical parameters as in Figure 4 for a hard and a soft ground. Even if  $b'_{lin}$  is most of the time slightly greater for a soft ground than for a hard ground, the effect of the ground type is not as important as the effect of the geometrical parameters.

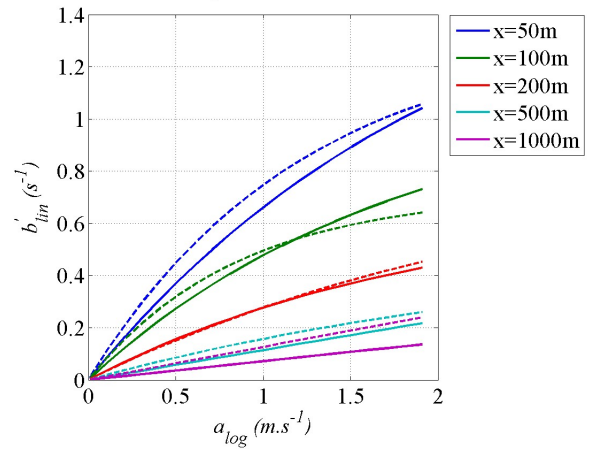


Figure 5: Relation between  $b'_{lin}$  and  $a_{log}$  for  $z=2m$  on a hard (solid lines) and soft (dashed lines) ground for different values of the horizontal distance  $x$ .

## 7 Error due to the linearization of the sound speed profiles

The linearization process gives  $b'_{lin}$  for each value of  $a_{log}$ . The error is calculated by subtracting the result of the Serie 1 calculation for a given  $a_{log}$  value (see §3) to the result of Serie 2 calculation for the corresponding  $b'_{lin}$  value.

Each  $b'_{lin}$  is determined to best estimate the level calculated with a pure logarithmic profile. In other words, as far as the process is valid, there is no better value than  $b'_{lin}$  to get a linear equivalent sound speed profile. Considering the error of this linearization process is then equivalent to evaluate the minimum error due a linear approximation of a logarithmic sound speed profile.

Table 1 shows the error in different cases. The mean and the standard deviation are calculated over the frequencies and the  $a_{log}$  values. It seems that the linearization leads to smaller errors on hard ground (less than 1 dB at up to 800m from the source) than on soft ground (up to 5 dB at 800m). The standard deviation of the error always increases with the horizontal distance and reaches very high values at large distances on a soft ground (7 dB at 800m from the source).

Table 1: Error mean and standard deviation for  $z=2m$ .

x	Hard ground		Soft ground	
	Mean (dB)	Std (dB)	Mean (dB)	Std (dB)
100m	-0.7	1.1	-1.5	1.2
200m	-0.5	2.5	-2.2	2.2
400m	-0.4	3.4	-3.4	3.8
800m	0.6	3.8	-4.8	6.2

## 8 Conclusion

The linearization of sound speed profiles has to be used in engineering methods. This paper proposes a way to estimate the best value of linear equivalent gradient by comparing reference results calculated using pure logarithmic sound speed profiles on one side, and pure linear sound speed profiles on the other side. This best value mainly depends on the geometrical parameters and at a second order on the ground parameters. A simple relation giving this best estimate can be established using a database of reference results.

Even when considering the best estimation of the linear equivalent sound speed profile, the linearization process generates sound level errors that cannot be neglected. Moreover, these errors seem to be greater on soft ground. These results can lead to the conclusion that the linearization of a sound speed profile cannot be considered discarding its associated error.

## References

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