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Nonlinear tissue mimicking phantoms characterization using the Nakagami statistical model: simulations and measurements

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In order to improve the tissue characterization, the probability density function of ultrasonic backscattered echoes which may be treated as random signals, is modeled by using Nakagami statistical distribution. Recently, it has been found that Nakagami statistical model constitutes a quite good model in tissue characterization due to its simplicity and general character.
In the present study, computer simulations and experiments on phantoms have been carried out to test the validity of Nakagami distribution in order to model the backscattered envelope of ultrasonic signals in the nonlinear case.
Experiments were performed using a 5MHz linear array connected to an open research platform. A commercially available phantom was used to mimic tissue backscatter. For different sizes and positions of the sampling window, the RF signals have been genrated at different frequencies and bandwidths, and the received echoes have been filtered around the center frequency and around twice the center frequency. Obtained signals have been analyzed in order to evaluate the Nakagami parameter (m), the scaling parameter (Ω) and the probability density function. These latter results have been compared to those obtained by using Field II software.

1 Introduction

Many researchers have used stochastic models to describe the probability density function (PDF) of the envelope of the backscattered echo of tissues which may be treated as a random signal. The parameters of these distributions depend on some characteristics such as the density (number of scatterers within the transducer resolution cell) and scattering amplitude related to the size of the scatterers. Among the commonly used distributions, we can quote Rayleigh distributions (square root of an exponential distribution), K-distribution [1] (square root of the product of a Gamma distribution with an exponential distribution), and Nakagami (square root of a Gamma distribution). The Rayleigh model which is commonly used [2], needs some conditions such as the presence of a large number of randomly located scatterers. Wagner [3] classifies the other models according to their Signal to Noise Ratio (SNR) compared with the SNR of Rayleigh distribution. The first class called pre-Rayleigh (SNR < 1.91) describes heterogeneous texture. The second, called Rayleigh (SNR=1.91), defines the homogenous texture class. The third corresponding to the periodic texture is the post-Rayleigh class (SNR > 1.91).

The K-distribution was shown to model pre-Rayleigh and Rayleigh texture [4, 5]. The two K-distribution parameters, provide information on the number of scatterers, the variation in the scattering amplitude and the average scattering amplitude. But it is not enough general to describe the statistics of the backscattered echo from range cells containing a periodic alignment of scatterers giving rise to post-Rayleigh. Recently Nakagami statistical model initially proposed to describe the statistics of radar echoes, was shown to be able to quantitatively characterize biological tissues thanks to its two parameters, Nakagami parameter (m) and scaling parameter Ω [6]. In addition to the scattering amplitude and density, this model can take into account the regularity of the scatterers spacing [7]. Nakagami statistical model has comparatively less computational complexity than the other models and is enough general to describe a wide range of scattering conditions in medical ultrasound, including pre-Rayleigh, Rayleigh and post-Rayleigh distributions. Although the Nakagami distribution can fit well with the PDF of the ultrasonic envelope, a multiple statistical distribution may be more appropriate to model the envelope statistics, because the ultrasonic signals returned from the tissues may contain contributions from more than one mechanism [8].

This paper is organized as follows. First, we introduce Nakagami model and its parameters. Secondly , computer simulations and experiments on phantoms are processed with the estimation of the PDF and the two Nakagami parameters. The comparison, the discussion, and some concluding remarks close up the paper.

2 Statistical model: Nakagami distribution

The probability density function, of the envelope f(R) of the backscattered signal can be described in terms of the Nakagami distribution and is defined by:

$$f(R) = \frac{2m^m R^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{m}{\Omega}R^2} U(R) \qquad (1)$$

where Γ(.) and U(.) are the Gamma function and the unit step function, respectively. Nakagami parameter (m) and scaling parameter (Ω) can be calculated as follows:

$$m = \frac{[E(R^2)]^2}{E[R^2 - E(R^2)]^2} \qquad (2)$$

and

$$\Omega = E(R^2) \qquad (3)$$

where E(.) is the statistical mean. The scaling parameter refers to the average power of the backscattered envelope. Moreover, the Nakagami parameter is particularly useful for characterizing the probability distributions of ultrasonic backscattered envelopes, including the statistical conditions for pre-Rayleigh, Rayleigh, and post-Rayleigh distributions. When the resolution cell of the ultrasonic transducer contains a large number of randomly distributed scatterers, the envelop statistics of the ultrasonic backscattered signals obeys the Rayleigh distribution. If the resolution cell contains the scatterers with randomly varying scattering cross sections having comparatively high degree of variance, the envelop statistics are pre-Rayleigh distributions. If the resolution cell contains periodically located scatterers in addition to randomly distributed scatterers, the envelop statistics are post-Rayleigh distributions. Because the values of m ranging between 0 and 1 reflect statistics ranging from pre-Rayleigh to Rayleigh distributions and larger values correspond to the PDFs of post-Rayleigh or Rician distributions, thus the

Nakagami parameter can be used to classify the properties of tissues. This has been validated in computer simulations experiments on phantoms [6, 8] and clinical measurements [9].

It is possible to see that the Nakagami distribution can be identified as belonging to the Gamma distribution class. If we define a new random variable A of Gamma distribution with parameters (α; β) and $R = \sqrt{A}$, the probability density function f(R) can be written as:

$$f(R) = \frac{2\beta^{\alpha}}{\Gamma(\alpha)} R^{2\alpha-1} e^{-\beta R^2} U(R) \quad (4)$$

which is a Nakagami distribution with parameter (m =α; Ω = α/ β). For convenience, we use the density given by the Eq. (4). as the density function of the Nakagami distribution with parameters (α; β). The second order moment of this distribution is then given by:

$$E(R^2) = \frac{\alpha}{\beta} \quad (5)$$

3 Experimental and theoretical results on phantoms

Both experimental measurements and computer simulations were carried out to explore the effect of some physic parameters on the estimation of the Nakagami statistical parameters.

The experimental arrangement includes an ultrasonic phantoms, a 5MHz linear array connected to an open research platform which is connected to a computer in order to visualize the machine interface where RF signals and B-scan ultrasound images are displayed, as well as the different windows allowing the selection of various physical parameters of the system such as the transmission frequency, the bandwidth, the number of time samples of the RF signals, etc…. For different positions of the sampling window, the RF signals have been acquired at different frequencies (from 2 to 6 MHz) bandwidths of the excitation waveforms. The received RF waveforms are then filtered around the center frequency and around twice the center frequency in order to detect an eventual non linear effect. Tissues are known to be a nonlinear propagation medium and generate new frequency components. To better characterize tissue properties, it is necessary to take into account the nonlinear behavior in the statistical model.

For the simulation, we used Field II software, developed by Jorgen Jensen [10] at the Technical University of Denmark, which is dedicated to the calculation of the pressure field at any point and whose major interest is to simulate probes of complex shape. By extension it allows the simulation of RF signals. This software has the advantage of simulating the contributions of the probe characteristics (shape, shooting strategy, excitation ...) on a 2D or 3D digital phantom and obtains the corresponding RF signals [11]. Ultrasound images are obtained by examining tissue which is defined by their reflectivity, with a probe itself characterized by its impulse response. Ultrasonic RF signals thus result from summing the responses of the diffusers during the ultrasonic wave propagation.

The results obtained from measurements and simulations were compared in order to study the effect of some physical parameters of the phantom on the sensitivity of the Nakagami parameters, in particular Nakagami parameter (m).

The theoretical and experimental results are obtained for two square sampling windows of size 0.3 cm^2 located at different positions inside the phantom, and for two different transmitting bandwidths (80% and 90%). The results show the same evolution of the Nakagami parameters according to the physical parameters. Changes in position of the sampling windows and bandwidth, does not influence too much on this evolution for the two transmitting ultrasonic frequency (Figures (1) and (2)). For the central frequency of 5 MHz, in most cases, the values of the Nakagami parameter are very close.

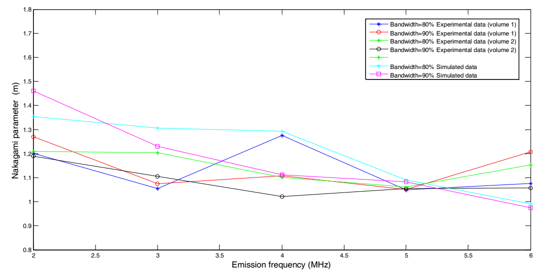


Figure 1: Comparison of theoretical and experimental values of Nagakami parameters versus the transmitting frequency for different transmitted bandwidths and without filtering the detected waveforms.

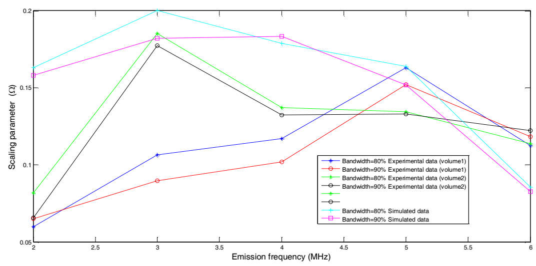


Figure 2: Comparison of theoretical and experimental values of the scale parameter versus the transmitted frequency for different bandwidths and without filtering the received signals.

The Nakagami parameter according to the transmission frequency and the position of the sampling window and of the bandwidth, increases with the filtering and it is more important in the case of a filtering around $2f_0$ (Figures (3) and (4)).

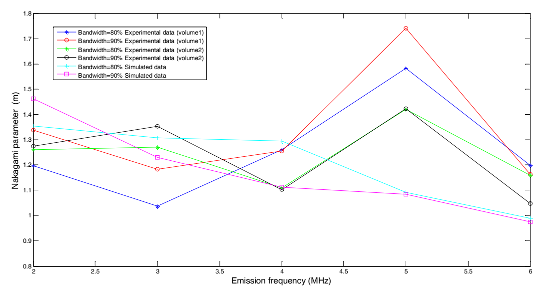


Figure 3: Comparison of theoretical and experimental values of Nagakami parameters versus the transmitted frequency for different values of the transmitted bandwidths and with a filtering of the received signal around $2f_0$.

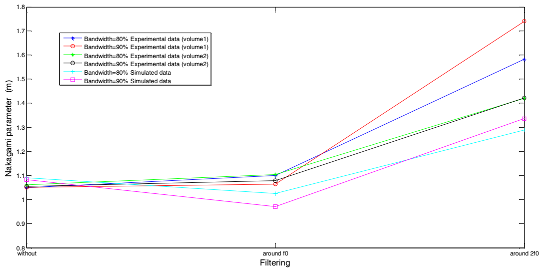


Figure 4: Comparison of theoretical and experimental values of Nagakami parameters versus the filtering in the case for a transmitted frequency f_0=5MHz.

Moreover we have tested the effect a nonlinear logarithmic compression on the envelopes of the echo signals, given by the expression [12]: $Z = \log_{10}(R+1)$.

We notice the same evolution of the new Nakagami parameters estimated by a non linear logarithmic compression when compared to the original parameters previously estimated. Nevertheless we can notice that the variation of the former values of m and Ω are respectively in the range 0.8 to 1.8 and 0.05 to 0.2 respectively, while after the logarithmic compression these values are respectively in the range 10 - 26 for m and 0 - 0.8 for Ω (Figures (5) and (6)).

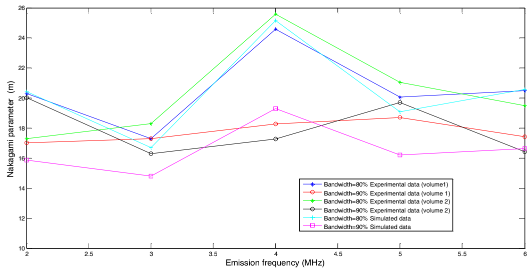


Figure 5: Comparison of theoretical and experimental values of Nagakami parameters versus the transmitted frequency for different values of the bandwidth after a non linear logarithmic compression of the envelope and without filtering.

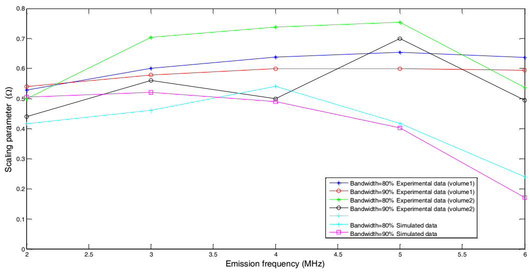
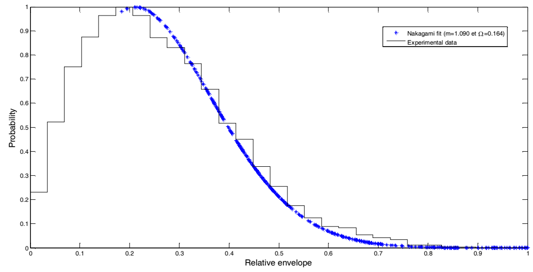
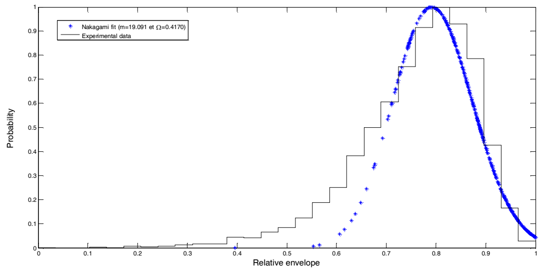


Figure 6: Comparison of theoretical and experimental values of Nagakami Parameters versus the transmitted frequency for different bandwidths after a non linear logarithmic compression and without filtering.

Figures ((7.a), (7.b), (8.a), (8.b)) represent the probability density function (PDF) of the backscattered envelope before and after nonlinear logarithmic compression for theoretical and experimental envelopes, respectively.

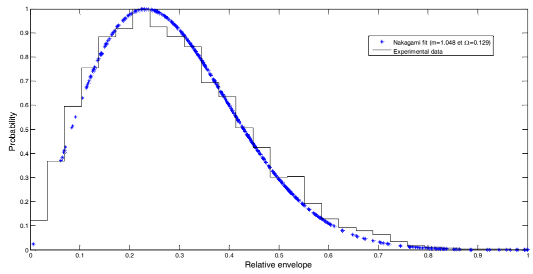


(a)

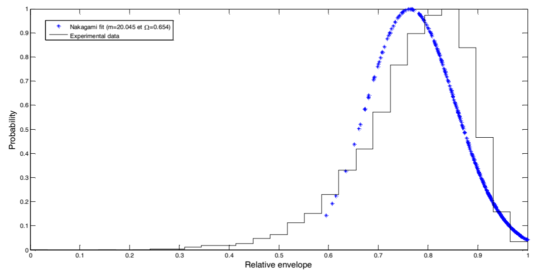


(b)

Figure 7: The probability density function (PDF) of the theoretical backscattered envelope.
(a)Before logarithmic compression; (b) After logarithmic compression.



(a)



(b)

Figure 8: The probability density function (PDF) of the experimental backscattered envelope.
(a)Before logarithmic compression; (b) After logarithmic compression.

4 Conclusion

The evolution of Nakagami parameters versus the physical parameters, such as the bandwidth, the emission frequency, the filtering around this frequency and the location of a sampling window in the phantom, is less important comparatively to the case when a nonlinear logarithmic compression is used. Thus the non linear logarithmic compression leads to Nagakami parameters more sensitive to the variation of the physical parameters, and then improves the sensitivity of the filtering around twice the centred frequency; this result would be used for improving the quality of images obtained by harmonic imagery.

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