Time reversal method for localization of sources of sound generated in viscous flows

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Localisation of sources of sound in viscous flows is an important issue in aeroacoustics. That topic concerns both academic and industrial communities. Time reversal method is widely used in linear acoustics for various purposes such as imaging or synthesizing complex wavefields. Nevertheless, time reversal method cannot be simply transposed to aeroacoustics due to the basic assumptions on which it relies. Here, an extended procedure is proposed to deal with the problems of reversing information in presence of flows and dissipation, two phenomena contradictory with the time reversal method. It is theoretically shown that time reversal method can be extended by using both the flow reverse theorem and the properties of the matched filter theory. Three cases are performed:steady flows with no dissipation, steady flows with dissipation and unsteady flows. For the first two cases, it is shown that time reversal method extension theoretically works. The latter configuration cannot be correctly treated theoretically. Then, numerical simulations supporting these results are presented. A special attention is paid to configurations dealing with viscous unsteady flows, which give good results contrary to what is predicted.

1 Introduction

Localisation of sources of sound in viscous flows is an important issue in aeroacoustics. That topic concerns both academic and industrial communities. The definition and the localization of the sources of noise in flows is still an open problem. Indeed, the concept of sources itself is fuzzy and no common definition is accepted by all the community. So, the localization of aeroacoustic sources from the knowledge of the far-field pressure remains a challenge.

One of the main difficulties is that different sources may induce the same far acoustic pressure field (no-uniqueness of the relationship between sources and fields [6]). This non-uniqueness of the solution explains the difficulty of that problem and the variety of methods proposed to solve it. One may cite the acoustic analogy approach, the causality approach or the direct resolution of the inverse problem.

The first group of methods relies on the famous Lighthill's analogy [14] or improved versions [10, 4, 13]. The application of such methods requires the choice of the source term and the propagation operator. Consequently, an *a priori* knowledge about the location and the nature of the acoustic sources is assumed. Then, they may suffer from limitations and ambiguities. For instance, previous turbulent jet noise investigations have not directly demonstrated that the noise sources actually consist in quadrupole-type sources. In this sense, based on acoustic analogy, equivalent mathematical sources may not correspond to real physical aeroacoustic sources.

The second group of aeroacoustic source localization procedures is based on the causality property. It consists in extracting the flow region which exhibits the highest correlation with the far acoustic pressure signal. In such a way, it is assumed that the source is defined in a statistical sense. These approaches may suffer from limitations due to the difficulty in *well* extracting the aerodynamic events or aerodynamic regions which are best correlated to the far pressure field [7, 8].

The third group of methods deals with solving the inverse problem directly from the equations without *a priori*. In this context, the time reversal method is appealing since it has been successfully applied in acoustics (without flow) for different kinds of problems: imaging, source localization, wavefront synthesis or communications [11]. The time reversal method appears to be a powerful method to solve inverse problems. The key idea is to use the reversibility of equations to back propagate information in time and retrieve original state for a given configuration. It relies mainly on two basic assumptions: conservation of energy and reciprocity of the medium [20, 21, 2].

Nevertheless, in aeroacoustics, the presence of flows breaks the property of reciprocity and flows are generally viscous.

At the glance, time reversal method seems not to be applicable. The present paper discusses the adaptations to perform to use it in aeroacoustics for viscous flows. It gathers the results of two recent publications [5, 9]. First of all, it is shown how to adapt the method to recover reciprocity, and to exploit the intrinsic qualities of robustness of the time reversal method. Three cases have to be distinguished: steady flows with no dissipation, steady flows with dissipation and unsteady flows. For the first two cases, it is shown that the time reversal method extension theoretically works. The latter configuration cannot theoretically be correctly treated. Then, numerical simulations supporting these results are presented. A special attention is paid to configurations dealing with viscous unsteady flows, which give good results contrary to what is predicted.

2 Theoretical analysis

The goal of this paper is to investigate the possibility to use an extended time reversal method to localize sources of sound in flows with dissipation processes. The sources can be created by different mechanisms. In the present paper, we deal with mass injection or with sound produced by the flow itself (nonlinear effect). The problem can be described by the Navier-Stokes equations:

$$\frac{\partial \rho}{\partial t} + v_i \frac{\partial \rho}{\partial x_i} + \rho \frac{\partial v_i}{\partial x_i} = m, \tag{1}$$

$$\rho \left(\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \right) + \frac{\partial p}{\partial x_i} = \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i, \tag{2}$$

$$\frac{\partial s}{\partial t} + v_j \frac{\partial s}{\partial x_j} = \frac{1}{\rho T} \left(-\frac{\partial}{\partial x_i} \left(-\kappa \frac{\partial T}{\partial x_i} \right) + \Phi \right), \tag{3}$$

where the flow fields are density ρ, velocity v_i, pressure p, entropy s and temperature T. The source of mass is noted m and the components of the source of force are f_i. The viscous tensor is assumed to be $\tau_{ij} = 2\mu \left(s_{ij} - \frac{1}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right)$, corresponding to a Newtonian fluid, with μ the dynamic viscosity and $s_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$. The thermal conductivity is denoted by κ and the viscous dissipation is defined by $\Phi = \tau_{ij} s_{ij}$. In the above equations, the Einstein convention is used. To complete this set of equations, an additional equation about the state of the medium is required. In what follows, the dissipation of the medium will be set through parameter μ (in dry air $\mu_0 = 1.8\ 10^{-5}$kg.m^{-1}.s^{-1}, this value provides the reference and $\mu^* = \mu/\mu_0$ is the dimensionless dynamic viscosity).

As mentioned in the introduction, two assumptions are necessary to use time reversal method for a given set of equations: reciprocity and energy conservation. Reciprocity is characterized by the possibility to exchange emitter and receiver: a signal emitted at point A and received at point B, is the same as a signal emitted at point B and received at point A. Obviously, the presence of flows breaks that property [17]: for a simple configuration such as the propagation of a pulse between two points (namely A and B) in presence of a uniform flow along direction AB, the signal emitted at point A and received at point B, is not the same as a signal emitted at point B and received at point A. In terms of source localization, a shift will appear between the detected source position and the original one. That shift depends on the magnitude of the flow. Consequently, the reciprocity is an essential relation and needs to be recovered.

For configurations with steady flows, it is possible to retrieve reciprocity by inverting the direction of the flows between the direct stage and the time reversal one. That relation known as reverse flow reciprocity was introduced by Howe [13] and used recently by Deneuve *et al.* [5]. Even if it seems very delicate in an experiment, from a numerical point of view, it is possible and consists in inverting both time and flow direction : $\mathbf{v}(\mathbf{x}, t) \rightarrow -\mathbf{v}(\mathbf{x}, -t)$ between the direct simulation and the time reversed one. Note that all other flow variables are reversed only in time ($t \rightarrow -t$). Thus, applying reverse flow reciprocity allows to recover the essential property of reciprocity required by the time reversal method. Unfortunately, it is not always possible to apply reverse flow reciprocity: it holds only for steady flows. Consequently, the extension of time reversal method proposed in the present paper is theoretically only valid for this kind of flows.

Conservation of energy is the second key property necessary for time-reversal method. Let's imagine waves propagating in a dissipative medium during the direct way evolution. In a time reversal experiment (numerical or experimental one), waves are back-propagated in the medium and undergo the dissipative effects twice. Note that dissipation does not break reciprocity. Indeed, in the last example, emitter and receiver can be exchanged. The effects due to dissipation are a loss of amplitude and a degradation of the original wave field. In acoustics, it has been shown that the quality of the focusing by time reversal method, is degraded [22]. The degradation consists in a reduction of the amplitude of the main lobe compared to a medium without dissipation but also in an increase of the levels of side lobes. Nevertheless, the maximum of amplitude is well-localized, even though energy conservation is broken. That is an illustration of the robustness of the time reversal method. The robustness is due to the fact that the time reversal method acts as a spatio-temporal matched filter [19]. That property means that even if energy is lost, using time reversal method ensures to back-propagate the maximum of amplitude at the appropriate point $\mathbf{x_0}$ and at the correct time $T - t_0$ where $\mathbf{x_0}$ is the original point of emission, t_0 is the original time of emission, and T is the time for which the time reversal method is applied. Note that the result is only valid for point $\mathbf{x_0}$. There is no guarantee on the control for the other points of the wave field, that is why side lobes may increase [19]. Hence, the matched filter property is very important and justifies the use of time reversal method in absorbing media. It shows that localization will be correct (in both time and space) even though the absolute level and eventually the side lobes will be degraded.

From the above considerations, three cases must be distinguished:

1. configurations with steady flows and no dissipation

   In this case, the time reversal method can be fully applied. Assumption of energy conservation is correct, and the assumption of reciprocity can be made by using the reverse flow reciprocity property. Consequently, in such a case, the time reversal method will retrieve the source with correct localization and correct level.

2. configurations with steady flows and dissipation

   In this case, again, it is necessary to ensure the reciprocity by reversing the flows. Then it is crucial to deal with the spatio-temporal matched filter property. Indeed, that property ensures that sources localization and time of emission of sound sources will be retrieved even if a slight degradation of the original state will occur. Consequently, the time reversal method can be applied to localize position and time of emission of sound sources to all configurations with steady flows even if dissipation exists, only the levels will not be retrieved.

3. configurations with unsteady flows,

   In this last case, it is not possible to justify theoretically the use of the time reversal method. Indeed, reciprocity cannot be retrieved by applying the flow reverse technique. That holds for viscous or inviscid flows. Nevertheless, as will be shown in the section devoted to numerical results, in some cases it is reasonable to use that method even though it is not theoretically justified.

3 Numerical Method

In order to illustrate the properties discussed in the previous section, various numerical experiments are proposed. These numerical experiments have to satisfy the model describing the problem: the Navier-Stokes equations and to be able to handle the time reversal method proposed above. In the next section the numerical solver used to solve these equations is briefly described, then the methodology for a numerical time reversal experiment is given.

3.1 Numerical solver

The Navier-Stokes equations (Eqs. 1, 2, 3) are solved in two-dimensional space with the code CAAmeleon (homemade code). This code is based on a reformulation of the equations under a pseudo-characteristic formulation proposed by Sesterhenn [18]. One of the main interests of the pseudo-characteristic formulation is that it enables an the implementation of efficient boundary conditions separating acoustic and hydrodynamic disturbances [16]. That property is very useful to implement time reversal method. Indeed, it requires to impose as inflow boundaries of the time reversal stage what has left the computational domain during the direct simulation (see next section for more details about the methodology).

The pseudo-characteristic formulation of the Navier-Stokes equations provides a decomposition of the pressure, velocity and entropy fluxes which enables a very simple and natural use of upwind schemes. To enforce both numerical stability and accuracy for wave propagation problems, a high order upwind Dispersion Relation Preserving (DRP) scheme is used. In the interior nodes the fourth order accurate upwind biased DRP scheme is used and this scheme is modified near the computational domain boundaries [15]. Time integration is performed using a third order TVD Runge-Kutta scheme.

3.2 Methodology of a numerical time reversal experiment

In the next section, different numerical experiments are proposed. All of them rely on the same numerical procedure:

1. **Direct Simulation** In the first stage, for a given configuration, all flow variables are computed by solving the Navier-Stokes equations. The flow variables are stored on the boundaries of the computational domain and inside the computational domain at the last time step.
2. **Time reversed simulation** In the second stage, the data stored during the direct simulation are used as inflow conditions on the boundaries of the computational domain. Before applying these boundary conditions, data are time reversed, and the direction of flows is also reversed.
3. **Data Analysis** During the second stage, the Complex Variables Method [5], [12] is used to track some particular acoustical events. In particular, this technique allows to determine the acoustical information in the aerodynamic flow without any post-processing.

4 Results and discussion

Using the numerical methodology described in the previous section, the three kinds of configurations are explored: localizing acoustical sources in (i) steady flow without dissipation, (ii) steady flow with dissipation, (iii) unsteady flow.

4.1 Configuration I: propagation in a steady flow without dissipation

It has been established in the previous section, than the extension of the time reversal method for aeroacoustics is theoretically able to localize the position and the level of source of sound in a medium with steady flow without dissipation. In order to illustrate that result, a numerical experiment satisfying these assumptions has been done: four gaussian spots of density have been imposed as initial condition in a medium with a shear flow. The shear flow is characterized by $U_0(x, y, t) = u_0 + \alpha y$ and $V_0(x, y, t) = 0$ where $u_0 = 100\text{m.s}^{-1}$ and $\alpha = 10\text{s}^{-1}$. U_0 is the x-component of the velocity while V_0 is the y-component. The direct simulation is done from 0 to $1000\Delta T$ as the reversed one (ΔT is the time step chosen to respect the CFL condition). Injection of mass is known to generate acoustical disturbances. These waves can be seen in Fig 1 where the pressure field is represented at three different instants (a) initial $t = 0$, (b) $t = 152\Delta T$ and (c) $t = 352\Delta T$. Starting from the four gaussian spots, the pressure field becomes quite complex due to the interaction of the acoustical waves with the shear flow. The right column of Fig 1 presents the pressure fields associated to the time reserved simulation at (d) $t = 648\Delta T$, (e) $t = 848\Delta T$, (f)$t = 1000\Delta T$. From these figures, it is clear that the time reversal method enables to recover both the localization and the amplitude of the source.

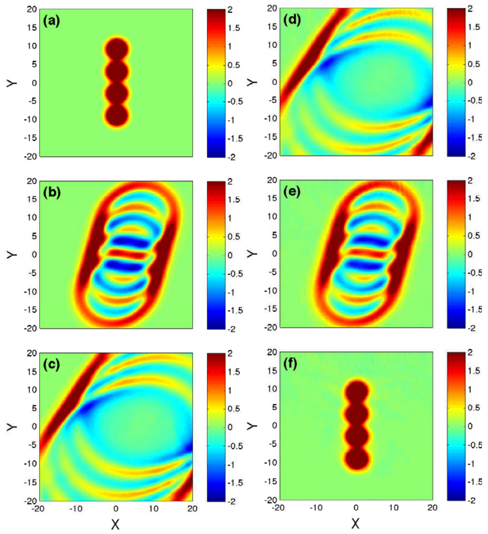

Figure 1: Direct (left column) and time reversed (right column) pressure waves due to initial gaussian spots of density injected in a shear steady flow at three different times ((a) $t = 0\Delta T$, (b) $t = 152\Delta T$, (c) $t = 352\Delta T$, (d) $t = 648\Delta T$, (e) $t = 848\Delta T$, (f)$t = 1000\Delta T$)

4.2 Configuration II: propagation in a steady flow with dissipation

To assess the performance of the method of localization through steady flow but with dissipation, the following configuration has been used: the steady flow is made with a vortex: $U_0(x, y, t) = -\alpha y \exp(\frac{1-r^2}{2})$ and $U_0(x, y, t) = \alpha x \exp(\frac{1-r^2}{2})$ where α gives the magnitude of the vortex and $r = \sqrt{\frac{x^2+y^2}{r_0}}$ describes the distance from the core (r_0 is the size of the vortex). The dissipation is chosen through parameter μ (here $\mu^* = 10$. Again, the acoustical source is due to the injection of a gaussian spot of density.

The left column of Fig. 2 shows that the cylindrical wave (the simulation is two-dimensional) emanating from the source is strongly distorted by the presence of the vortex. At this stage it is difficult to evaluate the effects due to the dissipation. The right column presents the time reversed simulation for three different instants (d) $t = 648\Delta T$, (e) $t = 848\Delta T$, (f)$t = 1000\Delta T$. It can be seen that the waves back-propagate to the good location: the distortion of the wavefront are well compensated and the original shape of the gaussian spot is retrieved. Nevertheless, the level of pressure at the source is not the same than the original one. Because of the viscous flow, the energy is dissipated during the first stage (direct computation) as well as during the time reversal stage. In this case, the only effect is on the amplitude of the pressure field.

Different simulations have been made for different values of parameter μ^* ($\mu^* \in [0, 5.5\ 10^4]$). For all these simulations, the location of the initial spot of pressure is well retrieved. Only the level is not correct except for $\mu^* = 0$ where the error is very weak (but exists because of the numerical dissipation). Fig. 3 shows the error between the original level and the one found by the time reversal method: the error grows with the magnitude of dissipation effect as expected. It is noticeable that the error function is not linear and this is due to the presence of the vortex [9].

These numerical experiments support the theoretical predictions: the extended time reversal method applied to localize sources in a steady flow with attenuation allows to find the correct position of the source but not its level. Even if this limitation is important, the position of the source is an important piece of information.

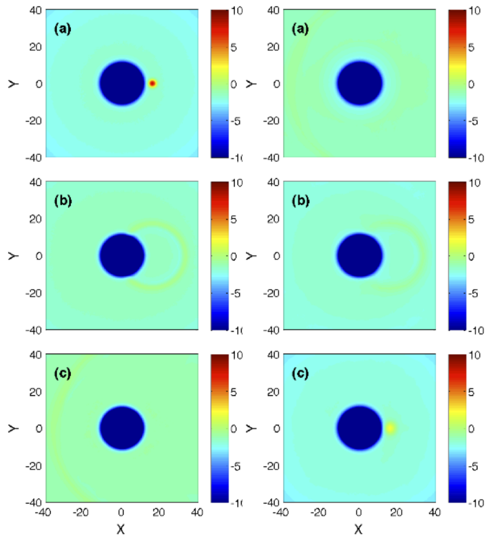

Figure 2: Direct (left column) and time reversed (right column) pressure waves due an initial gaussian spot of density injected in steady flow (one vortex) with $\mu^* = 10$ at three different times ((a) $t = 0\Delta T$, (b) $t = 152\Delta T$, (c) $t = 352\Delta T$, (d) $t = 648\Delta T$, (e) $t = 848\Delta T$, (f)$t = 1000\Delta T$)

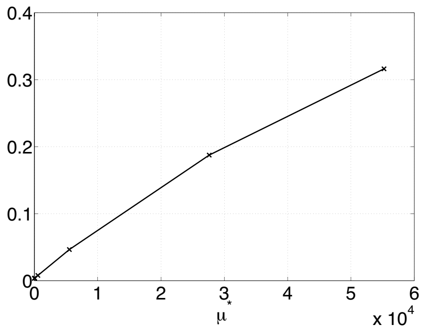

Figure 3: Relative error between the level of the initial source and the level predicted by the extended time reversal procedure

4.3 Configuration III: propagation in unsteady flow

The third case deals with the problem of finding source in an unsteady flow. A case of interest is the case for which the unsteady flow is the acoustical source itself. A two-dimensional plane mixing layer can generate sound [3]. In the present paper, the flow consists in two uniform streams with Mach numbers 0.5 and 0.25, respectively, for the upper and lower parts. The two regions are matched by a hyperbolic tangent profile for the streamwise velocity. Outflow conditions are specified at the top, the bottom and the left boundaries. Moreover, the flow is forced at the fundamental frequency predicted by the linear stability theory and its first, second and third subharmonics. The inflow condition used on the left boundary is similar to the one used in Colonius et al. [3]. With such deterministic inflow perturbations, two quasi-stationary vortex pairings are observed within the computational domain. The regions where pairings occur are associated with the sources.

Figure 4 shows at the same time the instantaneous pressure field at frequency $f_0/2$ (thanks to the complex variables method, not described here [5]) which is superimposed onto the vorticity field. Note that the vorticity field is obtained from the direct simulation.The first test case using $\mu^* = 0$ allows to recover previous results [5]: the pressure field converges towards the origin of the acoustic wave of $f_0/2$. This origin coincides with the first pairing observed in the instantaneous vorticity field known to be the region where sound is generated [1]. That result shows that the time reversal method enables to localize sources generated in a mixing layer. But because of the unsteadiness of the flow, it has been theoretically predicted that time reversal method cannot be used. Nevertheless, in that configuration, the region where flow is unsteady is confined in the zone of high vorticity. The extent of this zone is measured by the vorticity thickness (here $\delta_\omega \approx 1$). In comparison, the wavelength associated to acoustical waves (characteristic length for acoustic process) is higher. For the highest frequency considered (and so the smallest acoustical wavelength) which is $f_0/2$, the wavelength is about equal to 25. That difference between the characteristic scales of aerodynamic and acoustical processes explains why time reversal method seems to be working when the interest is focused on acoustical variables. Time reversed acoustical waves are only slightly altered by the noreciprocity which is essentially localized in a very small region of space compared to the acoustical wavelength.

5 Conclusion

An extended time reversal method to localize the source of sound in aeroacoustics have been proposed. Theoretically, three configurations can been distinguished: (i) localization in steady flows without dissipation, (ii) localization in steady flows with dissipation and (iii) localization in unsteady flows. Successive numerical experiments involving localization of acoustical sources by the extended time reversal method have been presented in the above configurations. For steady flow configuration, numerical results demonstrate that the time reversal procedure allows to detect the origin of the aeroacoustic source but the source amplitude is reduced due to the viscous energy loss. It is then confirmed that the application of the methodology permits to recover the shape and the location of the source of sound. Furthermore, it is emphasized that even if the presence of an unsteady flow (including viscous fluid) theoretically breaks the reversibility of the fluid motion equations, such an approach is robust and effective for the detection of particular aeroacoustic sources. The potential of the proposed methodology is then underlined and such an approach offers some new potential future applications in determining and analyzing the physical mechanism by which turbulence can generated sound.

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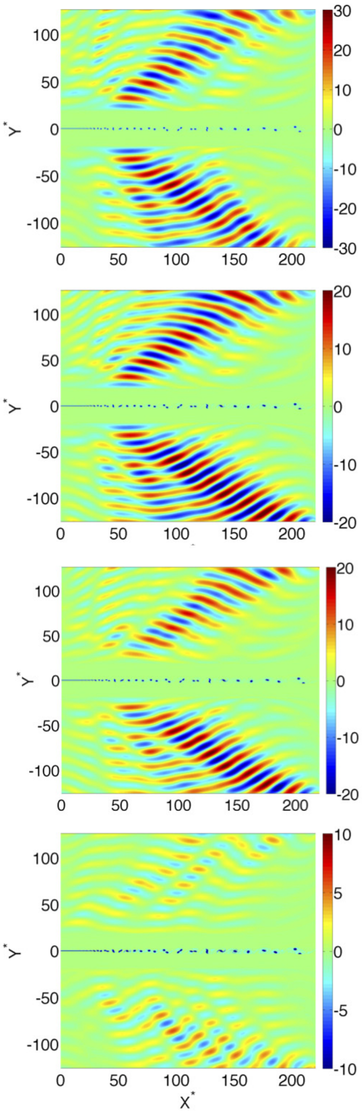

Figure 4: Time reversed simulations of plan mixing layers for different values of μ. Top to bottom: $\mu = 0$, $\mu = 0.0001$, $\mu = 0.001$ $\mu = 0.01$ U.S.I.

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