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On the usefulness of entropy in statistical energy analysis

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Statistical energy analysis is well-known method in the field of high frequencies. This method is founded on an analogy with thermodynamics and is largely inspired from results of statistical mechanics. But nowadays, the method is limited to the prediction of vibrational energies by application of the first principle of thermodynamics that is an energy balance on individual subsystems. In this study, we propose to extend statistical energy analysis by introducing the concept of vibrational entropy. The explicit formula given the vibrational entropies in terms of energies and number of modes is given. The entropy created during the energy echanges between subsystems is also given. Some examples are described which illustrate the meaning of vibrational entropy and vibrational temperature.

1 Introduction

Statistical energy analysis is a well-known method in the field of medium and high frequencies in structures and acoustics. The mean idea is to apply statistical physics to mechanical structures. Modes are random and excitation forces are white noises. It results in a particularly simple method based on application of an energy balance to individual sub-systems. The underlying assumptions are,

- White noise random forces mutually uncorrelated,
- Modal forces have same power spectral density,
- Couplings between sub-systems are conservative,
- Weak couplings between sub-systems,
- Large number of modes in all sub-systems,
- Light damping loss factors.

With this set of assumptions, the exchanged power between two adjacent sub-systems is found to follow the so-called coupling power proportionality relationship which states that power is proportional to the difference of modal energies.

In this study, we introduce the concept of vibrational entropy. Thus, we extend statistical energy analysis taking advantage of the application of the second principle of thermodynamics.

2 Energy and SEA

In statistical energy analysis, the structure is subdivided in n sub-systems. The primary variables of sub-systems are the vibrational energy E_i and mode count N_i for $i = 1 \dots n$ contained within a frequency band of width $\Delta\omega$ and central frequency ω (typically an octave band). The sources assumed to be white noise mutually uncorrelated, are characterized by their injected power P_i.

The dissipation of vibration occurring in sub-systems follows the law,

$$P_i^{\text{diss}} = \eta_i \omega E_i \tag{1}$$

which states that the dissipated power is proportional to the energy level. η_i is a dissipation factor which may include several types of dissipations processes.

The net power exchanged between sub-systems i and j follows the coupling power proportionality,

$$P_{ij} = \omega\left(\eta_{ij}E_i - \eta_{ji}E_j\right) \tag{2}$$

where the coupling loss factors verify the reciprocity relationship $\eta_{ij}N_i = \eta_{ji}N_j$. This law of proportionality of power and difference of modal energies constitutes the main result of SEA. It has been first derived from a modal approach in Refs. [1, 2, 3]. In Refs. [4, 5], its derivation is based on the geometrical approach with rays. The notion of statistical ensemble is of a great importance [6] particularly with regard to the question of variability of responses [7, 8]. Finally, in Ref. [9], some interesting arguments based the H-theorem are given in favour of this law.

The energy balance of sub-system i in steady-state condition reads,

$$\omega\eta_i E_i + \sum_{j\neq i} \omega\left(\eta_{ij}E_i - \eta_{ji}E_j\right) = P_i \tag{3}$$

This is a set of linear equations on unknowns E_i. It can be used for instance to calculate E_i from the knowledge of P_i.

3 Entropy and SEA

In this section are summarized all the equations on entropy in SEA. The proofs and comments may be found in Refs. [10, 11, 12].

Entropy of a sub-system with vibrational energy E and number of modes N is,

$$S = kN\left[1 + \log\left(\frac{2\pi E}{h\omega N}\right)\right] \tag{4}$$

where $k = 1.38\ 10^{-23}$ J K^{-1} is Boltzmann's constant, $h = 6.63\ 10^{-34}$ m^2 kg/s is Planck's constant and ω the central frequency of bandwidth in rad/s. The physical unit of entropy is J/K.

The vibrational temperature of sub-systems is defined as,

$$T = \frac{E}{kN} \tag{5}$$

with physical unit K.

In a driven sub-system, excitation forces tend to warm up it by injecting energy. The rate of entropy being injected in sub-system i is,

$$\frac{dS_i^{\text{inj}}}{dt} = \frac{P_i}{T_i} \tag{6}$$

where T_i is the vibrational temperature and P_i (W) the power being injected by driven forces.

Dissipation of vibration by natural damping processes tend to cool down the sub-system since damping is responsible of reduction of energy. The rate of entropy being extracted is,

$$\frac{dS_i^{\text{diss}}}{dt} = \frac{\eta_i\omega E_i}{T_i} = k\eta_i\omega N_i \tag{7}$$

Finally, during exchange of power between sub-systems, a mixing entropy is created. This is a irreversible process. The rate of entropy created at interfaces is,

$$\frac{dS_{ij}^{\text{irr}}}{dt} = k\eta_{ij}\omega N_i \frac{N_i N_j}{E_i E_j}\left(\frac{E_i}{N_i} - \frac{E_j}{N_j}\right)^2 \tag{8}$$

It can readily be checked that a global entropy balance holds,

$$\sum_{i=1}^{n} \frac{dS_i^{\text{inj}}}{dt} + \frac{dS_i^{\text{diss}}}{dt} + \sum_{i>j} \frac{dS_{ij}^{\text{irr}}}{dt} = 0 \qquad (9)$$

meaning that in steady-state condition, no entropy is supplied to the system.

4 Numerical simulation

The structure is shown in Fig. 1. It is composed of six plates coupled by their edges. The plates are made of the same material (aluminium) and have different thicknesses in order to impose a light coupling between them. The calculation are performed in the octave band 1000 Hz (6283 rad/s). The resulting modal densities and modal overlaps are shown in Tab. 1. It can be seen that all sub-systems contains several hundreds of modes and have large modal overlaps. These values ensure that a statistical energy analysis calculation is valid [14].

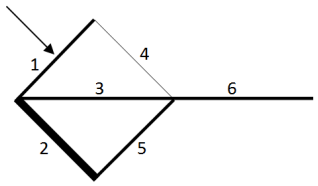

Figure 1: Test structure made of six plates coupled by theirs edges.

Table 1: Modal densities and modal overlaps of sub-systems.

Sub-system	Surface (m^2)	Modal density (s/rad)	Modal overlap
1	7.21	0.35	2.19
2	7.21	0.70	4.38
3	8.00	0.39	2.43
4	7.21	0.17	1.10
5	7.21	0.35	2.19
6	8.00	0.39	2.43

In Fig. is shown a graph of links between sub-systems. This system has been chosen in order to enforce a large number of energy propagation paths from source (sub-system 1) to receiver (sub-system 6).

In Fig. 3 is shown a result in terms of energy balance. Arrow thickness is proportional to powers while circle size is proportional to energies.

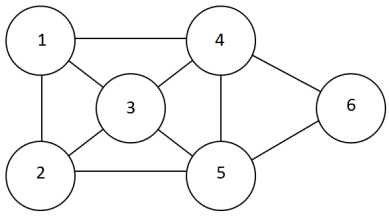

Figure 2: Graph of links between sub-systems.

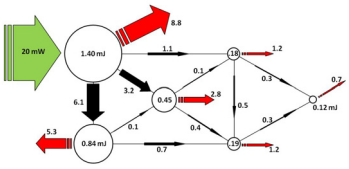

Figure 3: Exchanges of energy in the structure.

In Fig. 4 is shown the same result but now in terms of entropy. Arrow thickness is proportional to mixing entropy while circle size is proportional to entropies.

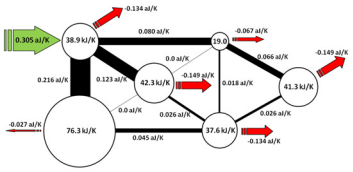

Figure 4: Exchanges of entropy in the structure.

Comparison of these figures leads to the following observations:

- Sub-system entropy levels are quite different from energy levels,
- Dissipated power is proportional to energy level (see Eq. (1)),
- Dissipated entropy rate is NOT proportional to entropy level,
- Dominant paths are more apparent with mixing entropy than with exchanged energy

5 Conclusion

In statistical energy analysis, it is possible to realize an entropy balance in addition to the more conventional energy balance. This new tool should improve the analysis of vibracoustical systems in high frequency domain and could be related to the question of energy path [17]. Two quantities are of interest, the entropy levels and the mixing of entropy. Both contain different information and provide a different regard on the same structure.

Acknowledgments

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