Active control applied to string instruments

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This study aims to control the soundboard’s vibrational modes in order to modify the timbre of string instruments. These structures are wooden plates of complex shapes, excited by strings through a bridge. In order to apply an efficient control on such systems, modal parameters must be identified using classic algorithms applied on experimental measurements. Then it is possible to design a feedback controller using these parameters and classic control methods. At first, a simulation of the control has been done for a rectangular spruce plate, boundary clamped and excited by a single string. This simplified case and first results in term of changes in eigenmodes are presented here.

1 Introduction

The main objective of active control is to reduce annoying noise. Two main fields have been concerned by active control in the last decades: acoustics and structural vibration. Different methods, depending of noise sources can be applied. More recently, active control has also been used to modify sound qualities instead of cancelling it.

A lot of studies have been made in order to modify the characteristics of musical instrument sounds since [1]. Usual methods have been used like PID, bandpass or notch filter control [2]. In most studies, string instruments or percussions are controlled. The active control applied to musical instruments aims to modify their sound characteristics. For instance, some people have tried to control the instrument excitation [4]. Other people have worked on structural vibration of musical instruments. In these cases, they have tried to decrease and to increase the instrument vibration. Different works can be found in the literature, for example on modal parameters of a guitar soundboard [3] or on vibration of a xylophone bar [5]. Applied to soundboards, this approach only allows the modification of instruments timbre and does not affect the pitch of the final sound.

In the following study, the modal control method is used to achieve a stable control of string instrument soundboards. The different steps of this method are: (1) the identification of modal parameters of the studied structure, (2) the use of these parameters to set the observer and the controller, (3) an optimisation of transducers dimensions and positions and (4) the application of control.

This article presents this method and its application to the simulation of a simple rectangular spruce plate under string excitation. Finally, first experiment of modal identification on this simple plate is presented.

2 Modal control

2.1 General principle

The main advantage of modal control is that it makes possible to target the control on modes of vibration. A multi-input multi-output system can be used to modify several modal parameters of string instrument soundboards. This method is based on the state space approach which starts from a system description using first order equations governing the state variables of the studied structures.

Figure 1 shows an exemple of state space approach feedback loop. In this case, the dynamic of linear system may be described by a set of first order linear differential equations [6]:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + K_w w(t) \\
y(t) &= Cx(t)
\end{align*}
\]

with
- \( x \) = state vector
- \( u \) = control signal
- \( y \) = measured signal
- \( w \) = disturbance signal

and
- \( A \) = system matrix
- \( B \) = input matrix
- \( C \) = output matrix
- \( K_w \) = disturbance signal input matrix

Structural vibration can be measured by sensors as shown in Figure 1. Control signal \( u(t) \), which is computed using the controller, is sent into the structure thanks to actuators. This control signal is added to the disturbance signal and involves a new structure dynamic. Matrices \( C \) and \( B \) match respectively the behaviour of sensors and actuators while matrix \( K_w \) corresponds to the modes amplitude at the application point of the disturbance signal \( w(t) \).

2.2 Control system

2.2.1 Controller

The function of the control system is to generate a control signal \( u(t) \) from the signal measured by sensors and to send it into the structure thanks to actuators in order to shift its modal parameters. This control signal is chosen to reach a new modal state of the structure. Several algorithms can be used to find the gain matrix \( G \) which gives the desired control signal when it is multiplied by the state vector \( x(t) \). Thus, it can be written as:

\[
u(t) = -Gx(t)
\]
Such a method allows to choose the pole position of each mode and has the advantage of being always stable. Thus, both a decrease and an increase can be applied to modal damping and frequency. For our study, a pole placement algorithm is used to find the gain matrix $G$.

2.2.2 Observer

However, this kind of controller needs the modal state of the structure as its input. No sensor is able to measure the modal state directly. So an observer is introduced in the feedback loop to estimate this modal state. It uses the matrices $A$, $B$ and $C$ in order to model the dynamics of the structure and the transducers behaviour. The matrix $L$ determines the convergence properties of the control algorithm [7]. It is multiplied by the difference between the measured signal $y(t)$ and the estimated signal $\hat{y}(t)$ and can be calculated in the same way as $G$. Then, an observer is implemented in the feedback loop to give an estimated vector state $\hat{x}(t)$ which is multiplied to the gain matrix $G$ in order to give the control signal $u(t)$. The equation of the observer is:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) \tag{4}$$

with

$$\dot{\hat{y}}(t) = C\hat{x}(t) \tag{5}$$

Then Eq. (3) becomes:

$$u(t) = -G\hat{x}(t) \tag{6}$$

It can be noticed that the controller and the observer design can be carried out independently. This is known as the separation principle [6].

2.3 Structural vibration control

2.3.1 Structural specifications

The modal control can be applied in several structures [9] or to other systems such as ducts [8]. In the present case, the structures under study are string instrument soundboards which are wooden plates with complex shapes. Soundboards bring unusual characteristics for active control. For example, wood is an anisotropic material and has a relatively high damping compared to metal (usual material in active control literature). Moreover, the disturbance signal is a string force applied on the soundboard through a bridge.

2.3.2 Identification

Modal parameters are necessary to achieve the control. Indeed, matrices $A$, $B$ and $C$ contain natural frequencies, modal dampings and eigenvectors of the structure. Classic methods are used to find these parameters [9]. Nevertheless, a modal analysis of the structure has to be done to find its eigenvectors. As some programs like Modalys give the modal parameters, identification algorithms included in this software are used. It’s important to notice that identification has to be done very carefully. Indeed, the control quality depends on the accuracy of modal parameters.

3 Control simulation

A simulation using analytical model has been realised in order to design the control system and to predict its effect.

3.1 Structure

The considered structure is an orthotropic wooden rectangular plate under simply supported boundary conditions. Mode shapes and natural frequencies of such a structure are given by (see for example [10]):

$$\Phi_{mn}(x,y) = A_{mn} \sin \left( \frac{m\pi x}{L_x} \right) \sin \left( \frac{n\pi y}{L_y} \right) \tag{7}$$

$$\omega_{mn} = \pi \left( \frac{1}{\rho h} \right)^{1/2} \left( D_1 \frac{m^4}{L_x^4} + D_2 \frac{n^4}{L_y^4} + (D_2 + D_4) \frac{m^2n^2}{L_x^2L_y^2} \right)^{1/2} \tag{8}$$

with $D_1$, $D_2$, $D_3$ and $D_4$ coefficients depending on mechanical parameters of the structure.

As a first approach, modal dampings are taken directly from the literature [11] with $\xi = 0.0063$.

Finally, modal parameters allow one to have state matrices and to build the model of the structure. As these matrices are given by an analytic method, they can be used in the control simulation to build the observer. So the identification step is not necessary in this case.

3.2 Transducers

In this paper, transducers effect is only modeled by their position. Indeed, electromechanical sensors and actuators coupling will be included in matrices $C$ and $B$ in further work.

3.3 Control

A pole placement algorithm is used to manage the control. A single-input single-output system is applied. Transducers are colocalised at the bridge position. The control signal $u(t)$ is given thanks to the gain matrix and allows one to reach the desired modal state for the structure. Nevertheless, this control doesn’t allow independent modifications of each mode. Several sensors and actuators must be used and well placed to perform an independent control.

It is then possible to compute the open loop transfer function and to compare it to the closed loop transfer function in order to observe the control effect.

3.4 Time simulation

Matrices $A$, $B$, $C$, $K_w$, $G$ and $L$ are used to do a time simulation of a plate model excited by a string. This excitation is created thanks to Modalys [12], a physical-based synthesis program, and is sent into the plate. Model of the section 3 is used in Simulink, a Matlab toolbox, in order to listen the effect of the control on the sound emitted by the soundboard. The string signal filtered by the controlled soundboard is then compared to the signal filtered by the uncontrolled soundboard.

3.5 Results

3.5.1 Control results

First, modifications of the frequency response function at the bridge position are observed. Figure 2 shows this frequency response before and after the control.

It can be noticed that it’s possible to modify the natural frequency and the damping of several modes. In Figure 2,
the damping of the second mode is increased as it is shown in Figure 3 (left). The damping and the natural frequency of the twelfth mode are respectively decreased and shifted as it is shown in Figure 3 (right).

3.5.2 Time simulation results

Figure 4 shows the effect of this soundboard control when the disturbance signal is a bowed string signal. The spectrum of the measured signal is shown before and after the control of the soundboard. The Figure 5 (left) shows the first harmonic which is decreased thanks to the control of the second mode of the soundboard. The Figure 5 (right) shows the fifth harmonic which is increased thanks to the control of the twelfth mode of the soundboard. These modifications of the string signal are done with only one sensor and one actuator. A control with more transducers is able to control more harmonics in the same time.

The gain variations on controlled harmonics between this two spectrums are about 10 dB thanks to control.

4 Experiment

In order to test the control method, a simple experiment is set. A modal identification of this experiment has been done so far.

4.1 Experimental setup

Figure 6 shows the experimental setup which is a rectangular spruce plate used by luthiers to make soundboards of acoustic guitars. The plate is 40 cm wide, 60 cm long, 4 mm thick and under clamped boundary conditions. A single string is tight on and connected to the bridge with a direction almost parallel of the soundboard plane.
4.2 Modal analysis

4.2.1 Identification

A modal analysis of the structure is made thanks to vibrometer measurements. The average frequency response is presented in Figure 7. The identification is done thanks to Modan and allows one to get the natural frequency, the modal damping and the modal shape of each mode of the soundboard from its experimental frequency responses. These informations are very useful for the transducers optimisation.

Figure 7: Average frequency response of the experimental setup.

Several measurements are done to study the effect of the string tension on the soundboard modes. These measurements show that this effect is very small. The string tension slightly shifts modal frequency to high frequencies and slightly changes modal amplitude. Modal shapes are not influenced by string force because the string is tight parallel to the soundboard.

4.2.2 Finite element model

Then a finite element model using plate elements is compared to the experimental results. This numerical study uses mechanical characteristics from the literature and clamped boundary conditions. Figure 8 shows the experimental and numerical modal shape of the sixth mode while table 1 gives experimental and numerical frequencies of the eight first modes. For most of modes, modal shapes and natural frequencies are very close between the experiment and the finite element model. The small differences are certainly due to differences in elasticity parameters.

This comparison shows that the finite element model can be computed without the string preloads when it is used for transducers optimisation.

Figure 8: Example of comparison between the sixth soundboard mode found with experiment and with numerical model.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Experimental frequencies</th>
<th>Theoretical frequencies</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>61 Hz</td>
<td>74 Hz</td>
<td>18 %</td>
</tr>
<tr>
<td>2</td>
<td>93 Hz</td>
<td>112 Hz</td>
<td>17 %</td>
</tr>
<tr>
<td>3</td>
<td>164.5 Hz</td>
<td>183 Hz</td>
<td>10 %</td>
</tr>
<tr>
<td>4</td>
<td>172.5 Hz</td>
<td>185 Hz</td>
<td>7 %</td>
</tr>
<tr>
<td>5</td>
<td>192.5 Hz</td>
<td>212 Hz</td>
<td>9 %</td>
</tr>
<tr>
<td>6</td>
<td>257.5 Hz</td>
<td>268 Hz</td>
<td>4 %</td>
</tr>
<tr>
<td>7</td>
<td>277.5 Hz</td>
<td>283.5 Hz</td>
<td>2 %</td>
</tr>
<tr>
<td>8</td>
<td>327 Hz</td>
<td>354 Hz</td>
<td>8 %</td>
</tr>
</tbody>
</table>

5 Discussion

This paper presents the design of a control of the soundboard’s vibrational modes. Modal control method is described. Then a numerical simulation of a simple case and its results are presented. Finally, a basic experiment is studied.

Next step of the study is the piezoelectric actuators and sensors design. Indeed, transducers must be efficient on the controlled modes. That’s why a finite element model including piezoelectric components must be developed to choose size and location in order to be highly efficient on the controlled modes [6], [14]. Finally, numerically designed controller have to be implemented in the real structure to validate the method.

However, some new problems appear. For instance natural modes of a wooden plate are very damped and a lot of control methods are used for rightly damped structure. Using of piezoelectric patches also bring problems as ceramic-wood coupling is small.

Main goal is to apply control to musical instruments. Then perceptive studies will be done to evaluate the control effect on the instruments sound [13]. Finally, other methods like semi-adaptive method will be tested.

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References


